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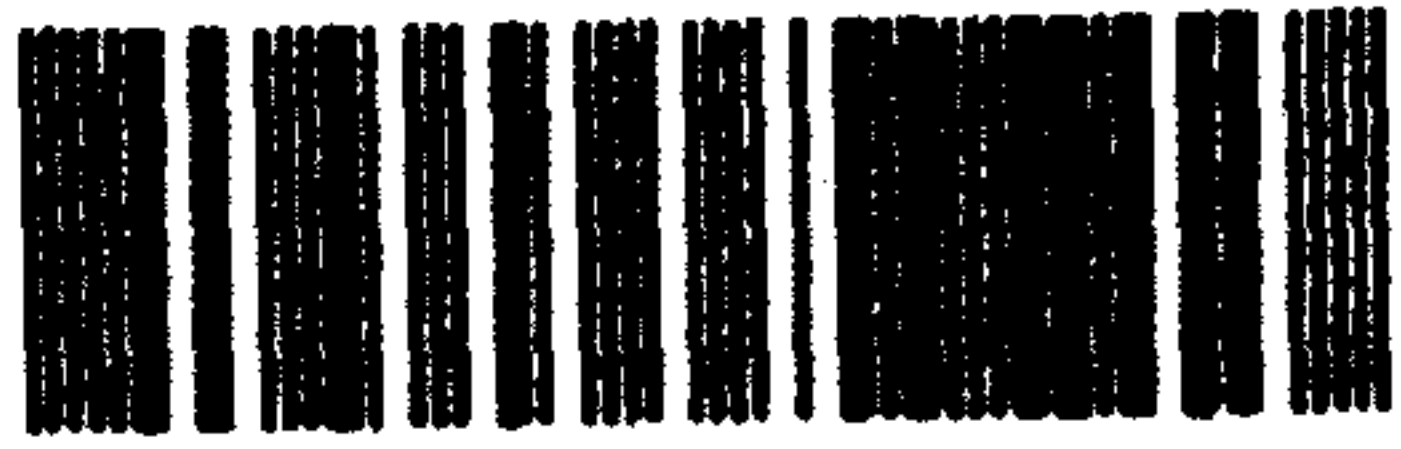
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**CHANGING MY OWN AND MY STUDENTS' ATTITUDES
TO CALCULUS THROUGH WORKING ON
MATHEMATICAL THINKING**

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BSc (Mathematics), MPhil (Mathematics Education)

A thesis submitted in fulfilment of the requirements for the award of the
degree of Doctor of Philosophy in Mathematics Education at the Open
University, United Kingdom

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ABSTRACT

Over the years, I have seen my students struggle to move from elementary to advanced mathematical thinking. I believe there is a need for an effective mathematical pedagogy in the learning of advanced mathematics that explicitly promotes mathematical thinking. Researches in mathematics education have contributed much towards providing theoretical perspectives for understanding thinking, learning, and teaching. It has also provided description on aspects of cognition as well as evidence on the viability and consequences of various kinds of instruction. In this study, I will describe my experience in translating some of the theories into classroom practice appropriate to both my students' and my own circumstances. The purpose of my research was to bring about improvements to the teaching of Advanced Calculus (Engineering Mathematics) to engineering undergraduates with the aim that it would also bring about changes on how students think about the mathematics. A framework that guides the design of classroom instruction and activities to support and encourage students' use of their own thinking powers will also be highlighted. A model of teaching was finally adapted based on various approaches designed to invoke students' ability to use their own thinking powers, enhance their problem solving skills and promote soft (generic) skills that can contribute towards students' acquirement of necessary attributes as an engineer.

The researcher will also be the teacher and thus the study was implemented using an action research and case study perspectives as various methods within these stances will ensure flexibility in responding to the dynamics of interaction between the teacher and her students. Every encounter with my students, within and beyond, the classroom, was considered and contributed to my reflection about my teaching, review of the strategies and consequently, changing the way I teach or interact with the students. The thesis will present a description of the challenges and obstacles to making changes from my own and my students' point of views. Analysis of the research findings will also add knowledge about the factors that influence students' learning. A model that depicts the process of change will be proposed.

Keywords: undergraduate mathematics, mathematical thinking, improving teaching practice

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Dr. Sabariah Binti Baharun, who was also a co-writer for the workbook project.

To my dearest husband, Awaluddin and my boys, Jazakallahumma khairan kathira.

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List of Abbreviations

UTM – Universiti Teknologi Malaysia (Malaysia University of Technology)

SPM – Sijil Pelajaran Malaysia (Malaysia Certificate of Education)

STPM – Sijil Tinggi Pelajaran Malaysia (Malaysia High School Certificate of Education)

GCE – General Certificate of Education

KBSM – Kurikulum Baru Sekolah Menengah (Secondary School New Curriculum)

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CHAPTER 1

INTRODUCTION

1.1 Introduction

Mathematics is an important subject in engineering education. In a report entitled, “The Mathematical Education of Engineers” (OECD, 1966), two core syllabuses were recommended, a short one for all engineers and a longer one to cater for engineers who would go into research and development. The main components of the shorter syllabus were: Algebra and Analysis (Calculus), Mathematics for Computation, Probability and Statistics. It was further suggested that the implementation of the curriculum should be based on the following objectives: (i) it should satisfy the mathematical needs of engineers, (ii) promote better collaboration between the engineering departments and the mathematics departments, (iii) increase students' motivation by teaching mathematics with applications-based examples in engineering and (iv) increase the appreciation of the relationship between numerical and analytical methods. Consequently, the report has been used as an important corner stone to review progress in the role of mathematics in engineering education.

“In identifying the role of mathematics within engineering education the OECD report clearly saw mathematics as being more than simply a calculation tool. Rather, it was seen as providing the means of investigating the nature of things and providing the engineer with a systematic and logical way of formulating and solving problems in engineering”

(Bajpai & James, 1985)

However, this study will be mainly concerned with Calculus as the main analytical component of the syllabus. It is an important mathematical tool in the study of motion and

change and it is thus, an essential part of the mathematics syllabus for these students. Currently there have been calls to review the contents of the Calculus syllabus and reform the way it is taught due to several reasons. Some cited the poor achievements of students and their poor problem solving skills (Tall, 1993; Selden, Mason & Selden, 1994; Roberts et al, 1996). Much of this failure, rightly or wrongly, was attributed to the way mathematics is learnt at secondary or high school, which allegedly, often stressed rote memorisation rather than understanding. Others were motivated by the availability of new mathematical software which provided varied approaches to Calculus. Researchers have explored and demonstrated the role computers can play in enhancing the teaching and learning of Calculus (Simons, 1988; Tall, 1986; Dubinsky & McDonald, 1998). Thus, pedagogy and mathematical content has become issues to be considered. What are the students supposed to learn since computer software can do most of the manipulation required?

The mathematical learning difficulties of engineering undergraduates have been well documented. Various problem areas have been identified. Some of these were, the difficulty of learning some specific mathematical topics, the difficulty in coordinating procedures and manipulating concepts, poor problem solving skills and the inability to select and use appropriate mathematical representations (Smith, 1979; Morgan, 1988; Tall & Razali, 1993; Artigue & Ervynck, 1993).

Consequently, there have also been several attempts to improve students' learning, and the teaching of mathematics, generally and specifically in Calculus through the implementation of specially designed courses and materials. Among the courses implemented were remedial classes, bridging courses, '*levelling-up*' courses (Gonzalez-Leon, 1979, Smith, 1979, Kurz, 1985, Howson et al, 1988) that were designed to help weaker students cope with the mathematics at university level. Other efforts concentrated on the organisation of mathematical content through curriculum reforms, student-centred

learning and teaching with technology (Morgan, 1988; Tall, 1991; O'Shea & Senechal, 1993; Roberts et al, 1996; Keynes & Olson 2000). Courses that taught problem solving explicitly and those that encouraged mathematical thinking were also recommended to help improve students' skills and build their confidence in negotiating their difficulties (Polya, 1981; Schoenfeld, 1985; Mason et al, 1982; Tall & Razali, 1993; Mohd. Yusof, 1995).

Furthermore, conclusions from research (Tall & Razali, 1993; Anthony, 2000) indicated that positive attitudes and good study skills could improve students' learning. There were suggestions that changes to the learning environment so as to promote good problem solving behaviour, to increase students' participation and that which provide more challenging ideas could stimulate active learning. The literature reviewed on students' difficulties and the various efforts to improve the situations also showed a general trend, moving away from remedial classes, popular in the late 1980's, towards teaching to increase understanding. Improving students' learning through the enhancement of their problem solving and mathematical thinking skills as well as through using technological tools to support conceptual understanding and problem solving methods are now thought to be more appropriate to enable them to cope with the mathematics needed for their engineering problems. Students should not only be able to understand the mathematical concepts and use its procedures in the mathematics classroom but should also be able to use this knowledge in solving real engineering problems.

1.2 Background of the Problem

My interest in the problems faced by undergraduate engineering students in the learning of mathematics came from my experience as an Assistant Lecturer in Universiti Teknologi Malaysia teaching engineering students in Diploma and Degree courses since 1980. In 1990, I did a Masters in Philosophy dissertation entitled, "A Study of a Mathematics provision for First year Engineering Undergraduates with Non-GCE A Level Entrance

Qualifications". It described a one year study of a special Mathematics provision to support and help students with non-GCE A Level entry qualifications. The aim was to upgrade their mathematical knowledge so that they would be able to cope with the teaching style at the university and come up to the standard required to continue with advanced mathematics in the second year. The study was carried out at a university in the United Kingdom. I had considered that the similarities between these students and my own, in Malaysia, were close enough to allow me to transfer any findings. I started out studying students' learning but ended up doing a thesis on curriculum development (Abdul Rahman, 1993). When I returned to Malaysia, I continued my work on studying students' learning difficulties and looking for ways to identify and help remedy them, in particular, focusing on my teaching methods (Mohd. Yusof & Abdul Rahman, 1997 (a) and (b)).

The prior mathematical knowledge of a majority of students entering UTM consisted of Modern Mathematics at the Sijil Pelajaran Malaysia (SPM) level. Some would have Additional Mathematics as an added qualification. In the past decade, various studies have been carried out on students' learning of mathematics in UTM. Findings from these research indicated that there are similar learning difficulties amongst the students (Liew Su Tim & Wan Muhamad Saridan, 1991; Tall & Razali, 1993; Mohd. Yusof & Tall, 1999) as have been reported elsewhere. There have also been several programmes carried out to overcome the said difficulties (Mohd. Yusof & Abdul Rahman, 1998). This section focuses discussion would be on mathematics learning and teaching with specific attention on the particular difficulties of students in UTM. However, it should be noted that Calculus is a core mathematics subject for the majority of students.

In a study of mathematics learning in UTM, Mohd. Yusof (1995) concluded that although lecturers would like students to develop positive attitudes towards mathematics and build good problem solving skills, they were not confident in their students' abilities to do so. Thus, they taught their students accordingly. Their teaching emphasised most on

developing effective algebraic manipulation skills and practiced on typical problems. The students themselves preferred this approach and set out to learn procedurally to be successful in routine tasks. There exist an overwhelming perception of mathematics as a subject most appropriately learnt by memorisation and rote. These skills had been important for their success in school and the students expected more of the same (Roselainy, 2001). However, the students' subsequent performance in Mathematics and Calculus, in particular, is not encouraging.

There are genuine concerns that there is a need to improve students' understanding of the mathematics that they have learnt. Previous studies have identified various difficulties associated with students' learning as well as the teaching situation. Most of the past efforts concentrated on remedial programmes to strengthen deficiencies in their basic mathematical concepts, particularly, those that they should have grasped at school level. At least for the time being, understanding of mathematics is measured as the ability of students to perform well in the examinations. However, students still had difficulties in using mathematics that they have learnt, in recognising the mathematics when found in other subjects, in translating real world problems into mathematical formulation, as well as solving these problems. These issues should be addressed to further develop meaningful learning and supplement past efforts.

In view of the students' difficulties described above, it is important that measures to improve students' facility with mathematical ideas and enhance their mathematical thinking must be planned. Suggestions to improve the teaching and learning situations should take into account the most immediate concerns of both students and lecturers. Amongst the more significant were, (i) students' overwhelming aspirations to perform well or at least succeed in examinations, and (ii) the limitations for successful teaching practice in terms of time constraints, large student numbers as well as career development demands for the lecturers (Roselainy & Yudariah, 2000). Findings from previous research

(Tall & Razali, 1993; Mohd. Yusof, 1995) have suggested that students who take up courses for problem solving had developed more positive attitudes towards mathematics. However, the changes would diminish without further encouragement or support in their normal learning environments.

The suggestion is taken up here, but with the logical consequent to incorporate into the routine teaching, strategies to invoke students' mathematical thinking powers. It would be more efficient for students to understand the mathematics taught rather than be trained to solve problems without real awareness. In addition, to be able to respond flexibly and creatively to new situations, the students themselves must be able to link and make connections between different mathematical ideas and reconstruct those ideas for themselves (Mason, 1999; Watson & Mason, 1998). The ability to apply only standard techniques to standard problems could not equip them to face real life and non-standard problem situations.

The lecturers' responsibilities would be to provide the necessary environment to ensure learning takes place. They must learn to appreciate how their students are struggling with the mathematical ideas especially with new and more complex mathematics. They should realise that the students need help in order to undertake for themselves the mental actions which experts find intuitive and natural. They would need support when working and struggling with the mathematical ideas. It would be difficult at first for most lecturers to change their teaching methods but not impossible. The techniques to encourage students' mathematical thoughts are in the main, based on the ability of the lecturers to think aloud and share their thinking with their students.

1.3 Statement of the Problem

The current efforts to remediate and improve students' learning of Calculus appeared to be insufficient to increase students' understanding of mathematics, using its procedures appropriately as well as fostering good problem solving skills especially in coping with

mathematics in students' own specialist areas. Thus, this study proposes to implement teaching integrated with strategies to invoke students' mathematical thinking powers so as to effect changes in engineering students' attitudes as well as increase their facilities with mathematical ideas.

1.4 Purpose of the Study

In the first instance, the research was to investigate changes in teaching practice and the effects of implementing teaching integrated with mathematical thinking strategies on students' learning behaviour. However, the study was implemented using an action research and case study methods and thus, the direction and focus of the research altered as a result of findings during the progress of the study. The research sought to answer the following research questions, which, in turn will guide the methods to be adopted for the research.

- What are the factors that affect students' learning behaviour?
- What are the students' attitudes and perceptions towards Calculus?
- What changes are required in the teaching practice?
- What are suitable strategies to invoke students' use of their own mathematical thinking powers?
- What are the changes in students' attitudes and learning behaviour?

1.5 Importance of the Study

My research studied the challenges and obstacles in changing my teaching practice to incorporate strategies to invoke students' use of their own mathematical thinking powers and how these strategies affected students' learning of Calculus. In adopting an action research and case study methodology, I tried to present a realistic portrayal of the teaching and learning situations. Several issues emerged in the process of managing the changes that encompassed certain management, administration and implementation aspects. These

are (i) difficulties in implementation in two main areas, (a) material and tasks design and (b) infra-structure support; (ii) changing in isolation; (iii) constraints in changing assessment; and (iv) negotiating with students to accept and embrace the changes.

The study also examined the students' responses to the teaching strategies. The assumption that underlies the teaching was that enabling students to become more aware of, recognise and invoke their own mathematical thinking powers would enhance their understanding and appreciation of the mathematics that they were learning. This report also describes students' responses, reaction to the strategies as well as the development of their learning.

1.6 Scope of the Study

The study investigated the teaching and learning of Calculus by engineering undergraduates. An early decision was to conduct a study on a selected group from a particular engineering stream studying Calculus in the first year and followed through to their second year of Calculus. However, due to events that evolved during the research, I refocused and concentrated on studying the learning of Engineering Mathematics (or Advanced Calculus) of engineering undergraduates from the Faculties of Civil and Electrical Engineering. The study will be presented in two parts, i.e., Part I covers the period of November 2001 – 2004, with the students from The Faculty of Mechanical engineering doing a course in Industrial Design. Part II covers the teaching of Engineering Mathematics (or Advanced Calculus) to students from the Faculties of Electrical and Civil Engineering in the first semester of the academic session 2005/2006. It also covers the observation of the teaching of Engineering Mathematics to students from the Faculty of Electrical Engineering in the first semester of 2007/2008.

1.7 Organisation of the Thesis

The thesis is presented in seven chapters to describe the development, implementation, application and analysis of action research in the teaching and learning of Advanced

Calculus of engineering undergraduates. Although the research process is not linear, the presentation is given in a linear fashion with certain features to capture the essence of the situations; in particular, through the presentation of my reflections and stories of selected students.

Chapter 2 reviews the literature on research of students' learning and the teaching of mathematics at tertiary level. It also summarises research on students' learning at Universiti Teknologi Malaysia (UTM) in order to draw attention to specific issues in teaching and learning at the institution. The research on students' learning is grouped under the categories of: (1) students' difficulties in the learning of specific concepts, (2) students' difficulties in algebraic manipulation and problem solving, and (3) students' beliefs and learning styles. Research on teaching will discuss issues related to (1) curriculum development, (2) learning and instruction theories, (3) the teaching environment, and (4) assessment issues. It will draw attention to reforms in the teaching of Calculus and the discussion on the organization of mathematical contents as the focus of changing the curriculum. In reviewing learning and instruction, theories on how students learn, mathematical thinking and how to design instructions will be presented. A brief appraisal of teaching environments and assessment concerns will also be included. The chapter will also include a presentation of research on teaching and learning in UTM focusing on students' background and the teaching situation. Finally, section 2.5 will present the basic guidelines that were used in implementing the teaching strategies to invoke students' use of their own mathematical thinking powers.

Chapter 3 explains the research perspective, methods and procedures as well as the analytical techniques that were adopted. In the discussion on the research implementation, the reasons for using action research and case study methods are presented as well as the measures taken to support the main features of the research, mainly, the study of teaching and learning in a natural setting, the concern and consideration of the participants' views

of teaching and learning as well as with students' development in the learning of Mathematics. As the development of the teaching strategies and materials were also an essential feature of the research, a description of the approaches used is included.

Chapter 4 presents a description of the first part of the implementation. Firstly, it presents the important ideas that I considered in changing my teaching practice, the development of the teaching strategies as well as developments in UTM that influenced the progress of the research. The teaching and learning situation covers several semesters and the teaching of Basic Calculus and Calculus II for various groups of students doing the course on Industrial Design of the Faculty of Mechanical Engineering. The presentation includes a description of certain classroom episodes as well as my reflections on the teaching, on the students as well as on the research process. As it is impossible to describe fully all the interactions, I had to choose the episodes that are included and so I try to make explicit the reasons for these choices. Due to changes in the policies of student recruitment, the groups that I studied changed from being 'typical' to 'atypical' students and thus I had to continue the research on a different group of students which is described in chapter 5.

Chapter 5 presents the second part of the study, the teaching of Engineering Mathematics to two groups of students in the first semester of the academic session 2005/2006 as well as my observation of the teaching of colleagues, Dr. Tee and Dr. Zee, to two more groups of students in the first semester of academic session 2007/2008. These students were recruited under new policies of higher education stipulated by the Ministry of Higher Education. Thus, there were changes to the curriculum and the courses as well as to the medium of instruction which now, must be in English. There were also changes to the teaching strategies and management of the students learning environment which were the results of the experience gained from the first part of the study. A product of the research was a workbook in Engineering Mathematics to support independent learning and a discussion on the use of the book is included. The workbook was written by my colleagues

and I, first published under the Sprint Print program of Pearson (Malaysia) but is now undergoing the editing process to be published sometime in December 2008 or January 2009.

Chapter 6 starts with a review of the theoretical foundations of the research and present the various issues that emerged and how the data from this research supports and build on the theories of learning and the factors that affect students' learning. The findings include a suggestion of a model that describes the process of changing students' attitudes towards Calculus through working on mathematical thinking.

Chapter 7 which is the final chapter will present the concluding remarks and recommendations of the research.

Note the difference in citing Malay Malaysian names which will occur in this report. For example, if the author's name is *Roselainy Binti Abdul Rahman*, with *Roselainy* being the first name, *Binti* stands for 'daughter of' and *Abdul Rahman* is the last name; it is cited in local publications as *Roselainy* (year) rather than by the last name. If the paper was published in international proceedings, the same name is cited as *Abdul Rahman, R* . Thus, in this report, there might different ways of citing the same name as it depends on whether the paper or book was published locally or internationally.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

My research interest has developed from my search for learning and instruction theories to support changes that I wish to make in my teaching of mathematics to engineering undergraduates and found that among the challenges I faced came from having to teach students for whom mathematics was not a priority subject. In particular, I will be looking at the teaching of Basic Calculus, Calculus II and Engineering Mathematics (or Advanced Calculus). During the course of the research, there were a few policy changes that changed the entry qualifications of the students, thus requiring modifications to the curriculum and to the names of the mathematics subject taught; these will be described in greater detail in Chapter 4. In reviewing previous research, I have categorised the various issues concerned with the teaching and learning of Calculus under the general headings of (1) research on students' learning, (2) research on teaching, (3) research into mathematics education in UTM.

2.2 Research on Students' Learning

In this section, the research on students' learning is grouped under the categories of: (1) students' difficulties in the learning of specific concepts, (2) students' difficulties in algebraic manipulation and problem solving, and (3) students' beliefs and learning styles.

2.2.1 Students' difficulty in the learning of specific concepts

The concept of function is one of the central ideas of modern mathematics. However, it also appeared to be one of the most difficult concepts for students to master. Although most students can graph simple functions, they usually have difficulties in interpreting information from the graphs of functions. Some students do not understand the graphs that

they themselves drew. They did not appear to grasp the relationship between the independent and dependent variables as described by the graphs. In fact even the role of the variable is not properly understood (Bennet, 1977; Eisenberg, 1991; Md. Nor Bakar, 1991; Tall, 1996). It would seem natural to study functions visually as well as analytically. However, in recent years, the analytic characterization of mathematics appeared to dominate the view of what constituted mathematics. Research looking at students' visualization abilities found that they could develop a better understanding of functions if the subject was developed visually. The students were also better at retaining the knowledge as compared to learning functions in an analytical framework only (Maselan Ali, 1996; Tall, 1996; Eisenberg, 1991). Amongst other difficulties associated with functions is the process of abstraction, the ability to understand and read the notation as well as manipulate the symbolic representations.

Cognitive difficulties were also found in other limiting processes, such as the concept of continuity, differentiation and integration. Students' spontaneous conception that was evoked by the common meaning associated with the word 'continuous' or 'continuity', have influenced the development of their understanding on the concept of continuity (Cornu, 1991). In the areas of differentiation and integration, the manner in which the topics were introduced at secondary schools and its subsequent treatment at universities has become a source of difficulty amongst the students. Furthermore, the interpretations of the common notations used for differentiation and differentials can in themselves create confusion amongst the students (Katz, 1986). Artigue (1991) listed several important findings from various research studies of the understanding of elementary Calculus. She noted that the students showed reasonable mastery of algorithmic algebra in terms of calculating derivatives and primitives but displayed some difficulties in conceptualizing the limit processes underlying the concepts of derivative and integrals. They also had difficulties in using relevant graphical representations and in interpreting the meaning of symbols used.

Essentially the educational process has a substantial influence in the development of students' concepts. The tendency to present Calculus using algebraic algorithms and theorems as the 'all-powerful' tools to solve problems sustained students' beliefs that theoretical considerations are not important and does not impair their problem solving. Discussion of the mathematical content and teaching practice will be given later in the chapter. The following subsection will first present some of the difficulties displayed by students in the learning and doing of Calculus.

2.2.2 Students' difficulty with algebraic manipulation and problem solving

Tall & Razali (1993) have identified possible qualitative differences between the able and less able students in mathematics. The more able students could flexibly manipulated concepts as objects or processes but the less able had a tendency to concentrate on coordinating processes. Thus, they preferred routine processes but were less likely to be able to relate ideas in a meaningful way. They also faced difficulties in organizing known facts and could not master the mathematical language and symbols. Furthermore, their main strategy in answering questions was to accumulate procedures and they showed a tendency to over generalize the use of certain mathematical procedures. Some students were unable to select and use appropriate mathematical representations, such as, visual, numerical, symbolic, algebraic and analytical. It was reported that the more successful students were able to use a variety of such approaches (Tall, 1993).

Another aspect of the students' learning difficulties to be considered was the findings that many students, even the good ones, were unable to solve non-routine problems (Selden, Mason & Selden, 1994). The authors concluded that the results showed that the knowledge the students had gained in their mathematics learning was appropriate for solving routine problems but not enough to assist them in problems requiring conceptual insights. A different facet of this problem, were the difficulties students faced in translating real world

problems into Calculus formulation. This is probably, another indication of students' poor problem solving skills but perhaps also related to the manner in which Calculus is taught.

2.2.3 Students' beliefs and learning styles

Various researches had explored the impact and influence of students' beliefs about mathematics and their mathematical performance. In a study by Tall and Razali (1993), it was found that students perceived mathematics as a subject wholly consisting of a conglomeration of facts and procedures. Generally, students expected to master mathematics by memorization and drill exercises as they considered the subject to be an objective and objectively graded discipline. It was also reported (Schoenfeld, 1989) that students believed that most mathematics questions should be immediately solvable and should not take up more 10-15 minutes. Good teaching practice should consist of the ability of teachers to ensure that their students could learn all the necessary rules and procedures. These beliefs have particularly influenced the students' problem solving behaviour. Thus, they would easily give up when faced with difficulties and showed great reluctance in persevering with new ideas and techniques. Students with poor track record in mathematics achievements in the past were over anxious when exposed to new problems and concepts (Tall & Razali, 1993). They showed little cooperation when teaching approaches that required their participation were instituted. Teaching and learning methods that promote understanding were effectively undermined by the students' beliefs that mathematics is best learned by rote and exercises. On the other hand, students who felt that mathematics was useful and would bring much benefit were more likely to persevere, put in more effort and were more motivated in their studies (Schoenfeld, 1989; Anthony, 2000). Such positive attitudes would also help to improve their ability to learn (Tall & Razali, 1993). Successful students believed that hard work was the key to success whereas students with poorer achievements believed that it was their inability that contributed to their failure.

Schoenfeld (1989) had also identified a strong correlation between students' positive self-concept and success related behaviours, such as persistence and efforts, to their achievements. There was a strong indication that motivation was also correlated to success. The better student was less likely to believe that mathematics was mostly memorizing and problems were always solved in step-by-step procedures. They were also more likely to find mathematics interesting and perceived themselves as working harder in mathematics than most other students. The weaker students believed that success was attributed to luck or inherent ability and that their failure was mainly due to their fault.

In addition, conclusions from research (Tall & Razali, 1993; Anthony, 2000) indicated that positive attitudes and good study skills could improve students' learning. There were suggestions that changes to the learning environment so as to promote good problem solving behaviour, to increase students' participation and that which provide more challenging ideas could stimulate active learning.

2.3 Research on Teaching

Issues on teaching will be discussed under (1) curriculum development, (2) learning and instruction theories, (3) teaching environment, and (4) assessment issues.

2.3.1 Curriculum Development

There are several aspects of curriculum development that deserves consideration. Here, reforms in Calculus teaching and its contents will be looked at specifically. In addition, the implementation of the curriculum and the teaching activities carried out will also be discussed.

2.3.1.1 Calculus Reforms

In recent years there have been strong movements for Calculus reforms in USA with several projects undertaken and reports made available (Roberts et al, 1996). Various motivations were given for the reforms such as to enhance the development of conceptual

understanding and to encourage students' active learning (Ross, 1996). Although the distinguishing features of the reformed Calculus course were not uniform, several themes had emerged. Concern about the importance of students' cognitive development saw the emphasis on

- (i) The multiple representations of concepts. Many felt that through this approach, students would gain a better understanding of the concepts as well as the ability to move between different representations as necessary to good problem solving.
- (ii) Content changes. Some felt that it was essential to re-examine the course content based on what was actually used in other mathematics courses and in client disciplines. Some changes became necessary due to the use of technology in the courses.
- (iii) Importance of concept development. It was felt that the over dependence on rote learning and procedural skills could not support students' facility with solving problems.
- (iv) Pedagogical considerations. The focus of the reforms was on students' learning. The choice of methods should be based on how it would encourage students' active participation and enhance students' understanding.
- (v) The use of technology. Some felt that the use of technology was essential as tools for the exploration of ideas, problem solving and a major support for multiple representations of concepts.

To ensure that students became more responsible for their own learning as well as to increase their active participation, the learning environment made more use of the following methods,

- (vi) Group work. Students were encouraged to work in small groups in or out of class. Sometimes it was a natural consequence due to the kinds of assignments given.

(vii) Projects. Projects were seen as a means for students to become responsible for the mathematics that they had to learn. It was necessary to expand students' experience in cooperative work, to learn how to communicate and present and write about the mathematics that they have learnt. Simulation of real-life problems could also be given as project work.

Related to the various issues discussed above was the need for more information to understand how people learn mathematics. Research on understanding and concept development should be carried out to support continued progress. Some reviews of projects carried out will be presented to highlight the design and variations in some of the new courses. One of these projects, *Calculus in Context* (O'Shea & Senechal, 1993), was implemented in 1987. The classes were structured so that general principles emerged from problems and real situations. Students were engaged in the construction and examinations of the mathematical models. A strategy that was used repeatedly was solving the problems with numerical methods before developing the analytical solutions. The presentation of topics also followed the course structure, for instance, systems of differential equations were presented early in the course. The authors were encouraged by the achievements of students who were considered as having weaker mathematics background. They performed just as well and sometimes, better than students with a more traditional background.

In another project, Brown (1996) described the planning and changes undertaken when reforms in the Calculus course were implemented. What was clear was that changes were not only made to the subject matter but also in the presentation, the organization of students' learning and assessment methods. One of the objectives of the new methods was to ensure students take responsibility for their learning. Brown observed that instructors had to put in a lot of intellectual effort and energy in helping the students learn how to

learn. It was also necessary for the instructors to study the students' ideas and understanding of the concepts taught in order to understand their difficulties.

Keynes & Olson (2000), reported on the progress and challenges of a new course, which they called the Calculus Initiative for period covering 1995-1999. The course was presented in a five-quarter sequence with changes in content and pedagogy. Teaching was conducted in teamwork and learning was student-centred which involved students working cooperatively in small groups with greater students and faculty interactions. Appropriate technologies were used in exploring mathematical ideas. The main objectives of the Initiative were to enable students to learn Calculus better and develop the necessary critical thinking skills in applying it to a variety of problems in Science and Engineering fields. In general, they found that most faculty members and graduate students increased their effectiveness as teachers as a result of teaching the sequence. There were also indications that students' mathematical knowledge and interest showed significant improvement as a result of their participation in the course.

2.3.1.2 Organization of mathematical content

Other researchers have outlined specific prescriptions on how to organize the mathematical content for the development of students' understanding and problem solving abilities. One of these programmes, due to Douady (1986, cited in Robert & Schwarzenberger, 1991) requires that the materials are presented in a cycle of (1) *explanation* of the role of the concept, (2) *institutionalisation*, that is, the teaching of the concept in the course, (3) *familiarisation* by means of further problems for reinforcement and (4) *transfer*, that is the ability to use the concept in a new context. The effectiveness of this programme depended on information about students' conceptual learning difficulties. In a similar approach, Artigue (1991) analysed the relationship between the qualitative and algebraic approaches in teaching differential equations. She implemented a four phase teaching programme which consisted of (i) introduction to the qualitative approach, (ii)

exploitation of the new concepts, (iii) comparisons between algebraic and qualitative methods and, (iv) concepts and fundamental theorems of the qualitative theory of differential equations. In another programme reported by Tall (1990), students' learning were presented in sequences which start from concepts meaningful to students at their current level of knowledge but which can be developed into formal mathematical concepts to be taught.

There were other efforts in determining suitable topics for a course in Calculus. Hallet (1996) wrote about the need to study the reasons for the inclusion or exclusion of specific topics and also to examine their usefulness for later courses or in other fields. Keynes & Olson (2000) redesigned the sequences in which the topics were taught. The main objectives were to help students to learn Calculus better and develop critical thinking skills with the underlying features of active learning, creative lectures, increased student/faculty contact and an increased exposure to the conceptual and visual aspects of Calculus.

Using the computer as an environment for the exploration of mathematical concepts was an interesting development in the use of technology in the teaching of mathematics. Heid (1984) conducted an applied Calculus course in which students used graphical and symbol manipulation software to perform routine calculations whilst she focused on the fundamental concepts. The students were allowed to construct their own ways to handle the concepts and her findings indicated that the students were more able to reconstruct facts from basic principles. Tall (1986) designed a software called *Graphic Calculus* to teach Calculus and differential equations using a cognitive approach to overcome known students' conceptual difficulties. Students were guided to use the software to explore graphical representations of various functions as an introduction to fundamental insights on the concepts of limits and differentiability. A different example of the use of the computer was the use of programming to encourage students to think mathematically about mathematical concepts. Dubinsky and his colleagues (1986) developed a special

language called ISETL (Interactive SET Language) for mathematics learning which supported most of the standard mathematical constructs with a syntax similar to mathematical notation. In writing procedures to express mathematical actions, it was expected that students would develop the intuitive insights and construct the mental processes to help them to understand the mathematical concepts being taught.

2.3.2 Learning and Instructional Theories

Researches in mathematics education at the tertiary level have provided various theoretical perspectives for understanding thinking, learning, and teaching. It has also offered explanations on aspects of cognition as well as reports on the viability and consequences of various kinds of instruction. In this sub-section, the review looks at research on how students learn, mathematical thinking for advanced mathematics and the instructional design that could promote and enhance students' understanding and facility with mathematical concepts and techniques.

2.3.2.1 How Students Learn?

What is meant when we want the learners to '*understand*' mathematics? Skemp (1987) described two ways of understanding mathematics, instrumental and relational understanding. He described the former as knowing '*rules without reasons*', which for most learners (and teachers too) was their learning goal, i.e., to know a rule and be able to use it. The latter meant that learners know both what to do and why to do. Learning relational mathematics consisted of building up a conceptual structure or schema from which a learner can produce, in principle, an unlimited number of plans for getting from any starting point within his schema to a finishing point. Thus, in this kind of learning, then, ends to be reached became independent of the means to reach them, the building of a schema is a satisfying goal in itself, and the more complete a learner's schema, the more confident he became of his ability to face new problems. However, a schema was never complete, it can only enlarge and thereby enlarging the learner's awareness of

possibilities. However, he cautioned against the possibility of two kinds mathematical of mismatches that could occur in mathematical learning; learners who only wanted to understand instrumentally taught by a teacher who wanted them to understand relationally and vice-versa.

Then, how do students learn? Were there differences in the learning of elementary and advanced mathematics? Gray & Tall (1994) were concerned with the manner in which individuals constructed concepts. The basis of their theory was on the duality of symbols as both process and concepts. They used the notion of *procepts* to explain the condition. They defined *procepts* as follows:

“An *elementary procept* is the amalgam of three components: a *process* which produces a mathematical *object*, and a *symbol* which is used to represent either process or object.A *procept* consists of a collection of elementary procepts which have the same object”.

(Gray & Tall, 1994)

Reflecting on the theoretical development on the construction of mathematical knowledge in elementary and advanced mathematics, Gray et al (1999) concluded that there were two different kinds of cognitive development in elementary mathematics: one focused on the properties of objects (as in geometry), the other, on the properties of processes and concepts in arithmetic, algebra and symbolic Calculus. However, in advanced mathematics, there would be a new focus of attention and cognitive activity. The emphasis would change to selecting certain properties as definitions and axioms and building up the other properties of the defined concepts by logical deduction. They suggested that less successful learners focused on the specific and associate it to real and imagined experiences that often do not have ‘generalisable’ and ‘manipulable’ aspects. High achievers focused increasingly on flexible processes and conceptual aspects of the symbolism allowing them to concentrate on mentally manipulable mathematical objects

that gave greater conceptual power. In a later paper, Gray & Tall (2001) explained further why some learners were successful in manipulating symbols, while some deal with them procedurally and yet, others who found mathematics totally overwhelming. They focused on the different ways that procepts arise in cognitive development. They suggested that new form of procepts have their own peculiarities thus making abstraction qualitatively different in each case and that different contexts posed different kinds of cognitive problems in both the nature of the procepts concerned and the *procedure-process-procept* spectrum of student activity. They observed that students struggled with cognitive reconstruction as they met new mathematical idea and objects. They recommended that mentors who were aware of the mathematics and underlying cognitive structures should support the students as they learn.

Dubinsky (1991) used the notions of objects, processes, and schemas to explain his theory of the development of concepts in advanced mathematical learning. The definition of a schema was as a coherent collection of objects and processes. He believed that learners would invoke a schema in order to understand, organize or make sense of a problem situation is his or her knowledge of an individual concept in mathematics. Thus, the learner will have a large number of schemas, which would be interrelated in a large and complex organization. He used the term process or mental process, to refer to the internal mental actions and the term object to refer to either mental or physical object. He suggested that we could infer a learner's ability to construct new mathematical knowledge, from his acts of recognizing and solving problems, of asking new questions and creating new problems. He considered constructing a schema a dynamic activity undertaken by the learner, and its existence was inseparable from its continuous construction and reconstruction. The description of a schema can be summarized as follows. It began with mathematical objects, which the learner would have constructed as some points in his mathematical development. He would use these objects for calculating at some points. First, the action would have been interiorized which meant that some

internal construction was made relating to the action. He called an interiorized action a process. Thus, interiorizing actions was one way of constructing processes. A learner could also use existing processes to make new ones, or compose or coordinate two or more processes to create new ones. In addition, it was possible to reflect on a process and to convert it into an object. Figure 1 (Dubinsky, 1991), is a graphical description of the construction of schemas.

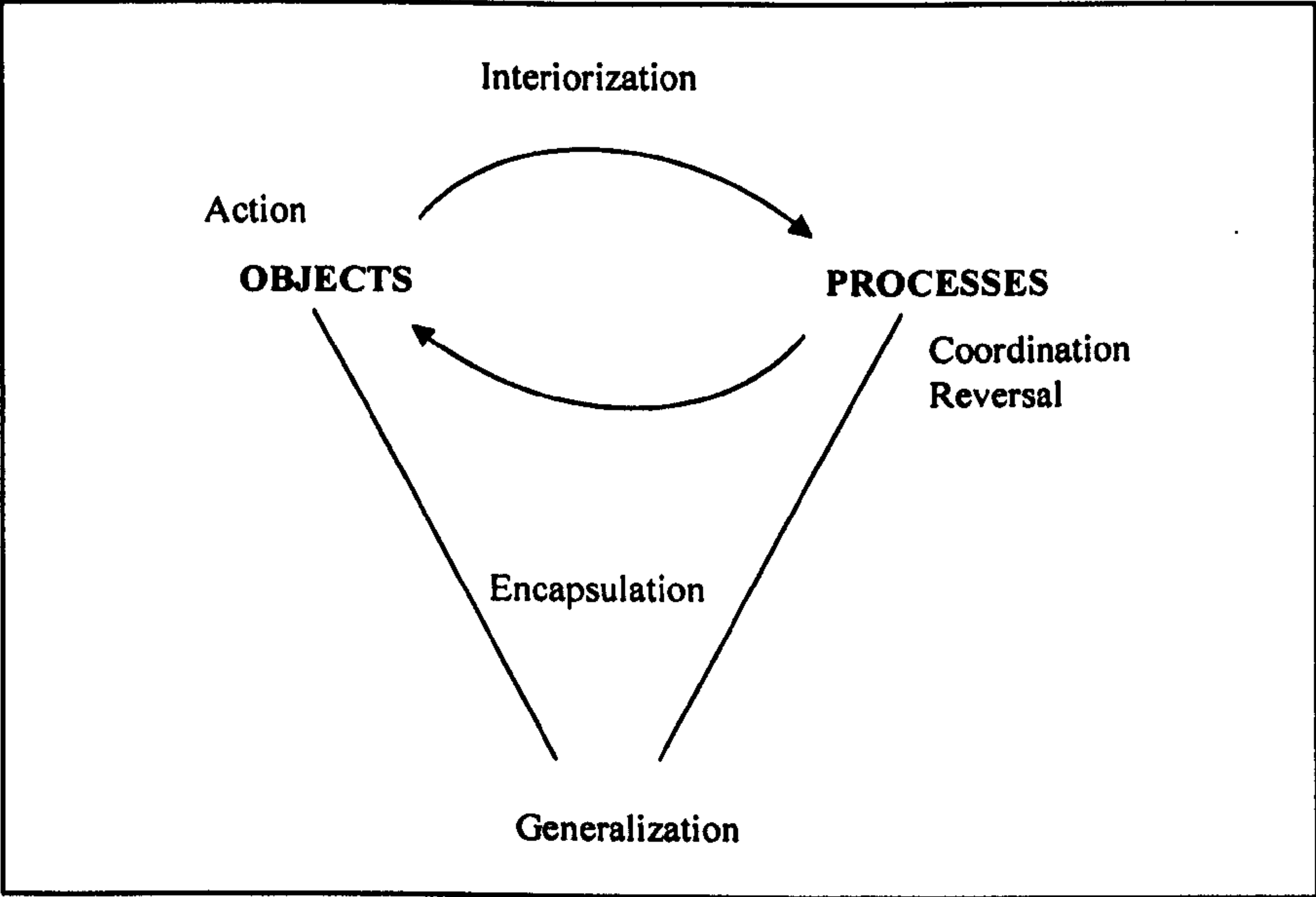


Figure 1: Schemas and their construction

In Dubinsky’s theory, the main concern was in the learners’ construction of schemas for understanding concepts. He suggested that the aim of instructions was to induce students to make these constructions and to help them in the process.

In advanced mathematics, learners should have the ability to construct abstract knowledge structures. Thus, Schwarz, Hershkowitz and Dreyfus (2002) considered the construction of abstract knowledge structures as crucial in mathematical education. In their theory, abstraction was defined as, “ ..an activity of vertically reorganizing previously constructed mathematics into new mathematical structure”. In their definition, abstraction is not an objective, universal process but depends strongly on context, on the history of learners

involved in making the abstraction and the objects available to the learners. Thus, they internalized or personalized the structure. They suggested that the process of abstraction passed through three stages, (1) a need for a new structure, (2) the construction of a new abstract entity, and (3) the consolidation of the abstract entity through repeated recognition of the new structure. The ability of learners to reconstruct the new structure, recognize it in different contexts and use it in the construction of further structures and the ability to articulate it verbally showed that they were in the consolidation stage. These abilities could occur because of problem solving activities and reflective activities. Therefore, it was important in their educational design to sequence the activities to lead to the abstractions and their consolidation.

In promoting the need for a problem solving approach to learning as contrasted to the learning of routine methods, Skemp (1993) suggested the kind of learning situation that could support the formation of mathematical concepts and the building of mathematical schemas. The learning situation should provide embodiments of the concept, reducing any irrelevant information while forming the concept. A number of examples of the concept were given, close together in time. One new concept should be introduced one at a time and lastly, ensure that the concept taught was one in which the learners had an appropriate schema so that they could connect the new concept to it.

2.3.2.2 Mathematical Thinking

In this section, I will be reviewing ideas about mathematical thinking with an emphasis on advanced mathematical thinking. I believe that the aims of mathematics learning for engineering students should include the ability to formulate problems mathematically, to work with several mathematical ideas and various representations and to select and use multiple procedures. Consequently, it is expected that they could make connections between relevant mathematical concepts and to transfer as well as use these knowledge in their engineering fields. It is clear that to do these, they will need a strong understanding

of basic concepts and the ability to adapt, modify and extend their mathematical knowledge appropriate to the problem situation. I found that my students need to have efficient strategies in coping with new mathematical ideas and thinking about mathematics. In the literature, problem solving and mathematical thinking are usually discussed together with various perspectives and definitions to describe them.

Schoenfeld (1985 & 1992) presented the view that problem solving can support better understanding of mathematics and required four categories of knowledge and skills. These were: *resources* which were the mathematical knowledge and skills that students had as they tackled the tasks; *heuristic* strategies and techniques that they used in solving problems; *control* and *monitoring* which were decisions that students made about when and what resources and strategies to use during the problem solving process and finally their *beliefs* about mathematics which determined how they approached a problem.

A Working Group on Advanced Mathematical Thinking was formed at the Conference of PME in 1985 which was concerned with extending the theory of the psychology of mathematics education to later age groups, starting at 16+. In a discussion paper for the group, Tall (1988) pointed out the difficulties in specifying a distinction between Advanced Mathematical Thinking and Elementary Mathematical Thinking. An initial problem was in interpreting advanced mathematical thinking, was it “advanced forms of mathematical thinking” or “thinking related to advanced mathematics”? He himself considered Advanced Mathematical Thinking “*to be any part of the complete process of mathematical problem solving, from the creative processes involving deductive and associative resonances between previously unrelated, or even undefined, concepts, through to the final ‘precising’ process of mathematical proof.*” Although, he believed that it was difficult to specify the features of Advanced Mathematical Thinking, he offered some characteristics to further discussion and these were: *(1) the abstraction of properties to provide concept definitions for mathematical concepts, (2) the use of abstract*

mathematical concept definitions to ease cognitive strain in thinking, (3) the insistence on logical proof rather than coherent justification, which involves (4) the deduction of properties of mathematical concepts (from given concept definitions) and (5) the implication that if certain mathematical properties hold then others follow.

In a report by the Advanced Mathematical Thinking Working Group for the 22nd Annual Meeting of the International Group for Psychology in Mathematics Education, North American Chapter (Heid, et al, 2000) presented three perspectives on the nature of advanced mathematical thinking. In the first perspective, Edwards and her team (Edwards et al, 2000) defined mathematical thinking as thinking that required deductive and rigorous reasoning about mathematical ideas that were not entirely accessible to the five senses. The thinking was not particularly tied to specific educational experience and levels of mathematics. Secondly, Rasmussen and his colleagues (2000) discussed advanced mathematical thinking in terms of practice which they called '*advancing mathematical activity*' to emphasise the progression and growth of students' reasoning in relation to their previous activity. They considered that mathematical learning occurs by participating in a variety of different socially or culturally situated mathematical practice. They emphasise the changing nature of mathematical activity and the progression of thinking in terms of horizontal and vertical mathematising. They described horizontal mathematising as transforming a mathematical or real world problem setting such that it could be analysed further mathematically. Vertical mathematising were activities that were grounded in or built on horizontal mathematising. The third perspective was developed by Harel (2000), who considered that advanced mathematical thinking should be considered as *advanced mathematical-thinking* which was advanced thinking in mathematics. Thus, advanced mathematical-thinking can be viewed as potentially starting at the elementary school. He defined advanced mathematical thinking by distinguishing a distinction between '*ways of understanding*' and '*ways of thinking*'. In his usage, '*ways of understanding*' referred to reasoning applied in a particular mathematical situation. Meanwhile, '*ways of thinking*'

referred to what governed understanding and reasoning that can be applied to the general mathematical situation. He identified three interrelated categories that influenced thinking, i.e., beliefs, problem solving and proof schemes. In addition, he discussed reasoning practices that can hinder advanced mathematical-thinking and looked at '*epistemological*' and '*didactical*' obstacles. Epistemological obstacles were rooted in the history and in the nature of the development of the mathematical knowledge itself whereas didactical obstacles arose from faulty instructions leading to problems in thinking and understanding. However, I also wanted to find out what the difference between elementary and advanced mathematical thinking was, where was the transition between elementary and advanced mathematical thinking and what were the implications to teaching?

The most obvious difference between elementary and advanced mathematics was the manner the subject was taught at the university. More new concepts were taught within a shorter time. There was a greater concentration on a small number of fairly similar topics where students needed to be aware of the possible relationship between the topics and be able to make connections, if and when necessary. The increase in the quantity of knowledge as compared to the limited time available for studying in the classroom meant that students had to augment their learning of the mathematics by themselves. Furthermore, they are expected to absorb formalised concepts very quickly and at the same time, be able to formulate and devise solution methods based on their understanding. The concepts themselves were presented differently as compared to their earlier experiences. At school, the emphasis was on the synthesis of knowledge, starting on simple concepts building from experience and examples to more general concepts. Whereas, teaching at the university emphasised the analysis of knowledge, beginning with general abstractions and forming a succession of deductions, which may then be applied to a wide variety of contexts (Tall, 1991).

In Section 2.3.2.1, I have quoted the ideas of Gray et al (1999) on the construction of mathematical knowledge in elementary and advanced mathematics; the different emphases as well as the different manner in which knowledge is presented. The literature has shown that the formalisation of mathematics at the university involved the construction of new mathematical mental objects, which were different and may be in conflict with previous objects developed in school. The mathematics was presented to students in various contexts such as numerically, graphically and symbolically. They had to develop the ability to use these different representations and at the same time, they also needed to make use of more abstract deductive thoughts. Thus, it has been suggested that the confusion and inconsistency in mathematical learning seen amongst first year students can be attributed to the above reasons and that these could become significant barriers to advanced mathematical thinking.

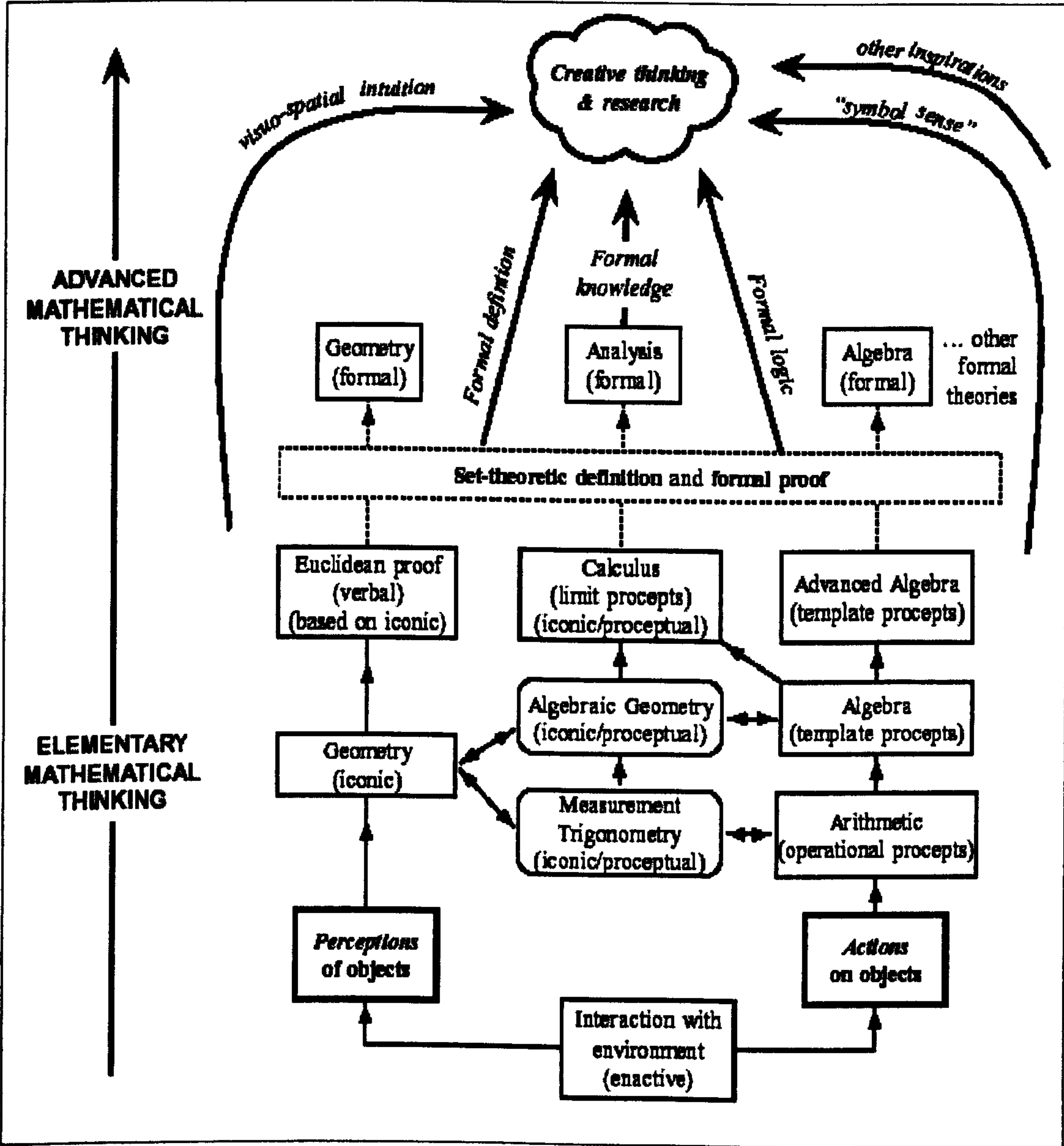
The first encounter in the move from elementary to more advanced mathematical thinking through the different mathematical topics has been described as the transition phase. For example, the move into calculus involves the idea of approximation, which is different from the concept of equality previously learnt. In advanced algebra, concepts such as vector product violates the commutative law of multiplication and the idea of four or more dimensions, break the visual link between equation and geometry.

This transition would require students to engage in a cognitive reconstruction. This can be seen from their initial struggle to understand the concepts being taught. Indeed, the move from elementary to advance mathematical thinking as described by Tall (1995), *“involves a significant transition, in particular, from describing to defining, from convincing to proving in a logical manner based on definitions. It is the transition from the coherence of elementary mathematics to the consequence of advanced mathematics, based on abstract entities which the students must construct through deductions from*

formal definitions". He described the actions and objects in the building of various mathematical knowledge structures in the following diagram (see Figure 2.1).

The figure outlines the different forms of representations and how they feature in different mathematical topics. It also shows the development of the visual-spatial to verbal in geometry, process and conceptual (proceptual) development in arithmetic and algebra and the relationship between them in measurement, trigonometry and Cartesian coordinates. Near the top of the figure are the subjects that identify the beginning of the transition to advanced mathematical thinking before the entry into more formalised mathematics.

Figure 2.1: The development of advanced mathematical thinking (Tall, 1995)



My next concern was “could teachers intervene, support or help” students in any way to change the way they think and solve problems in mathematics? The knowledge of how less successful students learn induces me to wonder about ways to support them to adopt more successful ways of learning and thinking about mathematics.

I also studied the ideas of mathematical thinking as proposed by Mason, Burton & Stacey (1982). In presenting those ideas, Burton (1984) described mathematical thinking as a way

to improve understanding and extending control over the study of mathematics. In particular, he described mathematical thinking from three aspects, the operations, processes and dynamics of mathematical thinking. Certain operations were identified as mathematical such as enumeration, iteration, ordering, making correspondence, forming equivalence classes, combining or substituting one from another to transform into a new state. These operations were independent of content area but very necessary for understanding and using mathematical ideas. Four processes were identified as central to mathematical thinking, specialising, conjecturing, generalising and convincing. Specialising is the exploration of meaning by looking at particular cases to make clear some common properties. Conjecturing should naturally follow as a student search for relationships that connects the examples and tries to express and substantiate any underlying patterns. Generalisation was the ability to recognize those patterns or regularity and making an attempt in expressing it mathematically. Convincing oneself and then another about the conjecture of the generalization that has been made encourages students to examine their ideas and explicitly communicate it first to themselves and then to others. In a book about developing strategies and skills to enhance problem-solving powers, Mason (1988 & 1999) showed how learning mathematics involved adopting a particular perspective of maximal involvement which he summarised as “learning is by doing”.

“Learning mathematics is a process of using the work of others to guide and inspire your own reconstruction of these ideas for yourself.”

And again,

“The most important thing learned from doing mathematics is not the facts and properties, but the increased sensitivity achieved each time you get stuck.”

(Mason, 1999)

In proposing strategies to provoke learners to become aware of mathematical thinking processes, Watson & Mason (1998) described a framework to generate and organized generic questions which can be asked about mathematical topics in various contexts.

These questions reflected the internal structures of mathematics and mathematical thinking and thus served the objectives of increasing learners' awareness of their own powers of thinking. Their framework for generating questions is the most important guide in developing my teaching strategies, in turning ideas into classroom tasks and activities.

Details as which of the ideas on mathematical thinking, classroom strategies as well as designing of tasks are adapted to form the basis of the teaching strategies that I have implemented is given in Section 2.5. It is extracted from a study that I did for my sabbatical from 1st July 2000 to 31st March 2001 in preparing for my doctoral research.

2.3.3 Teaching Environment

Research in this category has been concerned with the external environment to support mathematical learning and thinking. A desired objective was to encourage students to participate consciously in their own learning. Anthony (2000) found that students considered course design and organization as important factors that helped their learning. On the other hand, they preferred a passive approach to learning, to receive and be told of the important concepts and topics that would be tested or examined in the lectures. However, the students also cited poor delivery of lectures and boredom as reasons for their lack of involvement in the learning process. There were various ways suggested to provide the necessary environment to help students learn how to learn. It was conjectured that exposure to processes in mathematical thinking and problem solving would help students to invoke their own powers of mathematical thinking.

One method was to teach mathematical thinking and problem solving strategies explicitly as a separate course to support the mathematics teaching (Mohd. Yusof, 1995; Tall & Razali, 1993). Schoenfeld (1985) put forward a framework identifying the necessary elements to sustain effective problem solving. Another approach (Mason, 1999, Watson & Mason, 1998, Mason, Burton & Stacey, 1982) was to support students in their attempts to master the mental actions to learn, manipulate and to make connections between

different mathematical ideas. Students should be encouraged to learn by doing the mathematics and the lecturers should provide the necessary guidance and prompts to assist them in their efforts.

2.3.4 Assessment issues

Assessment practices have become a major influence in the implementation of the mathematics curriculum. By using narrowly designed tests and examinations, only limited aspects of students' learning can be assessed (Leder & Forgasz, 1992). Thus, the curriculum becomes guided by the tests and teaching is reduced to rote drill and practice of skills. In most mathematics classes, students' experiences consisted of learning routine, repeated and instrumental activities of applying fixed mathematical procedures to collections of structurally identical problems.

Generally, it was thought that learning outcomes at a higher level should encompassed broader objectives such as problem solving, reconstruction of concepts from basic principles, ability to transfer the learning and applying the knowledge as well as the capability to communicate the mathematics (Tall, 1991; Roberts et al, 1996). In particular, the different classification as suggested by Bloom's taxonomy (Bloom, 1956), was used as basic guidelines to identify major categories of cognition. In the preceding section, changes in the curriculum and the introduction of a variety of activities to promote conceptual and procedural understanding were presented. Consequently, there were suggestions of various assessment methods to evaluate students' learning (Roberts et al, 1996). Traditional methods such as tests, quizzes and examinations, were thought to be unable to fully inform instructors about the real state of their students' learning and should be supported by other techniques. Some of these could be: writing assignments, project work, laboratory work, oral presentations and portfolios.

The summaries from the different research and projects above, indicated that, although there would be possible difficulties in implementing the suggested changes and assessment

methods, there was a real need to effect changes so as to improve the teaching and learning of mathematics.

2.4 Research into mathematics education in UTM

In the past decade, various studies have been carried out on students' learning of mathematics in UTM. Findings from these research indicated that there are similar learning difficulties amongst the students (Liew Su Tim & Wan Muhamad Saridan, 1991; Tall & Razali, 1993; Mohd. Yusof & Tall, 1999). There have also been several programmes carried out to overcome the said difficulties (Mohd. Yusof & Abdul Rahman, 1998). In this section, discussion would be on mathematics learning and teaching in general. However, it should be noted that Calculus is major mathematics subject for the majority of students. As a start, a general description of typical students' profiles will be given. This will be followed by a brief report on current teaching practice. Together, they will provide the background to subsequent discussion and suggestions for the implementation of the guidelines to integrate mathematical thinking strategies in the teaching of Calculus.

2.4.1 Students' background

In reviewing the background of the students in UTM, two particular periods will have to be addressed. As this study spanned a number of years, there were some major changes to UTM's intake policies. In particular, students who entered UTM before July 2002, would generally have the Sijil Pelajaran Malaysia (SPM¹) qualifications. This group will be called Block 1. However, after July 2002, the entry qualifications must be post-SPM. In general, students had Matriculation² results, the Sijil Tinggi Pelajaran Malaysia (STPM³) and Diploma qualifications. Two groups will be studied and they are called Blocks 2 and 3. All students must come from institutions of learning approved by the Ministry of

¹ SPM – Sijil Pelajaran Malaysia (Malaysia Certificate of Education) equivalent to the GCE O-Levels

² Matriculation qualifications – from Matriculation programmes under the Ministry of Education

³ STPM – Sijil Tinggi Pelajaran Malaysia (Higher School Certificate of Education, equivalent to the GCE A-Levels)

Education and the Ministry of Higher Learning. Other equivalent qualifications were also considered on a case-by-case basis.

The prior mathematical knowledge of **Block 1** students consisted of Modern Mathematics at the SPM level. Some would have Additional Mathematics as an added qualification. It should be noted that the SPM is an important national examination. Schools followed a national curriculum and thus, there is some measure of homogeneity in the students' experience of learning mathematics. Passing the SPM examination is the most important consideration for both the teachers and students. They would have two years to prepare for this examination, the fourth and fifth form years, and teaching is directed towards ensuring students' success. Unfortunately, these concerns have spawned a teacher-led classroom culture of rote learning, drills and practice of typical examination questions with an over dependence on textbooks and study guides (Mohd. Yusof et al, 1999). Most students and teachers also shared similar preconceived notions about mathematics and the characteristics of what was considered good students. These were: mathematics was considered a hard subject; consisted mainly of facts and figures; a good memory was an important tool in mathematics learning. Students always expected that every question can be answered and that hard study, exercises and more exercises, was the only sure method for success (Lim, 2000). **Blocks 2 and 3** students have a more diverse experience of mathematical learning before coming to UTM. Although all students would have generally covered Algebra and Calculus, the Matriculation system, STPM and Diploma courses would have their own mathematical syllabi. To date, there is no documentation or research papers on the learning difficulties of these UTM students.

However, much of the literature reviewed about students' learning difficulties elsewhere have been about students with similar entry qualifications as **Blocks 2 and 3** students and thus, still relevant.

The following factors were culled from research on Block 1 students. However, from my personal observations of and from discussion with colleagues about the Block 2 and 3 students, we found that these issues were still relevant. A difficulty for the students in the first year is the change of learning and living environment. Among the major changes faced by UTM students are as follows (Yudariah & Roselainy, 2001).

- Adapting to the semester system at the university. The schools follow a term system and the SPM and STPM examination are set after two years of preparation. Matriculation centers followed a semester-style system but more learning time is given and the teaching (ref) is similar to the methods used in school. Only the Diploma students would have had some experience of the semester system similar to the one practiced by UTM.

At the university, students were expected to be able to absorb more complex ideas within the allocated 15 weeks (1 semester's duration). This change of pace would be new to all the students. Curriculum for Diploma courses were usually designed for a much slower pace of learning.

- Changes to the learning and teaching environment. Students were given more individual attention in school and much more closely monitored through the practice of teachers giving homework and special classes to prepare for examinations by working through special workbooks and past year papers. However, in the first year, they have to adapt to a learning culture where lecturers do not usually give them individual attention. They are expected to cope with the fast pace of learning and develop independent learning skills. They also have to cope with the change in the teaching methods. They were used to learning mathematics by rote and with intensive guidance from teachers. Now, they are expected to read more for themselves and sometimes work on problems in groups.
- Changes to the social environment whereby the students have to adjust to their new surroundings, physically and socially. There is much more freedom in the university

and with much more activities for students to follow. The ability to manage their learning and social life becomes very important.

- Changes in the presentation of mathematical content. School mathematics emphasised the synthesis of knowledge, starting from simple concepts, building up from experience and examples to more general concepts. The teaching was also focused on procedures and manipulation skills as it was aimed at preparing students for the national examinations. However, at university, although there is a similar tendency to reduce much of the mathematics into manageable procedures and algorithms, the teaching of mathematics usually begins with the presentation of theory, moving to general abstraction and making deductions from the theory to apply in a wide variety of specific contexts. Thus, the teaching objective appeared to be the analysis of knowledge.

2.4.2 Teaching Situation

A full description of the condition of mathematics teaching, the main teaching methods employed as well as programmes implemented to improve students' mathematical performance is given elsewhere (Yudariah & Roselainy, 2001). What follows will be a brief review of some pertinent considerations of the teaching situation. These are (a) teaching styles, (b) preference for teaching and learning mathematics procedurally, and (c) students' learning styles.

(a) Teaching styles

In general, most lecturers lecture! However, recent efforts have increased in supporting academic staff to practise more efficient teaching methods and assessment. The number of students in a given class is usually about 60. In most situations, lecture rooms or halls were not designed to facilitate group work although there have been recent developments where special classrooms have been provided for group work. In addition, lecturers are usually constrained for time, having to deliver their subject matter according to the

syllabus as well as carry out the necessary assessment in the given time. In particular, among the mathematics lecturers, the lecture method is considered to be the most suitable mode of delivery.

(b) Preference for teaching and learning mathematics procedurally

In a study of mathematics learning in UTM, Mohd. Yusof (1995) concluded that although lecturers would like students to develop positive attitudes towards mathematics and build good problem solving skills, they were not confident in their students' abilities to do so. Thus, they taught their students accordingly. Their teaching emphasised most on developing effective algebraic manipulation skills and practiced on typical problems. The students themselves preferred this approach and set out to learn procedurally to be successful in routine tasks.

(c) Students' learning styles

In the preceding sections, I have described the various factors that contributed to students' learning difficulties. The issue to be raised here is the overwhelming perception of mathematics as a subject most appropriately learnt by memorisation and rote. These skills had been important for their success in school and the students expected that it would be just as useful for their tertiary learning (Roselainy et al, 2001).

2.5 Guidelines to Integrate Teaching Strategies to Invoke Mathematical Thinking in Basic Calculus

At the beginning of the research, I took a five month's sabbatical to explore the various issues influencing and connected to the learning of mathematics of engineering undergraduates in UTM. The mathematics subject studied was Basic Calculus. However, I have used much of the strategies described and adapted them for the teaching of Engineering Mathematics too. These adaptations are described in the relevant sections in Chapters 3, 4 and 5.

The guidelines suggested here were mainly based on the frameworks developed by Mason and his colleagues (Mason, Burton & Stacey, 1982; Watson & Mason, 1998; Mason, 1999; Mason, 2000). In particular, the main underlying idea of learning mathematics is that 'the essence of learning is doing'. The central core of doing mathematics is: (1) specialising – the ability to construct particular examples to see what happens, and (2) generalising – the ability to detect a form, express it as a conjecture and then justify it through reasoned argument. It is considered important for students to perceive the fundamentals of doing mathematics, to be able to understand how others work out mathematics. Therefore, learning mathematics should involve more than just the ability to execute certain procedures, master some techniques and solve routine problems. The guidelines will encompass activities in preparing for teaching as well as classroom practice. All the suggestions are based on the particular difficulties and learning culture of students in UTM. It also takes into account the constraints that have been reported as restricting the implementation of teaching practice other than by the '*chalk and talk*' method.

Teaching

Preliminary preparations for teaching should include (a) finding out about students' background and their prior mathematical knowledge and facility with algebraic manipulations; (b) being aware of lecturer's own beliefs about mathematics and how it should be taught; (c) deciding on the objects and indirect objects of the topics. The direct objects refer to the learning of facts, skills, concepts and principles of Mathematics. The indirect objects are the transfer of learning, inquiry ability, problem solving ability, communicating mathematically, self-discipline and an appreciation for the structure of Mathematics.

In any learning and teaching situation, various manners of interaction will take place. In a typical classroom setting in UTM, the direction of communication is mainly from the

lecturer to the students. The main delivery method is expounding and explaining. However, in practice, it takes the form of presenting the relevant mathematical materials, in a fixed hierarchy of "definitions, facts, properties, procedures, techniques, examples followed practice of more examples and, finally, word problems". Other interaction styles and six important interaction modes (Mason, 1999, 2002 (a)) have been identified that will contribute towards effective learning and thus they should be used for effective teaching. These are:

(i) **Expounding** – attracting students into lecturer's realm of experience. Currently, this is one of the more predominant teaching styles and its usefulness must be balanced with other styles that would engage students in active learning. It could become overwhelming for students when it is the only method used.

(ii) **Explaining** – entering the students' world and working within that world. Lecturers have to acquire an awareness of the students' difficulties in absorbing mathematical concepts, ideas and in mastering specific techniques. These should be taken into account during the teaching.

(iii) **Exploring** – guiding the students in productive directions as they learn to make connections between the mathematical ideas themselves.

(iv) **Examining** – assessment activities to help students validate their own understanding. The roles of these assessment methods are to monitor students' learning and to supervise development of students' learning.

(v) **Exercising** – students should practice techniques and review connections between theorems, definitions and ideas.

(vi) **Expressing** – students should be encouraged to express their understanding. However, getting UTM students to express their understanding of the mathematics learnt is very hard. Various suggestions have been identified and will be presented in (see Section 5.2.4)

Identifying the thinking powers to be invoked

Below is a list of the various kinds of mental activities that represent mathematical thinking (Watson & Mason (1998)), grouped under six activity-headings for convenience.

Table 2.1: Action associated with mathematical thinking powers

Mathematical thinking	Ability/Action
Exemplifying Specialising	Recognise the specific features or attributes that makes an object or concept as an example; Construct special examples to see what happens
Completing Deleting Correcting	Know what to add, to remove or to alter to allow, to ensure or to contradict between expressions
Comparing Sorting Organising	Recognise similar attributes/properties as well as the different ones; Can organise or sort according to certain criteria
Changing Varying Reversing Altering	Altering to see resulting effects; Looking at other ways Do the converse Changing due to imposed constraints
Generalising Conjecturing	Detect a form or general pattern, moving from a few instances to making guesses about a wide class of cases Stating a general form but whose truth is not yet established

Table 2.1: Action associated with mathematical thinking powers (continuation)

Mathematical thinking	Ability/Action
Explaining	Elucidate
Justifying	Revealing an underlying structure or relationships that links
Verifying	Checking the relationships
Convincing	Ensuring your reasoning are plausible, convince yourself, a friend then an enemy
Refuting	Looking for contradictions

Identifying structures in Mathematics

A list of mathematical statements that exposes the structures of Mathematics is given below. I have used the statements to choose the structures thought to be suitable for Basic Calculus and compatible to the students' prior experience in mathematics learning. The selection was guided by my experience and an underlying concern of initiating changes to the students' views on Mathematics without inducing further anxiety on the part of the students. It must be remembered that more often than not, students were not aware of the structures themselves. If they do, it would be most likely, that they would have been told about them rather than made to see for themselves. Any new teaching approach that encourages them to think and explore would be a new experience for them, and they are usually very wary of anything new.

Structures in Mathematics:

- | | |
|----|----------------------------------------------------|
| 1. | Definitions |
| 2. | Facts, Theorems and Properties |
| 3. | Examples and Counter-examples |
| 4. | Techniques and Instructions |
| 5. | Conjectures and Problems |
| 6. | Representation and Notation |
| 7. | Explanations, Justifications, Proofs and Reasoning |
| 8. | Links, Relationships and Connections |

These different types of statements makes up a topic, provides the structure for that topic. Accordingly, I must be aware that I might have a structure for a topic different to the ones that my students have. The structure of the topic is based on which types of statements were stressed or in which ways the connections are seen. It is important to be familiar of the different aspects of a topic to be able to initiate students into thinking mathematically.

Using general questions and prompts

By considering the mathematical thinking activities, Watson & Mason (1998) developed several general questions and prompts that can be used to encourage the development of a 'sense of Mathematics' among the students. The list below (Table 2.1) provides an example of some questions generated and was used as a guide on how to choose suitable questions and structure the lecturer's preparations. They were not to be considered as fixed or used rigidly but should provoke similar questions among the lecturers and students themselves.

Table 2.2: Mathematical thinking and generic questions

Mathematical thinking	Generic questions
Exemplifying Specialising	Give me one or more examples of...? Describe, demonstrate, tell, show, choose, draw, find, locate an example of... Is ... An example of ...? What makes ... an example? Find a counter-example of ...? Are there any special examples of ...?
Completing Deleting Correcting	What must be [added, removed, altered] in order to [allow, ensure, contradict] ...? What can be [added, removed, altered] without affecting...? Tell me what is wrong with ...? What needs to be changed so that ...?
Comparing Sorting Organising	What is the same and different about ...? Sort or organise the following according to ...? Is it or is it not ...?
Changing Varying Reversing Altering	Alter an aspect of something to see effect. What if ...? Do... in two (or more) ways. What is quickest, easiest, ...?
Generalising Conjecturing	Of what is this a special case? What happens in general? Is it always, sometimes, never, ...? Describe all possible ... as succinctly as you can. What can change and what has to stay the same so that ...is still true?
Explaining Justifying Verifying Convincing Refuting	Explain why...? Give a reason(using or not using...) How can we be sure that...? Tell me what is wrong with ... Is it ever false that? (always true that...?) How is ...used in ...? Explain role or use of Convince me that

[Extract from *Questions & Prompts for Mathematical Thinking*, ATM, Derby. © Anne Watson & John Mason, 1998.]

By considering the mental activities and the mathematical structures, a grid (see Table 2.2) was produced to help in generating particular questions for the topic under study. Appropriate questions are developed as entries in the grid to help increase my awareness

of my own mathematical thinking and to support my ability to be explicit about it. In doing so, I hope to encourage, provoke and make students aware of their thinking abilities.

It is recommended, to start off with one question type, use it explicitly whenever possible so that, students can get a meaning of what it generates.

Table 2.3: A grid linking the list of mathematical structures to the mathematical thinking activities

	Exemplifying Specialising	Completing Deleting Correcting	Comparing Sorting Organising	Changing Varying Reversing Altering	Generalising	Explaining Justifying Verifying
Definitions						
Facts & Properties						
Examples & Counter- examples						
Techniques and Instructions						
Conjectures & Problems						
Represent- ation & Notation						
Explanation Justification						
Links relationships Connections						

[Extract from *Questions & Prompts for Mathematical Thinking*, ATM, Derby. © Anne Watson & John Mason, 1998.]

Using teaching strategies to facilitate learning

In this section, some teaching strategies were identified that could lead students to take the initiative in their learning, thus becoming more active participants, as well as encourage them to focus more on thinking mathematically. These teaching strategies were selected from a list developed by Mason (1998) and I have chosen some that I considered as important strategies.

Essential considerations

Be explicit about the enterprise – to be clear about the principal mathematical enterprise in the course: encountering new ideas, formalising old ideas; exploring consequences of axiom systems, developing techniques for use in other courses; algebraicising the geometrical, or geometrising the algebraic, etc.

Teaching contract and explicitness tension – there is always an implicit contract between lecturers and students, in that, students expect the lecturer will tell them what they need to know and that if they do the work, they will be able to learn. In changing my teaching and the way exercises were given, I needed to seek an explicit contract with my students so as to reduce tension in what they expect as compared to what I wanted to do.

Using the Mathematical Pedagogy

I believe that students' can develop their understanding and facility with mathematical ideas and techniques. A firm understanding of mathematical processes and problem solving skills are the most important attributes that they can carry over to their core study areas. The following ideas described the mathematical pedagogy that was used to achieve these aims.

Using mathematical themes – there are several themes that provide connections between different mathematical topics. By exposing the core mathematical themes and making

them more explicit, students may be able to integrate them into their sense of mathematical thinking (Mason, 2000). I have chosen the following themes to use:

Invariance amidst change – this forms the basis for many mathematical theorems. For example, differentiation is invariant under addition, multiplication, multiplication by a scalar, and composition. Invariance amidst change is an issue for students when they are uncertain as to what aspects of an ‘example’ are permitted to change, and what aspects are structural and invariant.

Doing and undoing – students are usually familiar with ‘doing’ questions, for example, “find the arc length of ...” or “differentiate a given function”. By reversing the question, as ‘an undoing’, students will have to identify the features or structures that can be varied, as well as the conditions for which the problems can be solved. Differentiation is a ‘doing’ process for which integration is an ‘undoing’.

General approaches to present the mathematical contents

The following approaches were selected as how to present the mathematical concepts and techniques in the classroom.

Directed - prompted - spontaneous – using the questions in a sequence of directed questioning then increasingly indirect prompting can eventually lead to students spontaneously use the questions themselves.

Exposing incantations – exposing the inner dialogues that I use when doing an example or using a technique on the board. The students will be aware that his lecturer has inner dialogues in trying to solve a problem and he will get to hear some of the shapes of those dialogues and perhaps appreciate the process of solving problems.

Balancing techniques and concepts in teaching – ideally, technique and understanding should develop together. Improvement in techniques free attention from understanding, and understanding will enable the students to reconstruct technique. Thus the teaching

strategies suggested here are variations of provoking students to reconstruct and re-express what they have learnt.

Addressing students' difficulties

Emphasising one thing per session – choose the most important point in the lecture and draw students' attention to it specifically.

Using common errors – collect the common errors that students make and these can be handed out to students and in each lecture, pick out one or two, and discuss them briefly in the class. Students could be asked to discuss them in pairs before explaining to the class what is wrong.

Specialising, generalising and counter-examples – the power to specialise a generality and to detect patterns in specific cases and express them as generalities, is considered an important mathematical thinking activity. It is important not to always particularise the general for the students, or always stating the general for them. Students are to be given the opportunities to state for themselves what is generalised from the topics under study.

Symbolic notation – is useful when it summarises or encapsulates a wealth of meaning; it is an obstacle if it presents a barrier because it lacks associations, images, and meaning. When students are introduced to notation, they need to know what is invariant and what can change. For example, different letters can be variables, not just x and y ; $\sin h$ can be differentiated with respect to h if it is convenient to label something as h , and $y = ax^2 + (a^2 + a)x + a^2$ can be treated as a polynomial in a as well as in x . Students require experience of the use of different variables in order to appreciate this fact.

Multiple notations – students should be offered different conventions used at different times for the same thing. The aim is for the students to realise that notations represent certain conventions in writing mathematics and can change over time. Students should be given some exposure to changes in notation so as to make them more aware that some

concepts and ideas are presented differently in different courses. This would help them to be more flexible and understand that there are different ways to think of the same concept.

Multiple representations – students need to see different ways of thinking about an object or a situation. For example, functions can be thought as rules for calculating values, as tables of values, as graphs or as ordered pairs. It can assist students to develop the ability to select and use appropriate mathematical representations, such as, visual, numerical, symbolic, algebraic and analytical.

Say what you see – one way to help students overcome uncertainty about notation is to get them to ‘say what they see’ of the notation. This can be expanded to include reading from diagrams, trying to express the meaning of certain expression, working through a worked example. This would help students to see and hear how others perceive the same item. This could be extended to the prompt, “see what you say” to invoke imagery in the students.

Introducing definitions – some examples and non-examples of a concept can be offered to students before introducing a concept. Let the students try to identify the features that are required in formulating a definition.

Introducing technical terms – if a term is similar to common usage and when it does not should be discussed explicitly.

Using unfamiliar examples – students can be offered unfamiliar examples. For example, on the concept of functions, choose an example which has aspects that disturbs the common notion of function, and be explicit as to which conjectures they provide counter-examples.

Forging links – after work on a topic or a problem in which a technique or approach has emerged, students should be encouraged to pause and imagine themselves working on another problem and imagine themselves using that technique again. The technique could

be mathematical or could be a sense-making technique such as looking for invariance amidst change.

Engaging the students

The approaches selected here are those that would encourage students to participate in their learning, to contribute to the lessons and be more willing to raise questions. Lecturers should bear in mind that any change should take into account students' anxieties about mathematics teaching and learning. The list is arranged according to what is considered most likely to succeed amongst the students. Hopefully, these could encourage more students to be comfortable with active learning.

Asking students to write down the least clear concept/ definition/ example/ technique – these can be asked to be handed in, or ask some students to share to the class. I think that this approach would be the easiest to start with and the one least intimidating for the students.

Asking students to write down what they thought were the salient points in the lessons - this approach will also help in keeping track of what students were attending to in the lectures as well as gain insight to what they were stressing or ignoring.

Using prompts and questions – there are several ways to use the prompts and questions such as (1) offer students a range of questions, (2) make the questions interesting enough to maintain attention, (3) repeating the questions often, with enough similarity for the students to become familiar with them, (4) encourage students to take responsibility of asking the questions themselves, (5) invite students to do, say or suggest something.

Invoking mental imagery – mathematical imagery does not arise spontaneously. Indeed many students seem not to be aware that they have control over the images, connections, diagrams and techniques which 'come to them' (or fail to come to them) in association with a technical term. Before announcing a theorem or drawing a diagram, use imperative

language (*imagine a cubic curve, imagine a chord drawn between two of the points, etc.*) to push students to construct their own mental image. Getting them to describe to each other what they are seeing, without using their hands or drawings, can help them to gain flexibility in moving from words to diagrams, and back, as a support for symbolisation.

Reconstructing and re-expressing – students can be encouraged to express in symbols something written in words, or to reconstruct commentary from just symbols. Students could draw a diagram from a verbal description, or to describe for someone else to draw a diagram they are presented with. The idea is to provoke students into using words, symbols, and diagrams, and to convert one to the other as necessary. They should try to expand technical terms and symbols into words with which they are comfortable and which they find meaningful but which are also mathematically correct. They should learn to read diagrams rather than simply look at them

Students' generated examples – ask students to make up (and do) their own questions of the same type; variants include asking for an easy and a hard question of that type; asking for or a question which shows they know how to do a question of that type (general); and asking for a description of what constitutes '*that type*'.

Talking in pairs – when saying a complicated sentence, or describing a complex idea, stop and ask students to say to each other what was just said. Be ready to give them time to respond. When they are ready to continue, ask if they have any questions that could be develop into a discussion. It would not be easy for the students to carry out this particular action but they should be encouraged to try this technique of expressing mathematics to each other.

As has been mentioned, the guidelines were specially prepared for the teaching of Basic Calculus, but much of it was also used in the teaching of Advanced Calculus or Engineering Mathematics.

2.6 Conclusion

The literature reviewed here has presented description of students' difficulties from various perspectives. I think that it is important that measures to improve students' facility with mathematical ideas and enhance their mathematical thinking must be planned. There were also views on how to improve the teaching and learning situations. The strategies that I have chosen take into account the most immediate concerns of both students and lecturers. It is also clear from the presentation that much of the literature reviewed is from the year 2000 and earlier. This literature review was undertaken prior to the start of the research. However, the nature of the research methods meant that at each cycle of the research, the experience, data and literature are reviewed. Thus, any additional literature studied and referred to will be mentioned in the relevant sections.

CHAPTER 3

RESEARCH METHODS AND PROCEDURES

3.1 Introduction

In Chapter Two, various research studies that probed issues related to the cognitive development of learners, pedagogical considerations, the psychology of mathematics education as well as curriculum development have been reviewed. Obviously, there are various factors or variables that will affect students' learning. In addition, my experience of teaching mathematics has shown up the particular difficulties faced by engineering undergraduates in UTM. These have also been reviewed in Section 2.4. In particular, I have identified the mathematical skills that I thought students needed in order to become independent learners, specifically, the ability to think mathematically.

In this study, I designed teaching strategies to support and encourage students' use of their own thinking powers and I examine the factors that seem to have influenced the implementation of these strategies and how it affected students' learning. Since I was both the researcher and teacher, appropriate research methods had to be adopted. A qualitative research perspective was chosen as various methods within this stance can ensure flexibility in responding to the dynamics of interaction between the teacher and her students.

There are many factors that influence students' learning, coming from within and without. Examples of factors coming from the students, are, their attitudes and beliefs about mathematics, their self-perception, motivation and personalities, the amount of prior knowledge they can call upon and their styles of learning. Examples of external factors are the teachers' teaching styles, classroom environment and support materials (Cocking & Chipman, 1988). An important element that definitely influenced the classroom interaction

will be the teacher, my actions and choices as teacher, my personality, my knowledge and how I interacted with students. These factors or variables are not readily controllable and how they interact with each other was not be easy to study. A qualitative approach allowed a study of the classroom in context and of the learning process in progress. However, I also paid attention to my changing teaching practice, as researcher and also teacher, which means that a greater awareness of the duality of roles, teacher and researcher, had to be invoked.

3.2 Research Implementation

At the outset, I thought that an ethnographic approach would be a suitable option as during the research period, the teacher would be actively involved in the teaching and learning environment. I started with the ideas of gathering data from the students' own perspectives, to capture in their own words their concerns and feelings about mathematics learning, i.e., a study of learning in a '*natural*' setting. I also thought that I would be building the theory from the data that would be accumulated in the traditions of 'grounded theory'. I started teaching with my new teaching strategies in November 2001, teaching a class of Industrial Design students Basic Calculus and realized some significant difficulties with the research methodology that I had chosen. These were:

- a) I had entered the teaching and learning environments with many assumptions about my students' abilities and difficulties. In fact, I had designed the teaching strategies based on these assumptions. Albeit, they were not all based on opinions and experience as some were drawn from research conducted on UTM students but not these particular students. The 'naturalness' of the situation has been compromised. Every action that I took was based on these assumptions and on assuming that I could support and encourage students to make changes in their learning behaviours.

b) The duality of roles, teacher and researcher, was not easily played out in the classroom. Wong (1995) had described an incident where he wanted to explore a particular child's understanding of a scientific phenomenon but in doing so, would have interrupted the pace of the lesson and ignored the rest of the class. He attributed the conflicts to the differences in the goals of teaching and research, "*the primary goal of research is to understand; the primary goal of teaching is to help students learn.*" I had difficulty in maintaining a research perspective as I was doing what I have been doing for many years past. At the beginning of the research, I was more the teacher and less of the researcher with the major difference that now I was paying more attention to my students' views and their behaviours in the classroom. I found Ainley's stance (1999) that teaching and research need not be in competition but could be complementary much more reassuring and I used this to make extra effort to be aware of these different roles that I would take.

As such, the research was then implemented based on action research and the case study methodology.

3.2.1 Using Action Research and Case Study Methods

There are various definitions of action research that emphasise different aspects of the methodology considered important by particular authors. Kemmis and McTaggart (1982) defined action research by highlighting that the linking of terms, action and research, identified the essential features of the method, "*...trying out ideas in practice as a means of improvement and as a means of increasing knowledge...*".

Dick (2000) believed that the distinction between theory-driven and data-driven research is more important than that between quantitative and qualitative research. He described action research "*... as a family of research methodologies that pursue the dual outcomes of action and research.*" He defined a theory-driven research as one that was based on

assuming that a body of existing knowledge and literature as given and worked towards extending or refining or challenging it. Data-driven research is one where researchers wanted to deal with the research situation and the people in it as they are, putting aside as many preconceptions as possible, so that they are more open to fully experiencing the research situation. Data-driven action research is considered more flexible and responsive to the situation under study. In researching my own practice, I studied and increased my understanding of the work situation and thus improve my own practice as well as the work situation itself.

Ainley (1999) takes the view that teaching and research could be complementary and she advocated that a teacher doing research must be able to recognize how the research informs on teaching, and when experience as a teacher also informs the research. The action research process has several features that were appropriate to my own research concerns. Firstly, action research is an intervention in personal practice with a commitment to educational improvement (Mcniff, Lomax & Whitehead, 1996). The teacher becomes the subject and object of research but with a greater awareness of the actions that are being carried out. It means that the teacher must investigate her actions and motives systematically, be critical of her interpretations and findings and be more open to alternative viewpoints. There should be a commitment to the actions implemented and the actions must be intentional. The process itself demands that the teacher becomes aware of the cycle of planning, action and review. She must alternate action with critical reflection, evaluating the research situation and back to the planning, modifying or changing if and when required. Secondly, action research will allow a detailed description of the classroom environment to be made, the teacher's actions as well as the students' behaviour in the class.

I have been a teacher who has seen her students struggling with new concepts in Advanced Calculus and facing difficulties with the need to use various mathematical techniques to

solve problems. Some of the techniques would be newly taught but others were taught in previous courses or some would have been taught when they were in school. I have also seen students who worked very hard but still have difficulties in understanding the concepts taught. They worked on many problems to increase their understanding but still found non-routine questions difficult. I have conducted many remedial classes and given extra tuition to 'weak' students and have come to realize that one of the most typical problems was that these students could not 'see' the 'general' through particulars nor 'see' particulars in the general. They could not see the essential features of a technique or recognize these when presented in different forms. I strongly feel that a key to help increase their understanding is the way they think about the mathematics. The purpose of my research was to bring about improvements to the teaching of Engineering Mathematics with the aim that it would also bring about changes on how students think about the mathematics. I am a teacher intent on changing the way I teach to help my students learn and understand more. Every encounter with my students, within and beyond the classroom, was carefully considered and contributed to my reflection about my teaching, review of the strategies and consequently, changing the way I teach or interact with the students. Discussions were held with colleagues in the Department of Mathematics as well as from the various Faculties of engineering, reflecting on students' developments, teaching strategies and curriculum concerns. I was already doing action research but I needed to heighten my awareness of the duality of the outcomes sought, action and research. I needed to be more systematic and methodical.

Kenny and Grotelueschen (1980) offered several reasons for choosing case study design for research. One reason was “ *to develop a better understanding of the dynamics of a program. When it is important to be responsive, to convey a holistic and dynamically rich account of an educational program, case study is a tailor-made approach.*” However, there are several definitions of case study research offered by various authors such as, “*an instance drawn from a class*” (Adelman, Jenkins & Kemmis, 1983); “*the examination of*

the instance in action” (McDonald & Walker, 1977); “*to reveal the properties of the class to which the instance being studied belongs*” (Guba & Lincoln, 1981); and Becker’s (1968) twofold definition: “*to arrive at a comprehensive understanding of the groups under study*” and “*to develop general theoretical statements about regularities in social structure and process.*” A significant strength of case study research is that it “...offers a means of investigating complex social units consisting of multiple variables of potential importance in understanding the phenomenon” (Merriam, 1988). As a case study would be anchored in real-life situations, it should result in a holistic account of the phenomenon and provide for a greater understanding of the situation.

Using the case study research methodology would allow me to study the implementation of the teaching strategies and how the class will respond to the changes. I hoped to discover and identify the many factors that would affect students’ learning and something of how these factors interact. I wanted to study factors that would or would not support students’ changing their learning behaviours. I taught several groups of engineering students, covering a general period from November 2001 to November 2006. The students were taught based on the semester system with a date in June or July as the beginning of the first semester of the academic session. Thus the academic session is referred to as 2001/2002, which means the period from July 2001 to March or April 2002. Generally, the research was conducted in three different phases thus involving various groups of students. These groups were further categorised into three blocks of students’ groups. In the first phase, the first block of students’ groups (Block 1) were various groups of students doing Industrial Design in the Faculty of Mechanical Engineering who were taught in Bahasa Malaysia (Malay Language). I taught these students Basic Calculus and Calculus II (see Chapter 4). The second block (Block 2) was made up of students from the Faculty of Electrical and Civil Engineering, who were doing Engineering Mathematics. They were taught in English. There were other attributes that were peculiar to each block and these will be discussed in the relevant sections describing the different groups (see Chapter 5).

In the event, there was another block of students [Block 3] that I would like to write about (see Chapter 5); these were students taught by my colleagues, using the same teaching strategies. I had made arrangements to observe the classes and conducted some interviews with willing students. The students were from two different classes of the Faculty of Electrical Engineering in the first semester of the academic session 2007/2008, taught by Dr. Zee and Dr. Tee. I will refer to the groups and Group G07 and H07. The different groups of students in Blocks 1, 2 and 3 will be identified in Section 3.6, where an overview of the research implementation will be discussed with an explanation of the various reasons for conducting the research in three parts.

3.2.2 Studying the Natural Setting

An important consideration was presenting a study that was realistic in describing the teaching and learning environment. As I have been a teacher with at least 20 years' experience at the beginning of the research, another important consideration was to heighten my awareness of the research component in the coming semesters. I also opted for research methods whereby I am both the subject and instrument of research. It was necessary to prepare myself for the duality of roles, teacher and researcher, to view the teaching and learning setting henceforth as,

“ ... a learning situation in which the researchers have to understand their own actions and activities as well as those of the people they are studying.”

(Burgess, 1982)

The techniques most associated with achieving the goals of working with subjects in their natural setting are briefly described in the following section.

3.2.2.1 Participant Observation.

Gold (1958) identified four different modes of participant observation such as complete participant, participant-as-observer, observer-as-participant and complete participant.

Meanwhile Ball (1985) discussed the 'hard-line' and 'soft-line' approaches in participant observation whereby in the soft-line approach, the researcher is an observer accepted by the group but not necessarily participating in the activities of the group under study. A hard-line approach is when the researcher is a full participant. In this research, the researcher is also the teacher, thus she is a full participant of the classroom but not in the same manner as the students. The researcher will be researching her own practice as well as observing the students' behaviour in the class. It becomes important to distinguish between the observation notes that were taken during the research period. It was not always possible to make notes when I was presenting my lectures and any event worth noting had to be written down as soon as it was convenient. In every class, I have organised tasks for students to carry out and during these times, I would write down my observations. Sometimes, the notes are written as soon as possible after the class. My first attempts were to write down everything that I thought pertinent thus there was a mixture of describing events as well as my comments or views on those events.

Mason (2002 (b)) suggested that the act of noticing is an important element of practice. He described three different levels of noticing: ordinary, marking and recording. In addition, he put forward the notion of disciplined noticing as an attempt to be systematic and methodical and suggested several elements to enhance the chances of noticing. These are: keeping accounts, developing sensitivities, recognising choices, preparing and noticing, labelling and validating with others. Adopting the ideas of disciplined noticing has heightened my awareness of researching my own practice. Thus in writing the accounts of the events, I tried to ensure that my personal opinions and judgments were not included in the descriptions of the classroom situation or other interaction with the students. There was also the difficulty in choosing events to write about. In the beginning, everything was considered worthy to be noted but during the later stages, there was an element of choice and I would try to make explicit the reasons for these choices as much as possible. As I was embarking on a research project working towards a degree, I became much more

regular in going over my notes at the end of class and reflecting upon my teaching and events that had occurred in the classroom. I also used the notes to identify students' responses to my teaching strategies, the presentation of the mathematical contents and techniques and to deliberate over changes that could be made.

Using the accounts to look for common themes became easier as I taught more classes. I was able to identify some themes emerging from the situations that were concerned with students' learning behaviour, the nature of the relationship between the students and me, my own awareness of my role and response to the students. A necessary part of using the accounts was to develop sensitivity to things that were happening in the classroom. I had thought that of all things, this would be the most difficult as I have been teaching for many years and have regarded the students as a generic group although you know you are teaching different people every time. I had to make a determined effort to suppress old judgements and adopt a more open minded and open hearted attitude to try to look at familiar situations with a fresh eye. One way was in noting down what I felt or thought about, including assumptions or opinions that I had about the events. A keen sense of feeling responsible to look for better ways to teach my students was useful in giving me the impetus to start changing and maintaining the change.

I had also decided to ensure that the students knew that I was researching my own practice and their class. The objectives of the research were shared with the students and they were reassured that any incidents or quotes that would be used would be anonymous to protect their identities.

3.2.2.2 Critical Reflection

A teacher engaging in action research brings with her all her understanding and experiences of the teaching and learning situation and thus has to reflect on and examine these prior knowledge and all her assumptions to be able to increase her awareness and insight into the research. Kemmis and McTaggart (1998) explained it as, "*plan, act,*

observe and reflect". It is important to alternate the action with critical reflection, which can be about the data and the interpretations made on it, a critique of the methodology to improve it and an examination of assumptions about factors that influences the study. Dick (2000) called the alternating action with reflection as the spiral nature of action research. He considered that the spiral provided the two main advantages of this method, which are, firstly, each turn of the spiral would offer chances to test the interpretations that have been developed against the data collected and secondly, with each turn, plans are developed to test the assumptions that guided them in action. It is this that would allow for the flexibility and responsiveness for effective change.

Other researchers talked about reflective practice in teaching which involved thinking about and learning from your own practice and the practice of others. The object is to gather new perspectives on the teaching and learning situations, improve judgement and review teaching strategies. Dewey (1933) defined reflective thinking as "*an active, persistent and careful consideration of any belief or supposed form of knowledge in light of the grounds that support it*". In examples chosen to demonstrate what was meant by 'reflective thinking', he chose one case of reflection at the time of action, another of reflection after the action, and a third which combines the two and also make use of public theory. Similarly, Schön (1983) identified two ways to carry out reflection: reflection-in-action and reflection-on-action. Reflection-in-action is sometimes described as 'thinking on our feet', and it involves looking at our experiences, connecting to our feelings, and attending to our theories in use. Reflection-on-action is carried out later after an encounter; where we have to spend time to explore why we acted in a certain way and what was happening in the group. Griffiths & Tann (1992) discussed the importance of uncovering personal theories in reflective practice and making them explicit. She felt that this particular process has not been given due attention in other theories on reflection. She suggested five different levels of reflection to support theorising such as:

- **Reflection-in-action: likely to be personal and private.**
 - 1) **Act-react (Rapid reaction) – at this level, reaction is immediate**
 - 2) **React-monitor-react/ rework-plan-act (Repair) – at this level, although there is pause for thought, it is on ‘the spot’ and very quick.**
- **Reflection-on-action: likely to be interpersonal and collegial.**
 - 3) **Act-observe-analyse and evaluate-plan-act (Review). At this level, thought and reflection are going on after actions are completed.**
 - 4) **Act-observe systematically-analyse rigorously-evaluate-plan-act (Research). At this level, observation becomes systematic and sharply focused.**
 - 5) **Act-observe systematically-analyse rigorously-evaluate-re-theorise-plan-act (Re-theorising and reformulating). This is the level of abstract, rigorous reflect which is formulated and reformulated.**

It is at the last two levels that engagement with public theories becomes more prominent. Her framework opened up the ideas on the roles of personal theories and public theories in teaching practice and more importantly, how theory and practice are related in education. She also discussed the differences in language used to articulate personal theories compared to the abstract terminology of public theories. She felt that it was essential to recognise the importance of public theories without losing sight of the importance of personal theories embedded in practice.

I found Griffith & Tann’s framework helpful in offering an efficient and practical guide for a teacher to undertake reflective practice as well as increasing her awareness of the research aspect of her work. In particular, reflecting upon my personal ideas and theories about students’ difficulties and constantly examining and comparing it with public theories was an important initial step in embarking on this study. My ideas were examined against

published literature as well as with other colleagues so as to gain a wider perspective on issues emerging. My colleagues and I have discussed and lamented students' difficulties in our mathematics classes over the years and in many ways, our views were similar and appeared to be supported by findings of certain published research works. These assumptions are clearly identified in this report and were used in the first instance in designing and developing the teaching strategies. Public theories on specific students' difficulties with mathematical concepts, how students learn, suggested strategies to overcome these difficulties were also referred to in creating the teaching strategies that were implemented.

However, at every stage, the teaching acts, students' responses, various issues emerging from the students-teacher interaction were reflected upon thus ensuring that the cycles of '*planning-implementation-review-modify*' were continually carried out. External influences (organisational, administrative, etc) were also considered as sometimes these provided information about changes to some initial assumptions about the students studied. Additionally, experiences within the research cycle sometimes necessitate changing certain notions about various aspects related to the students' such as abilities, prior knowledge, motivation, behaviour and attitudes.

3.2.2.3 Observation of Colleagues' Teaching

I started out the research working on my own in creating the teaching strategies. However, I had only focused on certain topics in Calculus namely, Multivariable functions and Partial differentiation. Throughout, I had colleagues who were supportive and were also concerned about students' development and we had numerous discussions about the progress of my work, the assumptions I was making as well as the tasks that I used to set for the students. They acted as evaluators for the work that I was carrying out. I made a decision to compile the teaching tasks in a workbook due to certain conclusions made (see Section 4.9) at the end of the academic session 2004. Thus, beginning in the academic

session 2005/2006, a workbook was designed to help students work on the mathematical tasks. However, the development of the workbook was done in a group where I was working with two other colleagues. For the workbook, we had to work out the tasks for all five chapters in the Calculus course, namely, Multivariable functions, Partial differentiation, Multiple integrals, Vector-valued functions and Vector calculus.

However, an early version of workbook was only ready for use in the academic session 2006/2007. My colleagues carried out the same teaching strategies and used the workbook in their classrooms. However, I was unable to observe their teaching during this period. Thus, I decided to observe their teaching in the second cycle of using the workbook, the first semester of 2007/2008. I did not have a chance to use the book as I was then teaching in Kuala Lumpur and was involved in teaching at the diploma level. The decision to observe their teaching would add objectivity to the evaluation of the teaching strategies as my role would be as a complete observer. My role was also made known to the students from the beginning. Contact with the students stemmed only from the research. I observed students' group discussion, looked at their work in the workbook and conducted some interviews with willing participants.

3.2.3 The Research is concerned with Participants' Views of Teaching and Learning

An important consideration of the research perspective was to find out what the students felt about the teaching and learning process, in particular, what they thought of about their mathematics learning and their supposed difficulties. Some of the methods adopted towards these objectives are as the following.

3.2.3.1 Interviews

Interviews are considered an effective method to solicit students' views about their learning, attitudes, and the teaching. I decided that the interviews would only be conducted with students who volunteered. As I was also the teacher, it was important that students

felt that they were not forced to participate. This study has adopted two specific interview styles. The first type was interviews that were aimed at finding out what students thought about their life at university, their learning and the teaching specifically about my teaching. The second type was more specific about their mathematical learning and might include some problem solving sessions. Both types helped in obtaining feedback on several aspects that I was concerned with such as students' views about their learning styles or difficulties, comments about the classroom practice, their ability to communicate mathematically and how they solve problems.

For the first type, the interviews were unstructured and non-directive in the mode of 'informant' interviews (Powney & Watts, 1987). The interviews were conducted in a conversational style and the students were given some freedom to impose on its structure though I had determined a general agenda. In the second type, the interviews were semi-formal and were focused on the mathematics that students have learnt and they were asked to solve some problems.

3.2.3.2 Conversations

Although some researchers regarded informal interviews as conversations or discussions (Woods, 1986), I have decided to differentiate the two forms of data collection. Even in the most informal interviews, some prior arrangement was made to set the occasion. However, conversations would usually be started up more naturally and would not be limited to any specific time, place or topics. There were several different forms of conversation or discussions that occurred:

- Group conversations or discussions during group work where I was a passive listener.
- Discussions where I might join in but keeping to the flow without steering the discussion.

- Conversations that were started because I had some questions or was seeking some explanations or information.
- Conversations initiated by the students themselves, within the classroom or at other times.

In most cases, the discussions were about the mathematics but as the semester progressed I was getting more cases of students coming up to talk to me about their personal problems which were sometimes about their financial, emotional, family or related to their studies. I will be reporting in relevant sections about how by becoming more interested in seeking out students' views, students became more encouraged to share their views as well as their problems.

The drawback in these sessions was in taking down notes as in so doing, it would disrupt the situation thus any notes were usually made after the events although I tried to write them down as soon as I could.

3.2.3.3 Questionnaires

During the course of the research, students were asked to evaluate the course and the lecturer. The students were given two questionnaires, (1) the course evaluation questionnaire given out by the University and (2) a course evaluation questionnaire that I prepared. The University's questionnaire is regularly given out to all students at the end of the semester and the students were to evaluate the lecturer's presentation and course materials. Some questions were also asked about the lecturer's relationship with the students. In the questionnaire I prepared, questions were more specific and were about the strategies that were used. Students were also asked to evaluate the progress of their learning.

3.3 The Research is Concerned with Students' Development in the Learning of Mathematics

The purpose of the study was to find out if implementing teaching designed to encourage students' use of their own mathematical thinking would effect changes in their attitudes and learning behaviour. Data was also collected that would display the students' facility with the mathematical concepts and techniques they were learning. Supportive data were collected using the following means:

- **Diagnostic test** – a diagnostic test was administered at the beginning of the study to the first group, A01. The test was designed to identify students' conceptual understanding or lack of it on pre-calculus topics. However, in subsequent semesters, I was working with students studying Advanced Calculus. I did not create any diagnostic tests for them but designed a simple problem-solving sheet on basic concepts in Basic Calculus.
- **Problem solving questionnaire** – the questionnaire consisted of 7 questions on differentiation and integration which the students would have studied at school and again in the first semester at university. This was given to students in groups D05 and E05.
- **Students' class worksheets** – consisted of questions specially structured to help students recognise mathematical thinking powers
- **Students' examples** – a collection of students' work that might show how they work on problems and help in the evaluation of the effect of the teaching acts.
- **Observations and conversations** – with students as they work on the mathematical problems.

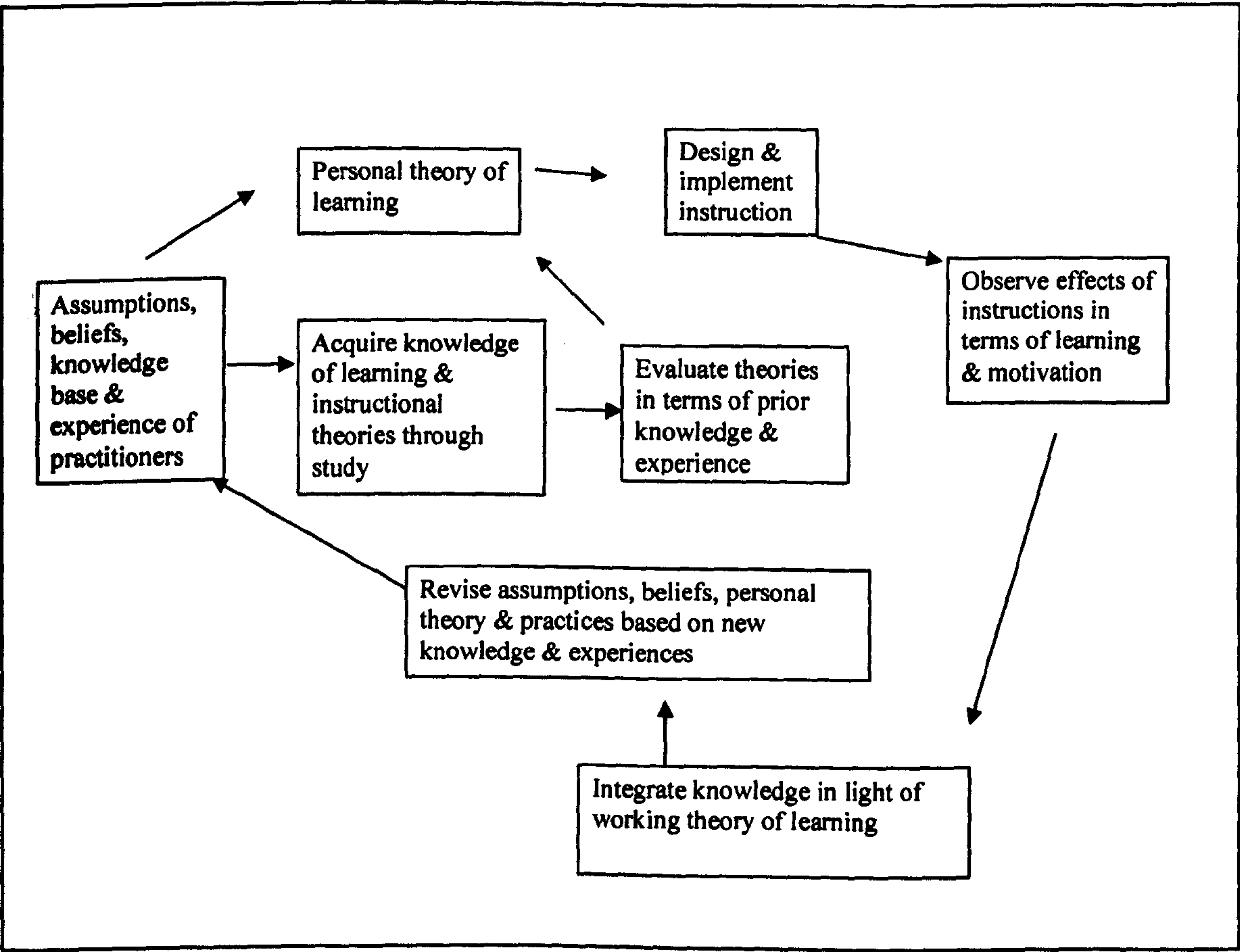
3.3.1 Development of Strategies and Teaching Materials

I had focused on finding teaching strategies that would support students' use of their own mathematical thinking powers as a means to increase students' understanding and improve

their attitudes in the learning of calculus. I had based my conclusions on past experience, documented research findings as well as evaluation made by some of my colleagues in the Department of Mathematics and Faculties of engineering at the university. I had used several theories and framework based on the works of various authors (Mason, Burton & Stacey, 1982; Skemp, 1987; Gray & Tall, 1994; Watson & Mason, 1998; Mason & Johnston-Wilder, 2004). The guidelines that were used to prepare the teaching strategies was presented in Section 2.5 and a more detailed explanation of the development of the teaching strategies will be given in Section 4.5.2.

However, the following model gives a succinct representation of the process of preparing the teaching strategies and their implementation. The model was adapted form Driscoll (1994).

Figure 3.1: Process of Designing and Implementing Teaching Strategies



The model depicts different phases that occurred during the planning, designing and implementation stages of the teaching strategies. However, I was researching my own practice thus this cycle of activities were within the cycles of the research process.

3.3.2 Designing Tasks to Invoke Students' Use of their Own Powers

One of the main objectives of the teaching acts implemented was to shift students' awareness gradually from rote learning towards understanding the procedures and recognising their own mathematical powers. The basic notions underlying the teaching acts were the explicit use of mathematical themes to provide linkages between mathematical ideas and to expose the structures of the mathematics. Some of the themes used were, *invariance amidst change*, which form the basis for many mathematical theorems and technique, *doing and undoing*, which can help students to identify features or structures that should be the focus of attention. Suitable prompts and questions were employed to engage students in, and to assess their grasp of mathematical ideas and techniques. The appropriate prompts and questions were designed by taking into account the mathematics that students were to learn and by considering how to bring about their awareness of the themes and powers within the topics. These prompts and questions was focused and direct in the beginning until the students became aware of the type of questions asked. As students became more aware of their powers, the prompts and questions were more indirect. This support served as a scaffold (Wood & Ross, 1976; Brown & Duguid, 1989), which encouraged students to become more aware of their own mathematical thinking powers and to develop learning independency, and which I intended to fade away as they took initiative for themselves.

My approach was to use examples structured in a manner that would lead towards a generality. Students had to work on typical examples and generic examples leading towards more general examples. To further strengthen their understanding and knowledge, non-typical examples were also given. Another teaching act that was used was to ask

students to make up their own examples. I used students’ own examples to assess what they were attending to in the topics taught.

I shall now illustrate the possible mathematical activities in finding the domain of Multivariable Functions. Examples were chosen and organised so that students became aware of the properties of the functions to be considered in constructing the suitable solution methods.

Firstly, a typical problem set in most textbooks is given.

Example 2: Given $z = 1 - x^2 - y^2$. Find the domain and range of the function. Sketch the graph of the domain

In my class, the same problem was modified and structured as follows.

<p>Example 2 (a): Given $z = 1 - x^2 - y^2$</p> <p>i. Evaluate $f(2, 1), f(-4, 3), f(0, -5)$ and $f(u, v)$.</p> <p>ii. Find the domain and range.</p> <p>iii. Sketch the domain of f</p>	<p>Questions and Prompts:</p> <ul style="list-style-type: none"> • Which pairs of variables are the input variables? • Which variable is the output variable? • Is there any restriction on the input variables for which the function is defined? • How do you represent the set of all inputs graphically?
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Table 3.2 (a): Example 2(a) - Finding domain, range and sketching a graph.

For this problem, I had identified the following themes and powers for students to focus on.

<p>Theme: Invariance amidst Change</p> <p>Sub-theme: Range of Change</p> <p>Activities: Specialising and Generalising, Characterising, Expressing</p>	
<p>Problem: Finding the domain of a function</p>	<p>Focus of Attention: property of function, values of domain and range, graph of function</p>
<p>Example 2 (a): Given $z = 1 - x^2 - y^2$</p> <p>i. Evaluate $f(2, 1), f(-4, 3), f(0, -5)$ and $f(u, v)$.</p> <p>ii. Find the domain and range.</p> <p>iii. Sketch the domain of f.</p>	<p>The <i>Questions and Prompts</i> were to direct students’ attention to the roles of the independent and dependent variables as well as to the property of the function, z.</p>

Table 3.2 (b): Themes, powers and mathematical activities of Example 2(a)

Example 2(a) is followed by two more examples, Example 2(b) and 2(c). In Example 2(b), only one aspect of the function was changed, by introducing a square root whilst in Example 2(c), the function in 2(b) was inversed. The objective of the questions is to draw students' attention to the importance of investigating the properties of functions in determining its domain. In this example, students would revise the procedure of finding the domain of a square root function and of an inverse function. In this way, the examples were directing students to look at various properties of functions as well as at the dimensions of possible variations.

Sub-theme: Range of Change	
Activities: Specialising and Generalising, Characterising, Expressing	
Problem: Finding the domain of a square root function	Focus of Attention: property of function, values of domain and range, graph of function
Example 2(b): Given $z = \sqrt{1 - x^2 - y^2}$ i) Describe and sketch the domain. ii) Determine the range. iii) Write down at least three possible values of $f(x, y)$.	Questions and Prompts: Compare 2(a) and 2(b). <ul style="list-style-type: none">• What remains the same?• What has changed?• What condition is necessary for the function to be defined?• How do you represent the set of all inputs graphically?
Example 2(c): Given $z = \frac{1}{\sqrt{1 - x^2 - y^2}}$ i) Describe and sketch the domain. ii) Determine the range. iii) Write down at least three possible values of $f(x, y)$.	Questions and Prompts: <ul style="list-style-type: none">• What condition is necessary for the function to be defined?• How does the condition affect the input variables? Output variable?• How do you determine the set of input variables? Output variable?• Compare (a), (b) and (c).<ul style="list-style-type: none">○ What is the same?○ What is different?

Table 3.2 (c): Invariance Amidst Change

I believe that the mathematics lessons should also provide opportunities for students to generalise as the ability to generalise would indicate that students could sense the underlying pattern even if they were not able to fully articulate it yet. Some initial assistance to ‘particularise’ were given to the students to help them see the ‘general in the particular’ and also to see the ‘particular in the general’. In the following examples, I used ‘specialising’ and ‘generalising’, so as to make explicit the structures and processes of mathematics which in turn will facilitate students’ move towards more advanced thinking, in particular, to ‘conjecture and justify’ as a natural extension.

Sub-theme: Range of Change Activities: Specialising and Generalising, Characterising, Expressing	Prompts and questions
Example 3: Let $f(x,y) = \sqrt{4-x^2-y^2}$. (i) Find the domain and range of f. (ii) Sketch the graph of the domain	Compare Examples 2(a) and 3. <ul style="list-style-type: none"> • What remains the same? • What has changed? • What was the property of $f(x,y)$ which required the condition $4-x^2-y^2 \geq 0$? • What information in Eg. 2(a) did you use to solve Eg. 3?
(iii) Could you give one example that is like Egs. 2(a) or 3? (iv) Please give another example? (v) Can you give a general example?	

Table 3.2 (d): Specialising and Generalising

My next example will illustrate activities to help students identify the important features in a technique, to sort and organise their information (See Table 3.3). To encourage mathematical communication, students were asked to work in pairs or in small groups. Working in this manner has helped to lessen their anxieties and provide a non-intimidating environment for students to communicate their mathematical ideas verbally.

Theme: Doing and Undoing Activities: Seeing the particular in the general, Imagining	
Problem: Finding partial derivatives	Focus of Attention: Important features in the technique of partial differentiation.
Doing: Given $z = x^2 + 2y^3 - 2x^4y$ find f_x . Now, find f_y .	Questions and Prompts: To find f_x , which variables were kept constant? To find f_y , which variables were kept constant?
Undoing: Given $f_x = 2x - 8x^3y$ and $f_y = 6y^2 - 2x^4$, how would you find $z = f(x, y)$?	Can you describe what you did to find f_x and f_y ? How can this help you to find $z = f(x, y)$?

Table 3.3: Doing and Undoing

My last example (see Table 3.4 (a) and (b)) will illustrate how students learn to make connections between technical terms, mathematical ideas and procedures. My experience has shown that there are students who have the knowledge but could not bring it forth during problem solving. These are typical instances of students who have learned their mathematics in separate pieces for which they could not construct the total picture for themselves (Tall & Razali, 1993). The examples help to encourage students to build on their intuitive mathematical ideas, raise them to a level of awareness until they can draw upon these ideas and articulate them. In simpler terms, they need to be aware of what they know, how to connect what they know, when and how to use what they know. This is how the prompts and questions were being used: to focus students’ attention and to help them learn how to make inquiries.

Table 3.4 (a): Making Connections

Theme: Doing and Undoing Activities: Comparing, Sorting, Organising, Characterising	
Problem: Finding areas using double integrals	Focus of Attention: setting up the integral, determine order of integration, determine limits of integration and evaluation of integral

Doing: Find the area of the region D, in the first quadrant bounded by $y = 4 - x^2$, $y = 3x$ and $y = 0$.	Questions and Prompts: <ul style="list-style-type: none"> • What do you know about area? • Does the order of integration matter? • How do you determine the limits of integration? • Would evaluating the integral in polar coordinates be more efficient?
-------------------------------------------------------------------------------------------------------------------------	-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------

In the question above, the student is expected to recall the formula for area, set up the integral and sketch the region of integration. Based on their sketch, they have to determine the order and limits of integration and lastly, evaluate the integral. For this question, they have to bring forth their knowledge of graphs of curves, sketching techniques, integration techniques and knowledge of alternative coordinate systems.

Table 3.4 (b): Making Connections

Theme: Doing and Undoing Activities: Comparing , Sorting, Organising, Characterising	
Problem: Finding areas using double integrals	Focus of Attention: setting up the integral, determine order of integration, determine limits of integration and evaluation of integral
Undoing: The area of a region, D is given by $\int_0^4 \int_{x^2}^{4-x^2} dydx$. i) Evaluate the integral with the order of integration reversed. ii) Can you give me one other example of the same kind?	Prompts and Questions <ul style="list-style-type: none"> • Can you describe the region by giving equations of the curves that form its boundaries? • Can you sketch the region of integration? • How do you determine the new limits of integration? • How do you evaluate an iterated integral?

In solving the Undoing question, students will now have to draw upon their earlier experience, re-organise their knowledge of procedures and focus on what were the important features stressed in the solution methods.

The design of the tasks was aimed at making explicit the mathematical processes and helping students to become more aware of their own thinking.

3.4 Analysis of Data

In literature on qualitative data analysis, conducting analysis simultaneously with data collection was considered one of its most prominent characteristics (Becker, et al, 1961; Glaser and Strauss, 1967). Dick (2000) used the metaphor of a spiral to describe action research and that each turn contained data collection, data analysis, action planning, action and evaluation.

The research was carried out based on a cycle of activities, '*planning-implementation-review-modify*', and thus at every stage data was collected, analysed and contributed towards further action planning. Actions were then implemented, reviewed and evaluated. Thus data was interpreted as they were collected and turned into action plans which were then acted on. Results of the action were reviewed immediately and informed the next turn of the cycle. Many conclusions were captured during the actions. However, I felt that it was necessary to allocate time to review the whole process and this phase of further reflection, much in the nature of Griffith & Tann's level 3 reflection was a time to look at the cycles of research, the actions implemented, conclusions made based on analysis of data within the teaching period and identify any other relevant factors and issues. The courses in UTM follow the semester system and thus the break in between semesters was a natural period to undertake this overall reflection so as to plan for the next period of teaching.

Several analytical methods that were common to many qualitative research perspectives were also used in this study (Miles & Huberman, 1994). These were:

- 1) Field notes - observations were written up as field notes as soon as possible after the events.

- 2) Noting reflection and remarks – reflection occurred at several levels following the designation made by Griffiths & Tann (1992). During the class sessions, many reflections were typically at level 1 [*Act-react (Rapid reaction)*] and level 2 [*React-monitor-react/ rework-plan-act (Repair)*]. This meant that some reactions to events or incidents in the classroom were quite immediate and were usually noted during class or as soon as possible after it. It also meant that the reflections were more in the nature ‘*thinking on your feet*’ and lacked the additional benefit of time to peruse over the incidents to gain greater understanding or clarity. There were times during the teaching when I noticed something or thought of something as these remarks were duly noted.

From time to time, reflections at levels 3 [*Act-observe-analyse and evaluate-plan-act (Review)*] and 4 [*Act-observe systematically-analyse rigorously-evaluate-plan-act (Research)*] were undertaken to gain further insights as well build a more complete picture of the research process, the observations and actions that have been carried out. It was also necessary to monitor and review any of the ‘in-action’ conclusions that led to modifying of actions. At each level, self-reflection (reflecting on my feelings, my attitudes, and my behaviour) were noted separately.

- 3) Sorting – the field notes were regularly read to identify patterns, themes, similar phrases, relationships between variables, differences or similarities in students’ actions and attitudes as well as my own.
- 4) Isolating patterns and themes – commonalities and differences are categorised. Some actions might be planned as deliberate attempts to further clarify or investigate them in the next cycle of data collection.

- 5) Assigning codes –. Tentative codes were identified based categories or themes that were identified by sorting and isolating themes and issues.
- 6) Gradually building up a more formal explanation of the research findings, examining these against public theories, and offering my own contribution to the body of knowledge.

I was concentrating on the appropriateness of the teaching strategies used, students' learning behaviour and my own behaviour. Thus at any one time, the focus of my attention would be shifting from one or other of these concerns. Analysis carried out during the research would also influence and direct the focusing and refocusing of teaching and research acts that were carried out. In this chapter, I had presented the strengths of an action research perspective but in Chapters 4 and 5, I will present an evaluation of the method based on my experience as there were difficulties in conducting research on your own practice.

3.4.1 Illustration of data analysis

In Phase 1 of the research, I taught two subjects, Basic Calculus and Calculus II, to four groups of students, A01, B02, C02 and D04 (see Table 3.1(a), p.60). In my observations, I tried to describe events as they were happening in class. However, certain themes were already identified based on findings from the literature review particularly from those that were conducted on UTM students as well as questions that guided the research. Some of these were:

- i) students' mathematical knowledge development: understanding of prior knowledge, basic concepts and current topics;
- ii) students' mathematical skills: algebraic manipulations, recall of previously taught techniques of differentiation and integration;
- iii) students' learning behaviour: participation in class in terms of asking questions, working on exercises; ways of working whether individually or in group, work on

tutorial questions or extra exercises, read up notes or text book; memorisation; drill and practise

iv) my teaching: teaching acts, coping skills and implementation of the teaching strategies

At the beginning, students' actions, my actions and thoughts were all written in the observation notes. Some notes were also added in after the class. It was not easy trying to teach, observe and make notes during the class sessions. Writing up the field notes at the end of each week helped in building up a more reasonable picture. These were read through and the themes were identified by simply marking them off the transcripts. Although the exercise was primarily guided by looking for themes already identified, other possible themes were also looked for. For the first group (Basic Calculus, A01) in Phase 1, the new teaching strategies were carried out only for about 4 weeks which consisted of 3 hours per week contact time. A simple analysis was carried out at the end of each week and I only focused on the more outstanding issues (see Section 4.5.4). However, the first foray helped to identify various shortcomings in my methodology that was adjusted for the next group (Calculus II, B02).

Illustration 1: From excerpts (Section 4.5.1, Group A01):

Week 3: Second session, 20/11/01 Sub-topics – Product, Quotient rules and the Chain rule	Comments/ Issues/ Themes
<p>I started the class with some examples of slightly 'harder looking' functions to differentiate. Each of the examples could be easily differentiated if simplified first. In the previous class, some students did not check to see whether functions could be simplified first. The product and quotient rules were used on expressions which conformed to the standard form which is, $y = uv$ and $y = \frac{u}{v}$.</p> <p>I felt that I would like to spend some time to draw the students' attention to some algebraic manipulations in simplifying expressions. I gave the following examples and asked students to solve the questions. I had hoped that it would trigger students to look and think about the expressions before using the rules.</p> <p>Example</p>	<p>Changes made based on students' response in previous class</p> <p>Addressing students' difficulties – algebraic manipulations</p>

Find the derivative of $y = \frac{(x-1)(x^2-2x)}{x^4}$.

As before, they were still students who tried to apply the quotient rule directly but found the numerator difficult as it had two factors. They felt that this was a hard question. I asked them to see if they could make it an easier question to solve.

There was some discussion in the class. However, some students decided to consult other students who had solved the problem or whom they thought could solve the problem.

I saw one student applying the quotient rule on the expression and then applying the product rule on the numerator. It was lengthy and his final solution was not simplified. However, the working was essentially correct.

I put up two more examples for the students to solve as below:

Example

Find the derivatives of the following functions:

a) $y = (x^2 + 1)\left(x + 5 + \frac{1}{x}\right)$

b) $r = (1 - t)(1 + t^2)^{-1}$

I allowed the students to work in their groups or by themselves for about ten minutes. Then, I put up the solution for the first example on the board

$$y = \frac{(x-1)(x^2-2x)}{x^4}$$
$$= \frac{x^3 - 2x^2 - x^2 + 2x}{x^4} = x^{-1} - 3x^{-2} + 2x^{-3}$$

$$\frac{dy}{dx} = -x^{-2} - 3(-2)x^{-3} + 2(-3)x^{-4} = -x^{-2} + 6x^{-3} - 6x^{-4}$$

Some students looked at the board but others were still working away. A student asked how to simplify the second example, $r = (1 - t)(1 + t^2)^{-1}$.

I showed her, $r = \frac{(1-t)}{(1+t^2)}$ but writing on the board for the whole class

to see. More students were now looking at the board. A couple of students asked, “when to simplify an expression and how to simplify an expression?” I said that they were good questions but perhaps they should discuss with their group members first.

We had 2- hours’ work on the rules of differentiation and since there was a little bit of time left, I asked students to give two examples of their own, one of each rule, of differentiation using the product and quotient rules. They did not have to solve their questions but they were to give one ‘easy’ problem and one ‘hard’ problem. During the class, 17 students handed in their work while the others chose to take home their work to be handed in later.

As I had asked for some work to be handed in, the students immediately focused on the new task and the

What are hard questions to students?

Peer group support

Procedural

“on-the-spot” decisions

Engagement with the tasks

Students’ own examples – what was their focus of attention?

Value of work = prompt response

<p>discussion on 'simplifying expressions' was abandoned. I felt that I had botched a good opportunity to explore students' skills in using algebraic techniques. I was focused on getting on with the syllabus and only realized the missed opportunity as I saw students abandoning the discussion for the new task.</p>	<p>– possible code: REWARDS</p>
<p>When they were asked to hand in their own examples, the first thing that happened was that some students requested the instruction to be repeated as they were not sure what was wanted. They had never before been asked to do anything like this and found the request surprising and were not sure how to respond.</p> <p>From the questions that they asked me, they were not sure '<i>what I wanted and meant</i>' by 'easy' and 'hard' questions.</p>	<p>Teaching opportunity vs finishing the syllabus - TENSIONS in teaching</p> <p>Teacher's approval??</p>
<p>From their general reactions, I could not ascertain whether they were taking the request seriously. There was a lot of "giggling" as they were talking to each other, looking at each other's work, while they were doing the examples. From personal observations and reflections on Malaysian students' behaviour, "giggling" had always been a 'defensive' mechanism for students to cope with various situations, pleasant or unpleasant.</p>	<p>Students' behaviour - interpretation?? To check with colleagues</p>
<p>In their written work, many students who handed in their work chose polynomial functions as examples with some choosing very simple functions. Thus, the 'easy' questions were too easy and their hard questions were not much harder. However a few also gave examples of other algebraic functions.</p>	<p>Students' choice, simple functions</p>
<p>I could also see that the new students who had just joined the class did not participate in the discussion.</p>	<p>Non-participation – why? how to motivate?</p>
<p>Up to this moment, I had been pleasantly surprised at the willingness of many of the students to contribute and participate in the class discussions.</p>	<p>Participation – why? "I am pleased" – why?</p>
<p>In UTM, students had to sign an attendance sheet for every class but during the last session, I had made a roll call to check their attendance so that I could identify the students who had not been participating. A few were from the first year and the others were mainly the repeat students. Now, in this class the new students had not participated as well.</p>	<p>Non-participation</p>
<p>Some students had to miss one of the three sessions, thus I had to designate that session as a tutorial so that they would not miss the lectures. They were asked to find a different time slot for tutorials or that they were to see me personally for tutorials.</p>	<p>Organisational problem</p>

In this excerpt, particular themes could be seen emerging and they were supported in subsequent data collected, for example, the themes, ‘seeking teacher’s approval’ and the need for ‘rewards’. In the next illustration, events in Group B02, showed similar themes appearing.

Illustration 2: B02 (see Section 4.6.2)

Week 1 – second session, 02/07/02, 4.00 – 4.50 pm	Themes/ patterns
<p>I gave a third example, $y = 1 - x^2$, the students who had responded earlier were quick with their response without any prompts. However, there were still a small number of students who did not participate. Then, I asked the students to discuss the examples using the prompts and questions: “<i>what is the same?</i>” and “<i>what is different?</i>” and “<i>what can change?</i>”</p> <p>There was a mixed response from the students. Students who knew me from Basic Calculus appeared to know what I meant and started discussing with students sitting next to them.</p> <p>Some of the new students were not sure of how to respond – they asked “<i>what do you want?</i>” or “<i>what do you mean?</i>”</p> <p>When they were invited to interpret the questions as they wished – they were not happy and a couple of students immediately inquired if they will be evaluated on this work. One of them said: “<i>have to work smart</i>” – which meant (upon enquiry) – “<i>do just enough of what is required to get the marks</i>”.</p>	<p>Students’ use of prompts and questions</p> <p>Teachers’ approval</p> <p>Rewards</p>

On the issue of ‘the need for teacher’s approval’, I had taken immediate action in terms of changing a particular aspect of the teaching, by providing worksheets for students to work on (see Section 4.6.2). The worksheets provided students with questions they could worked on thus appeasing their need for ‘teacher sanctioned work’ and helped to make the prompts and questions visible. I also collected some of the worksheets for assessment and grading purpose which satisfied their need for ‘rewards’.

At the end of the course, students’ comments in an evaluation questionnaire highlighted another theme, ‘rapport’. I was described as ‘*understanding*’, ‘*approachable*’ and ‘*helpful*’. Rapport was a significant concern for students as in the case of group D04 (see Section 4.8). I was asked by the group specifically because they thought that I could help them and that I had taught many of the students before.

In the case of Lily (see Section 6.3.3.3), a spur of the moment decision on my part to talk to her about her problems became an opening of an opportunity to support Lily's commitment to work on the mathematics and enough motivation to help her through a difficult time when she felt overwhelmed with the amount of revision she needed to do Calculus II. Thus, rapport was significant in her case as well as 'mediation' and negotiation' to encourage Lily's engagement with the mathematical tasks. Of course, Lily was already very concerned about her lack of performance and was intending to work hard but she was not sure where to start her work.

In Phases 2 and 3 of the research, I was working with students who had different entry qualifications and much more competent with their prior mathematics (see Section 5.1). The data were collected using various methods (see Sections 5.2.1 and 5.4.1). Issues that were considered relevant from Phase 1 were still monitored. In particular, students in Phase 3 were working consistently on the workbook thus they were involved in working on their mathematical thinking for all the topics. Even so, many students identified '*an approachable teacher*' as very desirable and motivated them to do more work (see Section 5.4.6.1). Other themes pertinent to supporting students' change in attitudes were also identified and culminated in the model of change suggested in Section 6.4.

The analysis of data follows the typical steps of writing up field notes, sorting, searching for themes or patterns, assigning codes and finally building up explanations of the research findings. In this particular research, undertaken over a period of seven years, some of the themes were also inputs into the '*planning, implementation, review*' cycle of the teaching itself.

3.5 Reliability, Validity and Generalisability

Criticisms against qualitative research focus on its subjectivity as a source of bias in the data reported or in any accounts produced thus I have to confront the issues of reliability, validity and generalisability of action research. What are the criteria for assessing action

research? Researchers take different positions in discussing these issues. Some researchers suggested that these concepts are irrelevant in a qualitative study (Stenbacka, 2001), whilst some offered different criteria deemed more suitable such as consistency or dependability and applicability or transferability (Lincoln & Guba, 1985). On the other hand, Dick (2000) maintained reliability and validity are still useful concepts in determining that the information collected is trustworthy and that the description given is based on some degree of objectivity.

In this study, I am using the ideas that are offered by Herr and Anderson (2005) which linked the goals of action research to their validity criteria. Their criteria were developed from their experience in insider action research studies. Table 3.2 shows the link between goals of action research, the criteria of validity and some comments that describes the meaning of the terms. The table was modified from the original table presented in Herr and Anderson (2005, p 55) meanwhile the comments are summaries of the explanations given.

Table 3.5: Validity criteria in action research

	Goals of action research	Validity criteria	Comments
1	The generation of new knowledge	Dialogic and process validity	The determination of the ‘goodness’ of research through peer review.
2	The achievement of action-oriented outcomes	Outcome validity	The extent objectives of study were met and the problems resolved.
3	The education of both researchers and participants	Catalytic validity	The understanding of all who were involved in the research was increased and they were moved to some action of change.
4	Results that are relevant to the local setting	Democratic validity	The accounting of multiple perspectives and interests.
5	A sound an appropriate research methodology	Process validity	The inclusion of multiple perspectives and the determination of what constituted as suitable evidence of the study’s assertions.

These criteria will allow the methodology and the results of this study to be evaluated to test for trustworthiness and relevance. Various methods to support reliability and validity were used and briefly described below.

- (a) The research methods implemented have been clearly described at the beginning of the research, they are reviewed regularly and discussions on the modifications are also presented. As the study is described in two parts within Chapters 4 and 5, with different classes being taught thus the review of methods is included in both chapters.
- (b) Triangulation – multiple strategies were used to collect data so that they can be checked and compared.
- (c) Reflexivity – I have attempted to address and identify my subjectivity by including my reflections as part of the cycles of research process. Assumptions and decisions made during the study are clearly identified as well as the basis for selecting informants, giving their description and the context of the setting chosen.
- (d) Peer validation – Peer reviews from other lecturers who have experience of teaching the mathematics subjects and similar students was sought so as to balance my subjectivity.

Could the results obtained be generalised to other situations? The research objectives of this study have excluded the idea of generalisation in the perspective of applying the research to other contexts. It is a study that seeks to describe a teaching and learning situation, enhanced the understanding of such a situation and also seeks to improve teaching practice. However, Mason (2002 (b)) put forward the idea of robustness in his Discipline of Noticing that can be connected to generalisability. Robustness refers to “*the variety of conditions under which sensitivities inform action effectively, and the variety of people who recognise and resonate with proposed distinctions*” (Mason, 2002 (b), p 188). He contends that an import consideration for other researchers is how to identify the

aspects that can inform future actions. Using such an idea of generalisability, I believe that there are certain aspects of this research that can be used in other teaching and learning research, for example, the teaching strategies described, the manner of how relationships with the students were developed and used to negotiate learning.

3.6 Overview of the Research Process

This section will present an overview of the research process and implementation. The research was conducted in three different phases where each subsequent phase was determined by changes to the students’ entry requirements, the curriculum as well as due to the review of the previous phase. An introduction of the different phases will be given and this will be followed by a general explanation and justifications of the decisions and changes made during each phase.

Introduction

The first phase of the study covered a general period from November 2001 to November 2004. I studied four groups of students from the Industrial Design Course (SRI) from the Faculty of Mechanical Engineering, which I had called Block 1. The groups are identified below, see Table 3.1(a). I taught two groups of students Basic Calculus and the other two groups, Calculus II.

Table 3.1(a): Block 1 – Groups of Students taught in the First Phase of Study

Group Name	Course/code	Subject Code	Faculty	Period	Subject
A01	Industrial Design (1 SRI)	SSM 1242	Mechanical Engineering	semester 2, 2001/02	Basic Calculus
B02	Industrial Design (2 SRI)	SSM 2242	Mechanical Engineering	semester 1, 2002/03	Calculus II
C02	Industrial Design (1 SRI)	SSM 1242	Mechanical Engineering	semester 2, 2002/03	Basic Calculus
D04	Industrial Design (2 SRI)	SSM 2242	Mechanical Engineering	semester 1, 2004/05	Calculus II

The second phase consisted of a study of two groups of students whom I taught Engineering Mathematics in their second year and in the second semester of 2005/06. These were the Block 2 students; see Table 3.1 (b).

Table 3.1(b): Block 2 – Groups of Students taught in the Second Phase of Study

Group Name	Course/code	Subject code	Faculty	Year/ Period	Subject
E05	Electrical Engineering (2 SEM)	SSE 1893	Electrical Engineering	Second year, semester 1 2005/06	Engineering Mathematics
F05	Civil Engineering (2 SAW)	SSE 1893	Civil Engineering	Second year, semester 1 2005/06	Engineering Mathematics

The third phase consisted of two groups of students taught by my colleagues, Dr. Zee and Dr. Tee, respectively. They were also studying Engineering Mathematics and were from Faculty of Electrical Engineering in the first semester of the academic session 2007/2008. These were students taught by my colleagues but using the same teaching strategies that I had developed. In this phase, I had made arrangements to observe the classes and conducted some interviews with willing students. Block 3 students are identified in Table 3.1 (c) below.

Table 3.1(c): Block 3 – Groups of Students Observed in the Third Phase of Study

Group Name	Course/code	Subject code	Faculty	Year/ Period	Lecturer
G07	Electrical Engineering (2 SEM)	SSE 1893	Electrical Engineering	Second year, semester 1 2007/08	Dr. Tee
H07	Electrical Electronic Engineering (2 SEE)	SSE 1893	Electrical Engineering	Second year, semester 1 2007/08	Dr. Zee

Details of Block 1 students will be presented in Chapter 4. Meanwhile, details of students in Block 2 and 3 will be given in Chapter 5.

Phase 1 – Students in Block 1, Nov 2001 – Nov 2004 (see Chapter 4)

Students in Block 1 had entered UTM before July 2002 and thus had entrance qualifications based on the Sijil Pelajaran Malaysia (SPM, *Malaysian Certificate of Education*) which are equivalent to the GCE O-Level qualifications. They were also taught in Bahasa Malaysia (*Malay Language*), the official medium of communication of UTM. However, UTM made very significant changes to various policies, such as, entry qualifications, medium of communication and future direction of the university in terms of staff development and research. In terms of changes that had a direct impact on students, such as the entry qualifications, all students who came to the university in July 2002 henceforth, had to have post-SPM qualifications such as Matriculation qualifications, Sijil Tinggi Pelajaran Malaysia (STPM, *Higher School Certificate of Education*), Diplomas from local colleges or other qualifications recognised as of similar standing. These qualifications are equivalent to the GCE A-Levels. The other significant change was that English Language became the medium of communication. These changes also brought about changes to the curriculum which were modified accordingly. Thus, in terms of the research process, I was engaged in studying my own practice on a group of students who had suddenly become ‘*atypical*’ and the last batch of an older system.

At the beginning of the research, the intention was to look at two groups of students, the first group would be taking Basic Calculus (SSM 1242) and then following them into the following semester in Calculus II (SSM 2242). I was aware that there were some differences in SRI Mathematics curriculum as compared to the mainstream Engineering courses. In particular, the SRI students will be doing a lighter mathematical load as they had a total of 6 credit hours of mathematics and statistics as compared to 14 credit hours for the mainstream engineering students. However, I had chosen SRI students based on

practical considerations in terms of the autonomy in planning teaching and assessment activities. Their subject code was different from the other Engineering students and thus they will have to sit for their own examination paper as well as the fact that there was only one section to be taught. In the first phase, I worked with students in Basic Calculus and Advanced Calculus with the main intention of allowing students to work on the new strategies for two semesters. I had hoped that I could observe students' use of the thinking strategies. However, the flexibility of the semester system meant that I always had a group of new students in my class. Thus, the strategies were familiar for some but not all of the students. The situation is described in Chapter 4.

Ongoing analysis of the data collected had contributed to changes made to teaching materials and tasks, however the basic ideas were kept intact but the tasks and delivery was modified and adapted. Some particular examples, was the use of worksheets which then became a workbook. When 'rapport' was identified as a significant factor by students, some personal connection and negotiation helped to support some of the students' efforts at making changes in the way they worked with the mathematics.

Significant changes in UTM's policies in terms of intake qualifications, desired graduate attributes and the curriculum meant that I had to study new groups of students which became Phase 2 of the research (Section 5.3). Experiences in Phase 1 showed that I had to focus and be more specific in my research. Thus, I had studied only 2 groups of students studying Engineering Mathematics (similar to Advanced Calculus). I had to review all the assumptions about students' difficulties and beliefs, and planned for better data collection and some validation of the strategies that I used. In phase 2, I had two colleagues to monitor and checked my teaching strategies, the tasks design as well as the progress of the research. However, a particular theme that was identified in both phases, 'rapport' which meant in this instance, 'students' perceived view of teacher's friendliness' raised the issue of whether the teaching strategies could be used by others. Furthermore, the teaching was

only carried out for some topics meant that the students and I were not working consistently based on the methods.

Consequently, I added Phase 3 to the research. Before Phase 3 was carried out, the worksheets that I produced were developed to become a workbook which would cover all the topics in the syllabus of Engineering Mathematics. Two colleagues worked with me to develop the book and finally they used it in their teaching in the first semester of 2007/08 (see Section 5.4). In this final phase, I became a complete observer. This gave me the opportunity to be more objective as well as evaluate the viability of the teaching strategies being used by others.

I had chosen an action research methodology thus the data collection and analysis at every semester and every phase were inputs into the next teaching sessions. My research truly helped me to modify, adapt and improve my teaching as well as contributed to the findings that I had finally reported in this thesis.

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CHAPTER 4

STUDY OF THE IMPLEMENTATION – PART I

November 2001 to November 2004

4.1 Introduction

Mohd. Yusof (1995, Section 2.2) research findings on students' and lecturers' attitudes towards problem solving struck a familiar chord in me as I had felt the same way about my students. I would like them to be able to become better thinkers and problem solvers but I am struck by the dilemma of how to present the syllabus within the time prescribed, ensure that they can do the problems to be able to pass the examinations as well as understanding the materials taught so that they can use the mathematics in their own specialist area. I was genuinely concerned and felt that there was a need to improve students' understanding of the mathematics that they have learnt. Mere instrumental memorization of techniques does not equip students to use mathematics effectively in their own discipline. Most of the past efforts in changing teaching practice had concentrated on remedial programmes to strengthen perceived deficiencies in students' grasp of basic mathematical concepts, particularly, those that they should have understood at school level. For the most part, understanding of mathematics was simply measured as the ability of students to perform well in examinations. However, in discussions with colleagues from the Faculties of Civil, Mechanical and Electrical Engineering, they claimed that students still had difficulties in using mathematics that they have learnt despite examination success, in recognizing relevant mathematics when found in other subjects, in translating real world problems into mathematical formulation, as well as solving these problems mathematically. Thus, these issues should be addressed so as to develop meaningful mathematical learning.

In view of the students' difficulties described earlier and the existing method of teaching (see Section 2.3 and Section 4.2.1), I felt that it was important to look out for ways to improve students' facility with mathematical ideas and to enhance their mathematical thinking. I also felt that suggestions for improving the teaching and learning situations should take into account my immediate concerns as well as those of the students. For the students, their main concern is to successfully finish the course and thus their main focus is on passing the examinations. This particular attitude is representative of the general culture of learning at school. Achieving good results were the main objective in learning. My own concerns were about changing my teaching practice, finishing the syllabus, ensuring that my students could perform well in the examinations as well as appreciate the knowledge that they were learning. I was also concerned about enhancing students' awareness about their ability to think and supporting the students in developing positive attitudes towards problem solving. Findings from previous research (Tall & Razali, 1993; Mohd. Yusof, 1995) have suggested that it was possible to intervene and support students to develop more positive attitudes towards mathematics. In Mohd. Yusof's work (1995), special classes to teach problem solving were held. However, further encouragement or support in their normal learning environment is necessary to maintain the changes in attitudes.

I believed that it would be more efficient for students to understand the mathematics taught rather than be trained to solve problems without real awareness. In addition, to be able to respond flexibly and creatively to new situations, the students themselves must be able to link and make connections between different mathematical ideas and reconstruct those ideas for themselves (Mason, 1999; Watson & Mason, 1998). The ability to apply only standard techniques to standard problems would not equip them adequately to face real life and non-standard problem situations in their discipline. Thus, to incorporate strategies to invoke students' mathematical thinking powers in the routine teaching would be a reasonable option.

In promoting these ideas, I contend that it is the lecturers' responsibility to provide the necessary environment to ensure learning takes place. They must learn to appreciate how their students would be struggling with the mathematical ideas, especially with topics, which for students would be new and more complex mathematics. They should realize that students needed help in order to undertake for themselves the mental actions which experts found intuitive and natural. The students would need support when working and struggling with these mathematical ideas. The techniques to encourage students' mathematical thoughts were in the main, based on the ability of the lecturers to think aloud and share their thinking with their students. I am aware that it would be difficult at first to change my teaching methods but not impossible.

How do I change my teaching methods and classroom tasks and activities to support my students to develop mathematical thinking skills? This question underpins my research considerations. The proposition is that by helping students to discover their mathematical thinking abilities, they will learn to appreciate and understand the mathematics that they have studied and be able to use the mathematical concepts and procedures in their own engineering subjects.

I chose to research my own practice, working with several groups of engineering students. It covered a general period from November 2001 to November 2004. The students were taught based on the semester system with a date in June or July as the beginning of the first semester of the academic session. Thus the academic session would be referred to as 2001/2002, which would mean the period from July 2001 to March or April 2002. In this chapter, I will present accounts of four different groups that I taught. These groups were in the first block of students (Block 1) to be considered. They were:

Table 4.1: Block 1 students:

Group Name	Course/code	Subject Code	Faculty	Period	Subject
A01	Industrial Design (1 SRI)	SSM 1242	Mechanical Engineering	semester 2, 2001/02	Basic Calculus
B02	Industrial Design (2 SRI)	SSM 2242	Mechanical Engineering	semester 1, 2002/03	Calculus II
C02	Industrial Design (1 SRI)	SSM 1242	Mechanical Engineering	semester 2, 2002/03	Basic Calculus
D04	Industrial Design (2 SRI)	SSM 2242	Mechanical Engineering	semester 1, 2004/05	Calculus II

These four groups of students had entered UTM before July 2002 and they were from the Industrial Design course in the Faculty of Mechanical Engineering. I taught two groups Basic Calculus in the second semester of the academic sessions 2001/2002 and 2002/2003. I taught the other two groups Calculus II in the first semester of the academic sessions, 2002/2003 and 2004/2005, respectively. The significance of the date ‘July 2002’ will be explained in Section 4.3 below.

From the outset, I wanted to focus on students’ learning but found that certain external factors such as, policy and curriculum changes also had a strong influence on my work. Thus, some discussion of these factors will also be given.

4.2 Changing my Teaching

Mohd. Yusof & Tall (1994) described a problem-solving course, which she conducted in UTM with the aim of providing students with opportunities to explore problems and reflect upon their actions. In each week during the semester, students had a 2-hour session to work on problems and a 1-hour seminar for reflection and sharing of ideas. Students were given problems to work on in groups and were encouraged to experience all aspects of mathematical thinking She found that, although students were resistant at first to the new work environment, they gradually became more appreciative and also displayed positive attitude changes towards problem-solving in mathematics. Their findings gave me great

motivation to consider that effecting some changes to my teaching might also produce beneficial results in the students.

An initial proposal to the Department to set up a separate problem-solving class was rejected due to concerns of timetabling and allocation of credits in the students' learning. Most of the undergraduate engineering courses were operating on maximum credits allocation. An alternative was to incorporate strategies to invoke mathematical thinking in the students within existing mathematical subjects. To investigate whether changes in the learning environment and teaching methods could effect changes in the students' abilities and attitudes towards studying mathematics became a research objective. Thus, I felt that it was important to search for learning and instruction theories to support changes that I wanted to make in my teaching. I wanted the changes to address both the existing needs of the students as well as ensure their progress through the university curriculum. I had to find teaching methods that would maintain a balance between helping students overcome their difficulties, and at the same time develop their understanding and skills in making sense of new mathematical knowledge, and enhance their competency in handling mathematical techniques according to the demands of the curriculum. I was interested in theories about the learning of advanced mathematics, in issues that dealt with how students learn and how these were related to the instructional design. I was concerned not only with developing teaching strategies that would help students overcome their current difficulties but those which could enhance their abilities to cope with more advanced knowledge.

My experience in teaching engineering undergraduates had enabled me to identify some typical difficulties that students had. It had also shown me the potential capabilities that they could develop. I had also identified the mathematical skills that they needed to have to become independent learners. In particular, they need to be able to think flexibly and be able to recall facts and procedures. They must have the ability to work out and reconstruct ideas from a few examples and be able to reconstruct techniques from a few core ideas.

They must be able to solve non-routine problems or at least make logical and reasonable attempts at solving problems. I needed to explore the various theories on how students could acquire these abilities. However, a first priority would be to find means to encourage and facilitate the students to shift from only wanting to get the right answers to wanting to understand the concepts. Ideas about mathematical thinking (Mason, Burton & Stacey, 1982; Watson & Mason, 1998; Mason, 1999), gave me the vocabulary to state what I felt the students should be able to do. In turn, these ideas provided guidelines in the preparation of my teaching notes and classroom tasks for students (Abdul Rahman, 2001).

4.2.1 Beliefs and Attitudes about Teaching and Learning Mathematics

I started by examining my own beliefs about teaching and learning as well as the various assumptions and beliefs about students' learning, which were culled from my own experience of learning and teaching mathematics and my own observations over the years. These were augmented by findings from various researches on UTM students' learning. These assumptions and beliefs are categorised as follows:

- ♦ **My own beliefs about teaching and learning Mathematics**

My own learning experiences at undergraduate and post-graduate levels were that of developing learning independence and thinking skills. I had always thought that my students could perform better if they would take time to think through their mathematics problems. I had exhorted students to think but did not consider that I could help in enhancing their ability to think. Additionally, in the past, I had been hampered with the conviction that it was difficult to change my teaching due to various reasons, such as, time constraints in delivering the existing syllabus, large number of students and the need to help my students succeed in their examinations. I, too, had resorted to 'drill & practice' sessions so as to ensure my students' success. Examination papers consisted of fairly routine questions, thus such a strategy was enough to ensure my students' good performance in the examination.

I saw mathematics as a hierarchical subject in which new concepts and ideas were built upon earlier basic concepts. Sometimes, I saw students struggling with new concepts in advanced mathematics. However, I felt that to understand the new concepts, they had to understand the previous ones. In many cases, the new concepts were similar but not quite the same as what was supposed to have been learnt previously. For instance, to understand the concept of partial differentiation for multivariable functions, students had to understand differentiation for single variable functions, which in turn, required them to recognize and be able to work with functions. How can I help my students recognize what was general, what ideas were similar and in which way were they different? These questions were always hovering at the back of my mind when discussing the students' performance with colleagues.

In most problem-solving situations, routine or non-routine, my students will need to work with multiple procedures and techniques. They should be able to make connections between several mathematical ideas, be able to move between different representations and operate complicated algebraic manipulations. How can I help students facilitate their recall of mathematical objects and facts, how to look for linkages and connections between different mathematical ideas and concepts?

My observations had always shown that many students especially those who felt that mathematics was difficult would plunge straight into a problem and randomly 'pull out' mathematical procedures to solve the problems. Findings from Mohd. Yusof's (1995) research, encouraged me to believe that learning mathematical problem solving skills could produce positive changes in students' ability to solve problems and change their attitudes towards problem solving. However, she also concluded that these abilities would diminish if not further supported.

Suggestions given by Tall & Gray (1994) and Schoenfeld (1989) also identified the need for students to be supported in class to overcome their difficulties and to develop their understanding and skills in making sense of new mathematical knowledge and

enhance their competency in handling mathematical techniques. These theories and findings provided an impetus for me to try to change the way I teach so as to support my students to develop better learning skills.

- ♦ **On students' abilities and difficulties** (Liew Su Tim & Wan Muhamad Saridan, 1991; Tall & Mohd Razali, 1993; Faridah Abd. Rahman, 1996; Mohd. Yusof & Abdul Rahman, 1997).

In these research papers, some of the students' difficulties identified were: poor understanding of basic concepts, poor computational competence, inability to effectively organize known facts, and problems in mastering the mathematical language and symbols. As such, the students did not have effective problem solving skills and showed a tendency to over generalize the usage of particular mathematical procedures. In general, these researchers concluded that there were indications that it could not be assumed that students mastered the KBSM Mathematics sufficiently for it to be considered as prior mathematical knowledge for the learning of advanced mathematics at tertiary level. In addition, although generally, the students were able to answer standard or routine questions, there were inconsistencies between their ability to answer questions and their understanding of the concepts and the mathematical procedures that they were using. In any particular class, there existed a wide range of mathematical performance and different levels of mastery of prior knowledge. In many instances, students found non-routine questions and problem solving difficult. Among the more noticeable difficulties were the poor mastery of algebraic skills and poor recall of mathematical facts when working with multiple procedures and building connections between different mathematical ideas.

- **On students' attitudes and perceptions** (Mohd. Yusof & Tall, 1994; Mohd. Yusof, 1997; Khyasudeen, A. Majid, Roselainy, Abdul Halim & Mohd. Noh, 1995; Yudariah & Roselainy, 1998).

Based on these reports, I would highlight students' attitudes and perceptions towards mathematics, their learning at university and life in general.

- **Towards mathematics** – it was found that students perceived mathematics as a subject wholly consisting of a conglomeration of facts and procedures. It was also reported that students with a poor track record in mathematics achievements in the past were over anxious when exposed to new problems and concepts. They would give up easily when faced with difficulties and showed great reluctance in persevering with new ideas and techniques. There was significant belief amongst the students that 'drill & practice' was the most successful way to master mathematics. So, not surprisingly, many showed resistance in adapting their learning styles and adopting suitable mathematical learning skills more appropriate for learning advanced mathematics. Consequently, they showed little cooperation when teaching approaches that required their participation were carried out.
- **Management of Learning** - Khyasudeen et al (1995) conducted an extensive survey on UTM students' study habits, attitudes and motivation, usage of library facilities, lecturers teaching approaches, classroom facilities and other general factors related to personal and social habits. The study involved 3554 students from a total population of 11,000 students. The results of the study indicated that 70% of the students had poor study techniques and did not display the necessary attitudes for studying in the university. For instance, they had poor time management skills, little peer group interactions, used lectures and tutorials ineffectively and had poor note taking skills. A large number of students had poor reading habits, showed no priority in buying books and showed little effort in searching for extra references and other materials. In addition, the students were not utilising available resources fully as indicated by

data on the poor use of library and the students support unit which was set up by the University to provide students with peer group and professional counselling.

- **Towards Life in General** – In the survey conducted by Khyasudeen et al (1995), they also found that 70% of the student respondents claimed to be highly motivated about their learning. However, this was not reflected in their learning behaviour. Responses from the section on study habits showed that they had poor class attendance, did not have complete lecture notes, and did not often participate in class or peer group discussion. In many cases, a student repeating a subject would still skip classes and did not make any attempt to discuss problems with their lecturers or academic advisers. Most of them also said that they were unable to build an effective working relationship with their peers and lecturers. These students faced many difficulties especially when a project or coursework was to be done as a group.

4.2.2 Developing a Mathematical Pedagogy and Teaching Strategies

Putting together all these beliefs and requirements and with selected ideas from the various theories of learning, I tried to design my teaching strategies. I have had experience in the past of providing remedial classes for weak students in UTM. Remedial sessions usually focused on helping students to overcome their existing difficulties with mathematical knowledge and competencies. However, students would still have difficulties with whatever mathematics they were currently learning. There were many factors to consider but I chose to focus on the development of mathematical thinking. I believed that if students were more flexible in their thinking strategies then they would have the basic skills to make sense of their learning.

I based my conception of mathematical thinking on the ideas of Mason, Burton & Stacey (1982). I used the description of how low and high achievers managed their mathematical

knowledge (Gray & Tall, 1994) and Skemp's (1987) description of the learning situation that would support problem solving. I needed a framework to design my classroom strategies to present the mathematical contents as well as design the classroom activities for my students to work on in order to achieve my teaching and learning goals. I found that Watson & Mason's (1998) ideas on what constituted mathematical thinking powers and mathematical structures were very useful in developing my lesson plans. I was very attracted to the ideas, such as, how to structure a topic (Mason & Johnston-Wilder, 2004) and how to use various prompts and questions (Watson & Mason, 1998) that could invoke students' use of their own thinking powers. That was the most appealing aspect of their approach, to support and encourage students' use of their own thinking powers.

The thinking processes, for example, specialising and generalising, ordering and classifying, imagining and expressing, abstracting and instantiating, conjecturing and convincing, were useful in guiding me to provide learning experiences where I could encourage learners to use these powers explicitly thus supporting them in recognizing the mathematical processes and structures for themselves. I decided to use "prompts and questions" (Watson & Mason, 1998) as a means to draw learners' attention to the mathematical processes and structures involved in facilitating the learners' understanding of concepts learnt. The use of the prompts and questions would also enable students to guide their own thinking and used as tools to engage with new problems. I felt that I would be able to provide students with a vocabulary to master their own thoughts as well as engage in new ones.

In addition, I took into account students' existing difficulties and their preference for instrumental learning. Thus, the short-term objectives of the teaching acts implemented were aimed at shifting students' awareness gradually from rote learning towards understanding the concepts and procedures as well as recognizing their own mathematical powers. The long-term objectives were that these strategies would help students develop

the ability to understand various related mathematical concepts; learn to reconstruct these concepts as parts of a whole and to make the connections between the parts. I believed that through this approach, I would also be able to support students during their struggle with mathematical ideas. Here, the short term objectives would mean those that I set at the beginning of the semester and I was hoping that some indications that students have attained some of the long-term objectives would be seen at the end of the semester.

Detailed description of the strategies used in teaching has been described in Chapter 2, Section 2.5. Here, I will summarise the strategies used in developing students' thinking and problem solving skills in the classroom. These included the following.

- (a) Structuring a topic – I focused on the mathematics that I want my students to learn – identifying the structures in mathematics (definitions, facts, theorems, properties, techniques, examples, etc) and making explicit the mathematical powers to be used (specialising, generalising, conjecturing, characterising, etc). In the beginning, the focus was on specialising and generalising, as the aim was to increase awareness of patterns, relationship between variables and the mathematical processes.
- (b) Using prompts and questions – I had selected some prompts and questions mainly to support students in specialising and generalising. Examples of questions used most often at the beginning were: *What is the same?*; *What is different?*; *What can change and what stays the same?*; *What happens in general?* Some examples of prompts that were used were 'Give me another example of the same kind' and 'Give me a counter-example'.
- (c) Using structured examples – the examples were presented in such a way so as to support the growth of the concepts as well as the understanding of the processes and procedures.
- (d) Working on specially designed tasks that encouraged students' use of their own thinking powers.

- (e) Providing linkages and connections between mathematical ideas – recognizing and using mathematical themes, using prompts and questions and using structured examples;
- (f) Providing and creating a favourable environment for active learning with an emphasis on mathematical communication.

The basic notions that would underlie the teaching acts were the explicit use of mathematical themes to provide linkages between mathematical ideas and to expose the structures of the mathematics. Some of the themes I had chosen were, **invariance amidst change**, which form the basis for many mathematical theorems and technique, **doing and undoing**, which could help students to identify features or structures to focus on and could help them in solving problems. Suitable **prompts and questions** were chosen to engage students in, and to assess their grasp of mathematical ideas and techniques (Mason, 2000). The appropriate prompts and questions were designed by taking into account the mathematics that students were to learn and by considering how to bring about this awareness. The idea was to make these prompts and questions more focused and direct in the beginning until the students became aware of the type of questions asked and should become increasingly indirect over time as they gradually use the questions for themselves. This support was to serve as a scaffold, to encourage students to become more aware of their own mathematical thinking powers and develop learning independency. Examples of how these ideas translated into lectures and tasks will be given in Section 4.5. The design of the teaching strategies is also part of the study and students' responses and mathematical performance in class were considered and had contributed to the modifying and improvement of the strategies as well. What is written here marks the ideas and notions that were held at the start of the study.

4.3 Developments in UTM

Developments in the curriculum and assessment of students were also significantly influenced by UTM's policies concerning student's intake, lecturers' career evaluation, developments in the manpower planning of the nation and new trends in education. A brief discussion of these changes and implications arising from them will be presented below.

4.3.1 Policy Changes

There were several policy changes that directly and indirectly influenced the teaching and learning environment. These were:

- **Students' entry requirements**

Prior to July 2002, the intake requirements were the SPM¹ qualifications. Students needed at least five SPM credit passes in relevant subjects whereby the compulsory minimum mathematics requirement was a good credit pass in KBSM² Mathematics whilst a credit pass in Additional Mathematics was considered an advantage, although not necessary. The Additional Mathematics syllabus covered basic calculus. In July 2002, UTM changed their intake requirements policy such that prospective undergraduates had to have post-SPM qualifications such as Matriculation³ qualifications, STPM⁴, Diploma or other equivalent qualifications from recognized institutions of learning. Students needed a minimum of 3.3 Cumulative Grade Point Average (CGPA) or its equivalent for entry. These changes were part of an overall change in higher education policies of the Ministry of Higher Education where various routes to higher learning were planned and implemented. Thus students with SPM qualifications were encouraged to go to Matriculation centres, community colleges, university colleges and other similar institutions of learning. All universities were

¹ SPM – Sijil Pelajaran Malaysia (Malaysia Certificate of Education, equivalent to the GCE O-Levels)

² KBSM–Kurikulum Baru Sekolah Menengah (Secondary School New Curriculum)

³ Matriculation qualifications – from Matriculation programmes under the Ministry of Education

⁴ STPM – Sijil Tinggi Pelajaran Malaysia (Higher School Certificate of Education, equivalent to the GCE A-Levels)

required to take in students with post-SPM qualifications. Accordingly, the curriculum and the courses had to be updated and reorganized.

- **Medium of communication for teaching**

Another important aspect of these two different periods was the medium of communication for teaching and learning in the university. The pre-July 2002 students were taught in Bahasa Malaysia (Malay language). Meanwhile, English was the medium for the post-July students. Thus, UTM had two parallel programs running, students taught under the old syllabus until they graduate and those taught with the new syllabus in two different languages. Meanwhile, Mathematics and Science were also being taught in English at schools but it was expected that these students would be the university's intake of 2007. Thus, the post-July intake of 2002 was made up of students who were still taught in Bahasa Malaysia in schools for all subjects other than English. Some difficulty in adapting to English as the medium of communication was anticipated not only for the students but also the lecturers.

- **Staff professional development demands**

UTM has determined that all academic staff must focus their efforts on seven main areas for professional development: teaching and learning, research and development, consultation, publication, students' development, management and service to the community. This requirement added extra pressure on staff because they must now balance their teaching workload with other activities for career advancement. This study was undertaken on a part-time basis so I was not exempted from these requirements.

4.3.2 Curriculum Development

The curriculum for the Engineering students had to change to accommodate changes in policy. The minimum duration of the course had changed from 4 ½ years to 3 ½ years. The curriculum for Engineering Education in UTM was reviewed and modified which meant

that the mathematics syllabus was also changed. However, in addition to these general changes, there were also specific differences to the mathematics syllabus for the group of students in the Industrial Design Course (course code – SRI) that I was studying. In particular, for students who had entered UTM before July 2002, there was an exceptional circumstance, in that students from the Arts stream at secondary school level were also accepted for the Industrial Design course. Usually, UTM only took in students from the Science and Technology streams. Thus, the Industrial Design mathematics syllabus was different in content and depth as compared to the other engineering courses. For instance, Basic Calculus for SRI students took the code SSM 1242, a 2-credit subject, whereas for all other engineering courses, the code was SSM 1203, which is a 3-credit subject. Similarly, Calculus II for the SRI students has the code SSM 2242, which is a 2-credit subject whereas for other engineering students, it was a 3-credit subject, Advanced Calculus with the code SSM 2283. It is important to note that each subject code must have its own examination paper. The subjects were usually offered in particular semesters and if a student failed or wished to repeat a paper to redeem a poor grade then they had to sign up for the subject when it would be offered next.

With the changes in intake policies, the post-July 2002 Engineering students' intake had a new curriculum and thus, the first year first semester mathematics subject was a 2-credit Calculus with the code SSE 1792 and the second semester subject is called Engineering Mathematics with the code SSE 1893, a 3-credit subject. Engineering Mathematics was comparable to the older Advanced Calculus. However, the mathematics subjects were offered in a 'mirror-image' mode. This meant that the subjects were offered in each semester for a certain number of students. For example, in the second semester, students from some faculties will be doing Engineering Mathematics and the rest will be doing Differential Equations. In the next semester, the first of the following academic session, they will be an exchange in that those who did Engineering Mathematics will be doing Differential Equations and vice-versa.

The detailed syllabi and other information about Basic Calculus, Calculus II and Engineering Mathematics, relevant to this study will be given in the appropriate sections and context of discussion.

4.3.3 Impact on my study

Much of the literature reviewed about UTM students' mathematical learning difficulties was studies conducted on students of the pre-July 2002 period. With the new intake, as the students had different entry qualifications, assumptions about their facility with prior mathematical knowledge and skills had to be re-examined. My own beliefs and attitudes had to be reviewed and revised as well. The effect on my teaching strategies will be presented later. However, general literature on the research findings about university students' difficulties in mathematics was still relevant. Students usually entered university elsewhere with qualifications considered equivalent to the Malaysian post-SPM level. Thus, the reading of Sections 4.1 and 4.2 above will have to be based on the awareness of these developments.

Three groups in the study were students in the pre-July 2002 categories and were doing the Industrial Design courses. For these students, mathematics at KBSM level was considered prior knowledge and it was assumed that they have had a collection of some basic mathematical skills and knowledge. Generally, all engineering undergraduates in the pre-July category would have to take the same mathematics subjects, which were Foundation Mathematics, Basic Calculus and Statistics in the first year, followed by Calculus II, Ordinary Differential Equations and Numerical Methods in the second year. Foundation Mathematics and Basic Calculus would include certain topics from the Additional Mathematics syllabus at secondary level. These were taught again to ensure that students would have opportunities to strengthen their basic mathematics. However, the content of each subject was modified further for the Industrial Design students. In general, the syllabus for the Design students was reduced and thus they needed less time for the

Mathematics. I also taught these students in Bahasa Malaysia and had to translate the ‘prompts and questions’ that I was using. Some of the ‘themes’ that I was using did not translate very well into the Malay language.

With the post-July students, teaching was slightly easier as it was in English but some students had difficulties with the language in terms of verbal communication and trying to understand the mathematics. The Malaysian Ministry of Higher Education put into effect the change in the language for Institutions of Higher Learning but these students had been doing the Secondary and post-secondary education in Bahasa Malaysia (Malay language) except for English, of course.

There were also changes to the syllabus. However, the change was not substantial as even though the name of the subject changed and to whom the subject was offered also changed, in the main, the mathematical content was relatively unchanged. Anyway, regularly reviewing my notes and the manner of presenting the mathematics was a part of the overall strategy that I was using in my teaching. Some discussion of these issues will be presented in greater details in the relevant sections of the teaching episodes for the particular categories.

4.4 The Study – Part 1

In this section, I will describe the teaching and learning experience that I have gone through when I taught the first four groups of students. I started out with a lofty aim of supporting and enhancing students’ learning of mathematics and mathematical thinking abilities. Although I had very clear ideas at the outset of how I would be conducting the class, what transpired was a revelation. Encouraging students to change and adopt different learning styles was not easy especially when I realized that the main player that had to change first was the teacher herself.

4.4.1 Choosing classes to study

An early decision was to work with a group of engineering students in their first and second year. They would be doing Basic Calculus in their first year; and Calculus II in their second year. I would be conducting the calculus classes. I hoped to introduce changes in the teaching and learning environment. If I could work with them for two years then they will have more time to become familiar with any new ideas that will be tried out. All of the students would have had exposure to the 'talk and chalk' method. Teaching, in school as well as in UTM, usually only uses lectures. I anticipated that it would not be easy to gain the cooperation and build students' confidence to be more active in the classroom. Another problem that I expected was in trying to change my teaching methods and putting the theory into practice. Unfortunately, I have also got into the habit of teaching through 'show and tell' sessions. Subsequently I am hoping to follow some of the students from the group into an engineering subject class that will have substantial mathematics content. I have talked to some lecturers in the Faculty of Mechanical Engineering; they claimed that students usually forgot all the mathematics they were supposed to have learnt and were usually unable to cope with the mathematics in their own engineering subject matter.

In the event, practical considerations prevailed. I chose a class in Industrial Design (1 SRI) under the Faculty of Mechanical Engineering, which took a Basic Calculus subject with a separate code SSM 1242 as compared to mainstream engineering groups whose subject code for Basic Calculus was SSM 1203. The significance of this choice was that I would be teaching the group alone. In UTM, each subject code will have its own examination paper. There are usually about 19 lecturers teaching the main engineering streams and as such it would be harder to initiate changes to the teaching as well as to the assessment. The number of students taking SSM 1242 was usually around 60, and usually taught by a single lecturer. Thus, I would have more autonomy in handling the teaching and assessment. This

meant that the next class that I would have to teach would be Calculus II with the code SSM 2242 for the same course group.

However, I also taught Basic Calculus to the SRI students in the second semester of 2002/2003 and due to events that have occurred during the teaching of the classes, I ended up teaching Calculus II to a fourth class of SRI students, in the first semester 2004/05 session. In the following sections, I will be describing my experience of teaching the different groups of students.

4.4.2 Subjects taught

I taught two subjects to the four groups of students. I taught the two groups Basic Calculus, subject code SSM 1242, which was a subject usually offered to the first year students in the second semester. I taught the other two groups, Calculus II, with the code SSM 2242, which was usually, offered to the second year students in the first semester. The teaching duration was of 15 weeks, which included a 1-week semester break. All the groups were doing the Industrial Design course with the Faculty of Mechanical Engineering. Information about the actual semesters and academic sessions is given in Section 4.1. They had to pass the Mathematics subjects to get their degree, but it turned out that several were repeating the course, and some had even passed the course for which this mathematics was the prerequisite! Thus the content was not actually vital to their career prospects apart from finishing the course as a requirement to qualify for the degree.

In the following sections, I will describe the teaching of the students in chronological order, i.e, Basic Calculus for group A01, Calculus II for group B02 and then Basic Calculus for group C02. I will present the description of group C04 separately.

4.5 The Teaching of Basic Calculus for Group A01

I started during the second semester of the academic session, November, 2001/02. This was the first class that I taught in which I tried out my teaching ideas. I was teaching Basic Calculus, which had the subject code SSM 1242.

4.5.1 Students’ Data

In the first week, during the first meeting, I met with only 32 students. Many of the other students only came to class in the 2nd and 3rd week. However, 6 students subsequently withdrew their registration of the subject. Thus, finally, I had a total of 49 students but only 24 were actually in the first year. The other 25 students were in the second, third and fourth years. Some were repeating the subject and there were a few who were considered as ‘repeat students who had not taken the subject’. See Table 4.2 below. This was a special category of students who had not taken the subject previously but for administrative purpose were classified as ‘repeat’ students. All the students should have taken Foundation Mathematics, as it was a pre-requisite subject for Calculus I. Thus, I began by assuming that they had this as prior knowledge. During the first meeting, I had also collected the students’ school mathematics and Foundation Mathematics results. This is given in the Table 4.3. I have the results of only 26 students and even though it is not complete, it gives an idea about the students’ mathematical learning background. From the data, it could be seen that the majority who came to the first class meeting were first year students.

Table 4.2: Number of students in 1 SRI

Course: 1 SRI, Industrial Design	
Course year	No. of students
1	24
2*	7
3*	15
4*	3
Total	49

Note: 2*, 3* and 4*: these students were repeating the subject.

Some of the 3rd and 4th year students were doing the Calculus for the fourth time.

Table 4.3: Students’ school mathematics Foundation Mathematics results

		Compulsory Mathematics entry qualification	Optional Mathematics entry qualifications	UTM: 1 st Semester
Students’ Number	Course Year	KBSM Mathematics	Additional Mathematics	Foundation Mathematics
17	1	1A	1A	A
19	1	1A	1A	A–
5	1	1A	1A	A
25	1	1A	1A	A
14	1	1A	1A	A–
7	1	1A	2A	B+
8	1	1A	2A	A
26	1	1A	2A	A
12	1	1A	3B	B+
4	1	1A	4B	A–
20	1	1A	5C	C–
23	1	1A	5C	C+
11	1	1A	7D	D
18	1	2A	5C	C
3	3	2A	6C	C
6	1	2A	6C	C+
2	1	2A	8E	C–
13	1	2A	7D	C
21	3	2A	7D	C–
10	1	2A	None	Not given
24	1	2A	None	D
16	1	3B	6C	C+
1	1	3B	8E	D
22	1	4B	8E	C+
9	1	4B	None	Not given
15	4	6C	8E	D

Notes:

(A) The results are presented in decreasing KBSM Mathematics score.

KBSM Mathematics is the compulsory Mathematics entry requirement.

Grades for SPM results are: 1A and 2A (Distinction); 3B and 4B (good credit); 5C and 6C (credit); 7D and 8E (pass). The Ministry of Education

usually normalise the results and actual marks assigned were never disclosed and were considered classified information.

Marks for the grades categories in UTM: A: 85-100; A–: 80-84; B+: 75-79; B: 70-74; B–: 65-69; C+: 60-64; C: 55-59; C–: 50-54; D+: 45-49; D: 40-49, E: < 39.

(B) For some students who were repeating the subjects, they would have taken Foundation Mathematics during their first year, which would at least be a year before this calculus class.

Mathematical qualification for entry into the course was a credit pass in KBSM Mathematics. A credit pass in Additional Mathematics was considered optional. In this group, there were some students who had taken Additional Mathematics and a small number who had not (see Table 4.3).

4.5.2 Teaching Strategies

In this section, I will describe some of the teaching strategies that were used and how some of the topics were taught. A general description of how the class was conducted will be presented as well as some description of the responses from the students. In the following sections, I will pick up several issues that came up during my teaching especially those related to the students’ learning behaviour.

The following Table 4.4 gives the syllabus for Basic Calculus (SSM 1242) organized into suggested weekly lectures schedule. This schedule was also handed out to students.

Table 4.4: Weekly lectures schedule for SSM 1242

Week	Topics	Contents
1	Introduction/ revision	Subject information, Revision on functions
2	Differentiation	Definition and differentiation from first principles, Standard derivatives of polynomials; Rules of differentiation: addition, product and quotient.
3		Chain rule for differentiating composite functions, differentiation of implicit functions

4		Differentiation of selected functions: polynomials, trigonometric, exponential and logarithm inverse trigonometric, hyperbolic and inverse hyperbolic functions
5		Differentiation of parametric function and higher order differentiation
6	Applications of derivatives	Finding equations of tangents and normals, finding errors, approximations and roots of equations. TEST 1 (25%, topics week 1-5)
7		MID-SEMESTER BREAK
8	Applications of derivatives	Rate of change, finding relative extremum and absolute extremum, points of inflection
9		
10	Integration	integration as the reverse of differentiation, Indefinite integrals, Standard integrals. Techniques of integration: (a) substitution methods, (b) integration by parts
11		Techniques of integration: (c) using partial fractions (d) integration using trigonometric and hyperbolic substitution
12		(e) integration of inverse trigonometric and inverse hyperbolic functions
13	Applications of the integral	Area – under the graph and between two curves. TEST 2 (25% - topics week 6-10)
14		Volume of revolution – disc method, washer method and shell method
15		

The credit rating for this subject was 2, which meant that I was given 2 hours' of lecture sessions and 1-hour tutorial session with the students. In real terms, each class meeting lasted only 50 minutes, as 10 minutes was the time assigned to students to move from one class to another. Traditionally, there would be no allowance for revision of any assumed prior knowledge. Students were informed of what was considered as pre-requisite knowledge and had to make any necessary revision for themselves. Previous experience had shown that many students would not have done the revision. Thus, I had decided at the beginning to incorporate short revision sessions if and when necessary within my lectures. These sessions would highlight some important concepts that were needed but students would have to carry out more complete revision themselves.

Week 1 – first and second class meeting

In the first week, during the first meeting, I gave out a test in pre-calculus algebra to determine the students' existing knowledge and algebraic skills. The rest of the class

session was spent on explanations about the subject, its syllabus, and the assessment that will be carried out. In the second class meeting of the same week, I spent some time revising the topic of functions. Lessons on the actual syllabus would only begin in the second week.

At first, the number of respondents who responded to the test was 32. However, 6 students subsequently withdrew their registration of the subject. The final number of students' work considered was 26. Many of the other students only came to class in the 2nd and 3rd week and did not take the test. The pre-test questions were adapted from a previous effort by a group of lecturers to determine students' recall and algebraic skills as they enter UTM (See Appendix A). However I added 10 questions (15-24) on the topic of functions, which should have been covered in the Foundation Mathematics course in UTM in the first semester. Other than their performance in the pre-test, I had also collected the students' school mathematics and Foundation Mathematics results. This is given in the Table 4.3 above. Results from the pre-calculus algebra are given in Table 4.5.

Table 4.5: Students pre-calculus algebra test results

Students' Number	Course Year	Pre-test (24 Questions)		Marks (%)
		Questions attempted	Right answers	
1	1	9	3	12.5
2	1	11	5	20.8
3	3	8	5	20.8
4	1	12	7	29.2
5	1	19	16	66.7
6	1	12	7	29.2
7	1	20	14	58.3
8	1	20	16	66.7
9	1	12	4	16.7
10	1	12	7	29.2
11	1	13	9	37.5
12	1	14	13	54.2
13	1	15	8	33.3
14	1	16	10	41.7
15	4	10	3	12.5
16	1	23	15	62.5
17	1	24	20	83.3

18	1	16	9	37.5
19	1	24	13	54.2
20	1	11	7	29.2
21	3	11	4	16.7
22	1	12	8	33.3
23	1	14	11	45.8
24	1	23	15	62.5
25	1	24	22	91.7
26	1	15	9	37.5

Conclusions from the pre-test

In the pre-calculus algebra test, questions 1 – 14 covered various topics that required students to use algebraic manipulation that they would have learnt in their school mathematics and in Foundation Mathematics, a first semester subject, such as, simplifying expressions, expansion of factors, factorisation and certain algebraic operations. Questions 15 – 24 were on functions, the definition of function and requested students to give examples of types of functions that they would have studied in Foundation Mathematics. In Table 4.4, I have noted the number of questions attempted by each student and the number of correct responses.

A brief analysis of the students' performance is given below.

(i) Most of the students attempted questions 1-14, although not necessarily getting the right answers. The least number of questions attempted was 8.

(ii) For questions 15-24, the number of students who did not answer any of these questions is 14, 6 students attempted 1 – 6 questions and 6 students attempted 7 – 14 questions. Those students who could not give examples of the required functions wrote that they could not remember.

Table 4.4 gives the results for the Foundation Mathematics as well as the Basic Calculus that the students got at the end of the course. These results were compared to their entry qualifications as well as to their performance in the Pre-test. A comparison of students' marks for their Foundation Mathematics and the Pre-test, given in Chart 4.1 showed that many were not prepared to take a test early in the semester and as they indicated in their

answers ((ii) above), many had forgotten the mathematics that they had learnt. Two students chose not to disclose their Foundation Mathematics results (number 9 and 10) and thus, they were not included in the data for the charts. Two students were consistent in their performance for the Foundation Mathematics and the Pre-test.

		Compulsory Mathematics entry qualification	Optional Mathematics entry qualifications	UTM: 1 st Semester	UTM: 2 nd Semester
Students' Number	Course Year	KBSM Mathematics	Additional Mathematics	Foundation Mathematics	Basic Calculus
17	1	1A	1A	A	A
19	1	1A	1A	A–	A–
5	1	1A	1A	A	B
25	1	1A	1A	A	A
14	1	1A	1A	A–	A–
7	1	1A	2A	B+	A–
8	1	1A	2A	A	A
26	1	1A	2A	A	A
12	1	1A	3B	B+	A
4	1	1A	4B	A–	C+
20	1	1A	5C	C–	E
23	1	1A	5C	C+	D+
11	1	1A	7D	D	E
18	1	2A	5C	C	C–
3	3	2A	6C	C	D+
6	1	2A	6C	C+	E
2	1	2A	8E	C–	E
13	1	2A	7D	C	D+
21	3	2A	7D	C–	E
10	1	2A	None	Not given	E
24	1	2A	None	D	E
16	1	3B	6C	C+	C+
1	1	3B	8E	D	E
22	1	4B	8E	C+	B–
9	1	4B	None	Not given	E
15	4	6C	8E	D	E

Table 4.6: Comparison of students' results: Entry qualifications, Foundation Mathematics and Basic Calculus.

Comparison of Foundation Mathematics and the Pre-test results

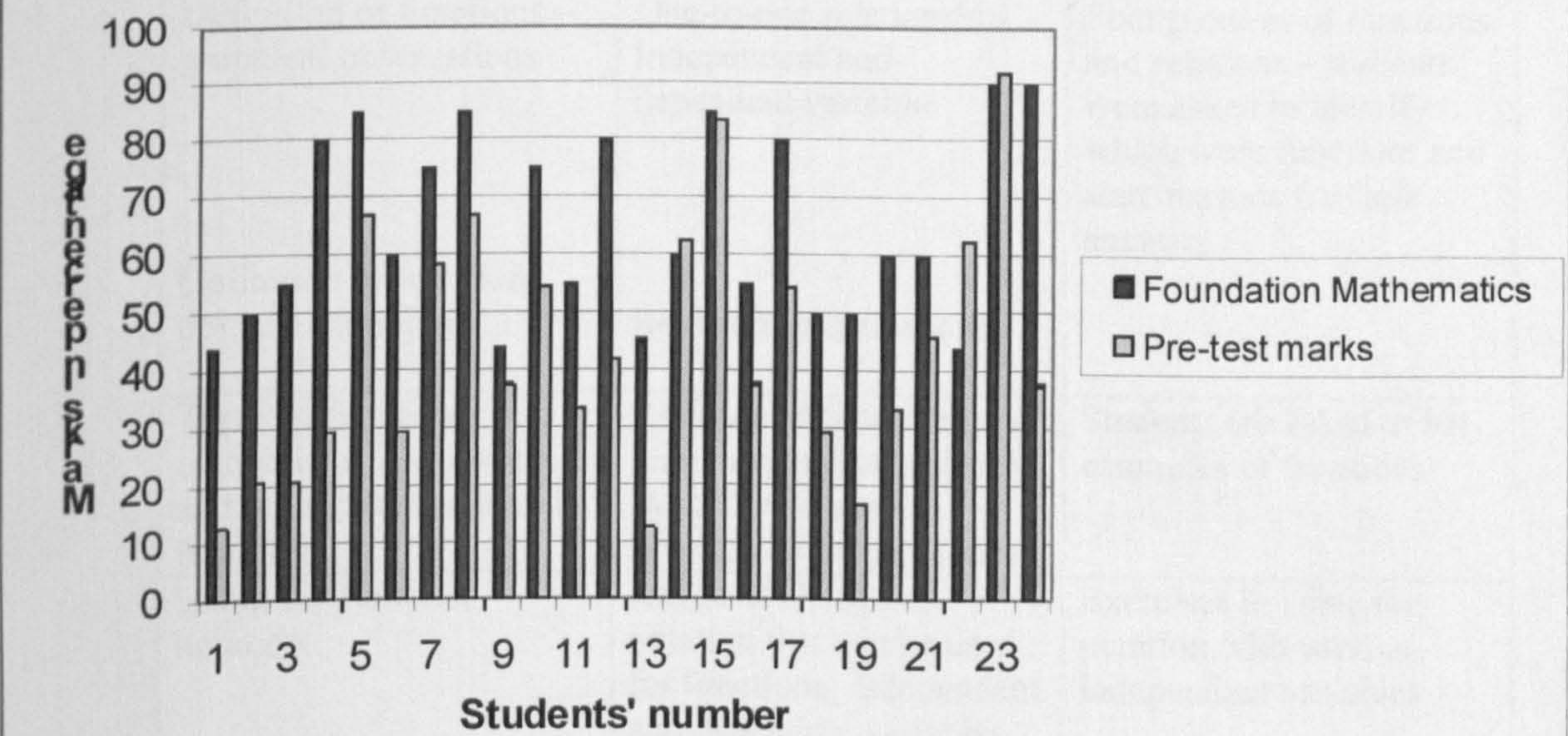


Chart 4.1: Comparison of Foundation Mathematics to Pre-test results

I had more or less anticipated the results of the pre-algebra test on the topic of functions and had chosen functions as the first revision topic. Past experience had shown that students’ understanding of functions played an important role in facilitating or impeding their progress in Basic Calculus as well as Calculus II. To determine which particular aspects of the topic will be focused on during the revision, I studied documented research findings on students’ difficulties in the learning of functions (Maselan Ali, 1996; Tall, 1996; Tall & Razali, 1993, Eisenberg, 1991; Md. Nor Bakar, 1991). I also reflected on my own teaching experience and tried to anticipate what students would need to recall about functions in future lessons. The revision focus is given in Table 4.7.

Table 4.7: Revision Focus on the topic of functions

Main theme: Invariance Amidst Change		
Sub-theme: Range of Change		
Mathematical structures: Definitions, Notations, Representations		
Activities: Specialising and Generalising, Characterising, Expressing		
Contents:	Focus of Attention:	Tasks/Problems
Definition of functions – graphical observations	One-to-one relationship Independent and dependent variable	Four pictures of functions and relations – students were asked to identify which were functions and state reasons for their answers
Definition of functions, domain and range	Revision of definitions	
Types of functions: algebraic, trigonometric and other transcendental functions	Revision of functions and their graphs that students should have met in Foundation Mathematics	Students are asked to list examples of functions
Using the function notation	Range of change of notation that can be used for functions; independent and dependent variables	Exercises in using the notation with various independent variables
Evaluation of functions	Notes and examples were given for students to read in their own time	Some problems were set and the solutions were also given
Operations on functions		
Composition of functions		
Inverse functions		

I used the second class session in the first week for the revision. There was no tutorial session in the first week. Students were given handouts with notes and a few problems to work on. I started with four pictures of functions and relations. The students were asked to identify which were functions or relations and the reasons for their answers. Short notes on the definitions of functions, domain and range were given. Some time was also spent on the notation for functions. Activities included changing the independent variable and the use of different letters for functions and variables. It seemed a simple exercise but many students in the past had difficulties in recognising functions written differently than the common form of 'y as a function of x'.

An example of an exercise on notation of functions that students were asked to work on is given below.

Strategy: Understanding definitions and recognising multiple notations

Activity: Recognising the independent variable and how to use the notation of functions.

Question:

Given that $f(x) = x + 2$ what is $f(x + 1)$?

Given that $g(x) = \sin x$ what is $g(3x)$?

Find $h(t)$ if $h(t) = f(t) + g(t)$. What is $h(3x)$?

This is one of the exercises where I could begin to put my ideas into practice. For this class, I wanted to focus on enhancing students' noticing of the various symbols that are used in functions and its relation to the meaning of functions. I also wanted to enhance students' ability to communicate, generally and mathematically, verbally as well as in written form.

At the beginning of the exercise, I asked the students to recall the definition of functions and notice '*what was the same?*' and '*what was different?*' between the given questions. The questions were written on a transparency for the overhead projector (OHP). As I walked around the class while they were working, I would repeat the prompts and questions to the students as necessary. The students were allowed to discuss with each other while working. The seating in the class was such that students were sitting in rows of chairs that were bound together in groups of four. Each chair had its own collapsible writing table. Thus, students could talk to their neighbour on either side or to someone in the row behind or in front. The furniture made moving around in the class very difficult. I encouraged the students to work in pairs but they were free to work in bigger groups of three or four people. Some students chose to work alone; many decided to work in pairs but a few chose to work in groups of three or four. However, as I walked around, I noticed that the students actually worked individually on a question and then compared their work with their team members. These are revision questions and some students were able to

answer the above questions. The prompts that I used were often ignored as the students worked on the problems.

This revision session was carried out in the first week. I did not realize that more students would be registering. Thus, I had new students coming to class up to the third week. The latecomers were mostly repeat students.

I had decided on a plan of teaching that would accommodate my research objectives and students' concerns. Students, especially the repeat students, were mainly concerned in passing the course. I was explicit in stating my objectives in changing some of the ways that we were going to work on the topics in the class. I would be presenting a mixture of lectures, class activities and review sessions. Thus, I decided that all three hours assigned to me would be used in this way and there would be no special lectures and tutorial sessions. The students did not object to these changes.

In Week 2, we tackled the topic of Differentiation with the following sub-topics: definition and differentiation from first principles, standard derivatives of polynomials; rules of differentiation: addition, product and quotient.

For this topic, I decided to focus on the main theme of 'invariance amidst change' with the sub-theme of investigating the range of change possible in looking at the definition of differentiation, its notation and the different representations of the functions. The particular activities focused more on specializing as I thought it would provide opportunities for most of the students to participate in class discussion and think of examples. For the topic of Definition of differentiation, I conducted regular lectures to cover this sub-topic in the syllabus. When we reviewed the basic formulas of differentiation, I decided to give the students some activities to foster their understanding of the use of the formulas.

Week 2 – first class session

Strategy: Understanding definitions, symbolic notation and its meaning

Topic of lesson: Basic formulas of differentiation

Constant Rule:

$$f(x) = c \quad \Rightarrow \quad f'(x) = 0$$

I asked students to give examples of constants to be differentiated. There were some responses from the students and some of the examples offered were:

$$y = 5 \text{ and } y = -21$$

I had used the notation of functions in terms of $f(x)$ but I noticed that the students gave me examples in terms of y .

I then asked, could $y = -\frac{301}{11}$ be an example?

Many of the students answered that the derivative was 0.

I asked, could $y = \pi$ be an example?

There was some low-voiced response that I could not hear. So, I asked them to give their response in one of three ways; “Yes”, “No”, or “I do not know”. In this way, I reasoned that every student could give me an answer. Then, they were clearly some responses of a ‘yes’, there was also some ‘no’ and a few said that they were ‘not sure’.

I asked, could $y = e$ be an example?

I had similar responses. I gave brief explanations about π and e . I asked them the same questions, ‘whether $y = \pi$ and $y = e$ could be examples of the constant functions? The replies sounded more confident and they said that the derivatives for both were 0. I offered the different constants as experience has shown that students were usually more comfortable with integers and sometimes forgot about constants such as π and e .

We moved on to the formula

$$f(x) = x^n \quad \Rightarrow \quad f'(x) = nx^{n-1}$$

I also used

$$y = x^n \Rightarrow \frac{dy}{dx} = nx^{n-1}$$

I could see that the students had an affinity for the symbols of functions in terms of y . I asked students to write down their own examples of functions “*of this form*” in their book. This time, I walked around to see whether students were writing down examples. Most of them did although they were a few who did not write anything down. Some of the examples that students wrote were:

$$y = x^4; \quad y = x^{-4}$$

Most of the other examples were similar except that the exponents were of different numbers, usually whole numbers.

I asked, could $y = x^{-1/2}$ be an example?

There were some affirmative answers.

I moved on to the possible range of change in the letters used for the variables. Past experience had shown that students had some difficulties in using other letters for functions and variables and in identifying the derivative with the changed notations.

If y is a function of x then its derivative is $\frac{dy}{dx}$.

Therefore, if $r = s^4$, then its derivative is $\frac{dr}{ds} = 4s^3$

If $w = l^{1/2}$, then its derivative is $\frac{dw}{dl} = \frac{1}{2}l^{-1/2}$

I asked students to hear the way the functions and their derivatives were read. I asked them to tell me which was the independent and dependent variables for each example. I asked

them to use $y = f(x)$ as an expression to refer to. Many of the students could identify the variables correctly.

There were still some students who were not participating in the class. I have not singled out any of these students during the lessons or made any personal contact with them yet.

Week 2 – second class session

I started the lesson by introducing the formula;

$$y = cf(x) \quad \Rightarrow \quad \frac{dy}{dx} = cf'(x),$$

with the following example:

$$y = 5x^4 \quad \frac{dy}{dx} = 5(4)x^3 = 20x^3.$$

I asked students to offer some examples of their own to the whole class.

Some of the examples given were:

$$y = 3x^4 \quad \frac{dy}{dx} = 12x^3$$

$$y = \frac{1}{2}x^2 \quad \frac{dy}{dx} = x$$

There were other similar examples, i.e., with positive integers exponents, which showed that students were familiar with the formula.

I used the example, $y = 5x^4$ again, but focused on the possible range of change with the use of the question, ‘What can change?’ For this first session, I conducted this segment as a lecture. I wrote on the board,

$$y = 5x^4 \quad \frac{dy}{dx} = 5(4)x^3 = 20x^3;$$

I also used an example offered by one of the students in the previous session, and wrote it under the first example,

$$y = 3x^4 \quad \frac{dy}{dx} = 12x^3$$

I asked them to compare the two examples and identify ‘*What has changed?*’

The students responded correctly and identified that 5 was changed to 3 in the second example. Many were smiling and nodding as they gave the answers. I kept these examples on the board and wrote another one,

$$y = 3x^6 \quad \frac{dy}{dx} = 18x^5$$

I used the same prompts, ‘*What has changed between this example and the previous one?*’

These prompts were verbal and not written down.

The students responded correctly that the exponent 4 was changed to 6. However, students did not actually use the word exponent or power, they just said “4 has become 6”.

Then I changed the example and asked, “*What is the derivative of the following function?*”

The function was written on the board but the question was asked verbally only.

$$r = 3x^6 \quad \Rightarrow \quad ?$$

Someone started saying “ $\frac{dy}{dx}$ is...” but stopped. A few said the answer is “ $18x^5$ ” but did

not offer to name the derivative. Then someone said, “ $\frac{dr}{dx} = 18x^5$ ”.

But there was no unison of voices in offering the answer for this example. I asked them to look at all the examples again for a minute or two and said that they could discuss the new example with their friends.

I wrote a new example and asked, “*What is the derivative of the following function?*”

$$r = 3t^6 \quad \Rightarrow \quad ?$$

A few students confidently gave the answer as, $\frac{dr}{dt} = 18t^5$. However, a few comments were heard, such as, “*what is this?*”, “*I’m confused*”, “*why?*”. The students who gave the right answers started to explain. There was a sudden eruption of conversation, questions were being asked and explanations sought, but from each other. It was the hardest thing I ever did on that day, to stay quiet!

I decided to put up another example on the board,

If $r = 3t^6$, can you find $\frac{dr}{dx}$?

The students were asked to discuss this question. Some students put up their hands to ask for my help. I went up to one group and they said that they were not sure about what they were supposed to do. I asked the other groups that had put up their hands for help if they had the same problem. They said yes so I decided to address the whole class for the last problem. These last two examples were impromptu questions that I decided to give the class after seeing their responses. I asked them to identify which was the variable of differentiation and what was the independent variable for the function. There was a mixed response as some students could identify the independent variable but were not sure what the variable of differentiation was. I referred to the previous examples that were on the board and identified the independent variables and the variable of differentiation for each of the problems. Then, I asked students to look at the last problem again and see whether they could identify which was the variable of differentiation and what was the independent variable for the function. They managed to give the right answers. However, I was not sure whether the right answer was because they recognized the pattern in the structure of the terms or whether they had become aware of the role of the independent variable as the variable of differentiation. I could see that most of the students were participating in the class activities. We did not manage to cover the sub-topic, Rules of differentiation in the class.

Week 3 – First session:

In the first lecture, there were new students in class. They told me that they had registered for the class at the Academic Department of the Faculty of Science. This was one of the administrative difficulties that I faced. Students could register for class either with the concerned lecturer or the Academic Department. However, information about the students' registration will only be made available upon request or sometime in week 5. I decided that I would talk to the students after class and that meant that I had to finish slightly earlier.

In this class, I wanted to teach the following sub-topic. This topic like the previous ones would be revision for those students who had taken Additional Mathematics at school. I was not sure that I could do all the four rules in this class.

Sub-topic 1: Rules of differentiations: Addition, subtraction, product and quotient rules

I decided to put the students to work on exercises in using the formulas of the four differentiation rules: addition, subtraction, product and quotient. The formulas were introduced using the $f(x)$ and $g(x)$ notation. The notes given to students were as given below.

If $f(x)$ and $g(x)$ are differentiable functions and a, b are any real numbers, then the functions $[f(x)+g(x)]$, $[f(x)g(x)]$ and $f(x)/g(x)$ (provided $g(x) \neq 0$) are also differentiable and their derivatives satisfy the following formulas:

1. Sum rule and difference rules

$$\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx} f(x) + \frac{d}{dx} g(x) = f'(x) + g'(x)$$

$$\frac{d}{dx}[f(x) - g(x)] = \frac{d}{dx} f(x) - \frac{d}{dx} g(x) = f'(x) - g'(x)$$

2. Linear rule where a and b are constants

$$\frac{d}{dx}[af(x) + bg(x)] = a\frac{d}{dx}f(x) + b\frac{d}{dx}g(x) = af'(x) + bg'(x)$$

3. Product rule:

$$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$$

4. Quotient rule:

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}; g(x) \neq 0$$

I put up some examples using simple power functions for them to work with. We have been mainly working with polynomials. However, I decided to introduce some examples of algebraic functions with negative integer exponents. I decided that I would go through all the differentiation rules using polynomials and simple algebraic expressions so that they could focus on the techniques first. I would be introducing more examples of algebraic functions with negative integers and fractional exponents as well as the differentiation of the trigonometric, exponential and hyperbolic functions and the inverse trigonometric and inverse hyperbolic functions separately. For each category of functions, we would be solving expressions using all the rules we have learnt and that would mean that students could reinforce their practice in using the rules but at the same time only focusing on one category of functions.

The following were some examples they had to work on in their groups.

Examples: Differentiate the following functions:

$$\text{a) } y = x^3 + \frac{4}{3}x^2 - 5x + 1$$

$$\text{b) } p(x) = (3x^2 - 1)(7 + 2x^3)$$

$$\text{c) } f(x) = \frac{4x - 7}{3 - x^2}$$

$$\text{d) } g(x) = \frac{2}{3x^2} - \frac{x}{3} + \frac{4}{5} + \frac{x+1}{x}$$

As I walked around to observe their work, many were using the product and quotient formulas in terms of u and v . So, they had to identify which will be u and which will be v . I had used different letters for the functions but the students could answer the questions easily. However, I noticed that many students did not simplify the expression for question (d) above before finding its derivative. They used the quotient rule for the first and last expression and then put the answers together but they had the correct answer. I noted that and felt that I would like to address 'simplifying expressions' in the next class.

After the class, I talked to the newcomers and as expected they were repeat students. They said that they just found out that they could sign up for my class. When we checked through our timetable, some students found that they could not come to one of the class session as it overlapped with another subject. I explained that I would like to use all three contact hours as lectures with working on problems integrated within the sessions and said that in the next class, we will try to find another time slot that would be agreeable to all. However, I did not have much expectation that it was possible. This meant that I have to revert to the 2 hours' lectures and 1-hour tutorial class sessions.

Week 3: Second session

Sub-topics – Product, Quotient rules and the Chain rule

I started the class with some examples of slightly 'harder looking' functions to differentiate. Each of the examples could be easily differentiated if simplified first. In the previous class, some students did not check to see whether functions could be simplified first. The product and quotient rules were used on expressions which conformed to the standard form which is, $y = uv$ and $y = \frac{u}{v}$.

I felt that I would like to spend some time to draw the students' attention to some algebraic manipulations in simplifying expressions. I gave the following examples and asked students to solve the questions. I had hoped that it would trigger students to look and think about the expressions before using the rules.

Example

Find the derivative of $y = \frac{(x-1)(x^2-2x)}{x^4}$.

As before, they were still students who tried to apply the quotient rule directly but found the numerator difficult as it had two factors. They felt that this was a hard question. I asked them to see if they could make it an easier question to solve. There was some discussion in the class. However, some students decided to consult other students who had solved the problem or whom they thought could solve the problem.

I saw one student applying the quotient rule on the expression and then applying the product rule on the numerator. It was lengthy and his final solution was not simplified.

However, the working was essentially correct. I put up two more examples for the students to solve as below:

Example

Find the derivatives of the following functions:

a) $y = (x^2 + 1)\left(x + 5 + \frac{1}{x}\right)$

b) $r = (1-t)(1+t^2)^{-1}$

I allowed the students to work in their groups or by themselves for about ten minutes.

Then, I put up the solution for the first example on the board

$$y = \frac{(x-1)(x^2-2x)}{x^4}$$

$$= \frac{x^3 - 2x^2 - x^2 + 2x}{x^4} = x^{-1} - 3x^{-2} + 2x^{-3}$$

$$\frac{dy}{dx} = -x^{-2} - 3(-2)x^{-3} + 2(-3)x^{-4} = -x^{-2} + 6x^{-3} - 6x^{-4}$$

Some students looked at the board but others were still working away. A student asked how to simplify the second example, $r = (1 - t)(1 + t^2)^{-1}$.

I showed her, $r = \frac{(1 - t)}{(1 + t^2)}$ but writing on the board for the whole class to see. More students were now looking at the board. A couple of students asked, “when to simplify an expression and how to simplify an expression?” I said that they were good questions but perhaps they should discuss with their group members first.

We had 2- hours’ work on the rules of differentiation and since there was a little bit of time left, I asked students to give two examples of their own, one of each rule, of differentiation using the product and quotient rules. They did not have to solve their questions but they were to give one ‘easy’ problem and one ‘hard’ problem. During the class, 17 students handed in their work while the others chose to take home their work to be handed in later. As I had asked for some work to be handed in, the students immediately focused on the new task and the discussion on ‘simplifying expressions’ was abandoned. I felt that I had botched a good opportunity to explore students’ skills in using algebraic techniques. I was focused on getting on with the syllabus and only realized the missed opportunity as I saw students abandoning the discussion for the new task. When they were asked to hand in their own examples, the first thing that happened was that some students requested the instruction to be repeated as they were not sure what was wanted. They had never before been asked to do anything like this and found the request surprising and were not sure how to respond.

From the questions that they asked me, they were not sure ‘*what I wanted and meant*’ by ‘easy’ and ‘hard’ questions. From their general reactions, I could not ascertain whether they were taking the request seriously. There was a lot of “giggling” as they were talking to each other, looking at each other’s work, while they were doing the examples. From personal observations and reflections on Malaysian students’ behaviour, “giggling” had

always been a 'defensive' mechanism for students to cope with various situations, pleasant or unpleasant.

In their written work, many students who handed in their work chose polynomial functions as examples with some choosing very simple functions. Thus, the 'easy' questions were too easy and their hard questions were not much harder. However a few also gave examples of other algebraic functions.

I could also see that the new students who had just joined the class did not participate in the discussion. Up to this moment, I had been pleasantly surprised at the willingness of many of the students to contribute and participate in the class discussions. In UTM, students had to sign an attendance sheet for every class but during the last session, I had made a roll call to check their attendance so that I could identify the students who had not been participating. A few were from the first year and the others were mainly the repeat students. Now, in this class the new students had not participated as well.

Some students had to miss one of the three sessions, thus I had to designate that session as a tutorial so that they would not miss the lectures. They were asked to find a different time slot for tutorials or that they were to see me personally for tutorials.

Week 3 – third session

The next topic was the Chain rule. I started the lessons with some examples that I had on some transparencies. It was conducted as a lecture. The notes are reproduced here.

Example: Find the derivative of $(x^2 - 4)^3$.

First we will find the derivative of $(x^2 - 4)^2$.

Relating derivatives:

$$\text{Let } y = (x^2 - 4)^2 = x^4 - 8x^2 + 16$$

Thus, $\frac{dy}{dx} = 4x^3 - 16x = 4x(x^2 - 4)$

However, $y = (x^2 - 4)^2$ is also the composite of the functions,

$$y = u^2 \quad \text{and} \quad u = x^2 - 4$$

We can get: $\frac{dy}{du} = 2u$ and $\frac{du}{dx} = 2x$

We can see: $\frac{dy}{du} \cdot \frac{du}{dx} = 2u \cdot 2x = 2(x^2 - 4)2x = 4x(x^2 - 4)$

Thus: $\frac{dy}{du} \cdot \frac{du}{dx} = \frac{dy}{dx}$

Theorem 2.4: The Chain Rule

If f is differentiable at the point $u = g(x)$ and $g(x)$ is differentiable at x , then the composite function $(f \circ g)(x) = f(g(x))$ is differentiable at x . And

$$(f \circ g)'(x) = f'(g(x)) g'(x)$$

In Leibnitz's notation, if $y = f(u)$ and $u = g(x)$, then

$$\frac{dy}{du} \cdot \frac{du}{dx} = \frac{dy}{dx}$$

Then, I went back to the original question and worked on one more similar example where the students had to identify the different composites of the given function.

The original problem was

Example: Find the derivative of $(x^2 - 4)^5$.

The other example that I used:

Example: Find the derivative of $(3x + 5)^4$.

Most of the students did not have any difficulty doing the two examples. Some of the students who came in later looked a bit lost and I resolved that I would try to give them some personal attention during the tutorials.

Then, I presented another way to remember the Chain rule.

From the notes: One way to remember the Chain Rule:

$$\text{Say, } y = \underbrace{f}_{\text{outside function}}(\underbrace{g(x)}_{\text{inside function}}) \text{ then } \frac{dy}{dx} = \underbrace{f'(g(x))}_{\text{derivative of the outside}} \times \underbrace{g'(x)}_{\text{derivative of the inside}}$$

In other words, to find $\frac{dy}{dx}$, differentiate the “outside function” of f with respect to the “inside function” $g(x)$; then multiply by the derivative of the “inside function” $g(x)$ with respect to x .

I solved an example using this approach

Example: Find the derivative of $f(t) = \sqrt{1-t^2}$.

I invited the class to discuss the problem first and to try to identify what was the same and different between this problem and the previous ones. Some of the students who responded managed to recognise that t was the independent variable in the new examples whereas it was x in the other one. They also managed to state the composite functions as

$$y = u^{\frac{1}{2}} \text{ with } u = 1 - t^2$$

Then, I showed the students a solution using the ideas of the ‘outside function’ and the ‘inner function’

Solution.:

$$\frac{d}{dt}\sqrt{1-t^2} = \frac{d}{dt}(1-t^2)^{1/2} = \underbrace{\frac{1}{2}(1-t^2)^{-1/2}}_{\text{derivative of the outside function}} \times \underbrace{(-2t)}_{\text{derivative of the inside function}} = \frac{-t}{\sqrt{1-t^2}}$$

Two more examples were given for the students to work on in their own time. I suggested that they could try to identify the outer and inner functions.

Example:

Find $f'(x)$ given $f(x) = \left(\frac{x-5}{2x+1}\right)^3$.

Example

Find $\frac{dy}{dx}$ if $y = \frac{u-1}{u+1}$ and $u = x^2$.

I took some time to resolve the allocation of the 3 contact hours, as there were students who could not attend one of the time-slots allocated. As expected, we could not find another common time slot and I promised to revert back to the '2 hours lecture and 1-hour tutorial' mode. Students who could not come to tutorials were expected to work on their own. However, they were reminded that they could make appointments to see me if they had any difficulties. I also said that we would discuss the above examples during the next tutorial class.

Week 4 – Tutorial class

Only 20 students attended the tutorial class. I wanted to know whether the students had been doing any extra problems in their own time. I had given them tutorial sheets for all the topics already discussed. Only a small number of students, five of them, had done some extra work and they were mainly the first year students. I asked the class to work on some problems in the session while I walked around and tried to talk to each student. I had only a couple of minutes to spare for each student. Most of the repeat students were busy doing the exercises from the sheets. We did not discuss the problems that were set in class as most students were working on the tutorial questions.

Week 4 – second session: Sub-topic – Implicit differentiation:

By now, I am running behind my own lecture schedule and I am worried about finishing the syllabus but decided to continue in the same manner at least for the next topic of implicit differentiation. Past experience had shown that many students found this technique difficult and confusing especially in using the notation. For example, given the following

expressions, $\frac{d}{dx}(3x^2y)$ and $\frac{dy}{dx}$, many were not sure of what is the difference between

$$\frac{d}{dx}(f(x)) \text{ and } \frac{dy}{dx}?$$

I start out by giving examples of explicit and implicit functions. I emphasized strongly that “ y is a function of x ” in both cases.

Explicit Functions

Examples

$$y = x^2 - 4x + 2, \quad y = \sqrt[3]{2x - 5}, \quad y = \frac{x^2 + 1}{x^2 - 5}, \quad y = x^3(5x^2 + 1)$$

Implicit functions

Examples:

$$x^2 + y^2 = 4y, \quad \frac{1}{x} + \frac{1}{y} = 1, \quad \sqrt{xy} + 1 = y, \quad x^2 = \frac{x + y}{x - y}$$

The students were asked to look and compare the given examples of explicit and implicit functions, looking at what was the same and different about them.

Then, I started with an example of a function to be differentiated:

Example:

$$\text{Find } \frac{dy}{dx} \text{ if } y^2 = x$$

The solutions were given in two forms:

Solution:

Differentiating explicitly:

$y^2 = x$ defines two differentiable functions

$$y = \sqrt{x} \quad \text{and} \quad y = -\sqrt{x}$$

Thus the derivatives are

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}} \quad \text{and} \quad \frac{dy}{dx} = -\frac{1}{2\sqrt{x}} \quad \text{for } x > 0$$

Differentiating explicitly:

$$y^2 = x \quad \text{then} \quad \frac{d}{dx} y^2 = \frac{d}{dx} x$$

$$\frac{d}{dx} y^2 = \frac{d}{dy} y^2 \frac{dy}{dx} = 2y \frac{dy}{dx}$$

$$\text{Therefore: } 2y \frac{dy}{dx} = 1 \quad \text{which gives} \quad \frac{dy}{dx} = \frac{1}{2y}$$

which gives the derivative for both explicit solutions.

My next example was similar but slightly more difficult.

Example:

$$\text{Find } \frac{dy}{dx} \text{ if } 5y^2 + 2y = x^2$$

The solution was given to the class with the procedure of finding derivatives for implicit functions clearly identified.

Solution:

Differentiate both sides with respect to x :

$$\frac{d}{dx}[5y^2 + 2y] = \frac{d}{dx}[x^2]$$

$$\underbrace{5\left(2y \frac{dy}{dx}\right)}_{\text{by the Chain Rule}} + 2 \frac{dy}{dx} = 2x$$

$$\frac{dy}{dx}(10y + 2) = 2x$$

$$\frac{dy}{dx} = \frac{2x}{10y + 2}$$

In both examples, I focused on the simplest implicit differentiation so as to highlight the use of notation and the role of the variable of differentiation and the use of the Chain Rule.

I then put up a series of expressions:

$$\begin{aligned} \frac{dy}{dx} \\ \frac{d}{dx} y^2 &= 2y \frac{dy}{dx} \\ \frac{d}{dx} y^3 &= 3y^2 \frac{dy}{dx} \\ \frac{d}{dx} y^4 &= 4y^3 \frac{dy}{dx} \\ &\vdots \\ \frac{d}{dx} y^{10} &= 10y^9 \frac{dy}{dx} \end{aligned}$$

Question: What is $\frac{d}{dx} y^{20}$? What is $\frac{d}{dx} y^n$?

However, the exercise was just in seeing whether the students could detect patterns in using the formula. I was using only positive integer exponents and the students found the exercises easy. I then use negative integers as exponents:

Question: What is $\frac{d}{dx} y^{-1}$? What is $\frac{d}{dx} y^{-15}$?

Again as expected, the students could do the exercises easily.

The procedure for implicit differentiation was given to the students as follows.

Procedure for implicit differentiation

Given that y is an implicit function of x and is differentiable at x . To find $\frac{dy}{dx}$:

- 1) Differentiate both sides of the equation with respect to x , treating y as a differentiable function of x .
- 2) Collect the terms with $\frac{dy}{dx}$ on one side of the equation.
- 3) Factor out $\frac{dy}{dx}$.
- 4) Solve for $\frac{dy}{dx}$.

I moved on to questions that would need a combination of differentiation rules in finding the derivative.

Example:

(a) Find $\frac{dy}{dx}$ if $2xy + y^2 = x + y$

(b) Find $\frac{dy}{dx}$ if $x^3 + y^3 + 3xy^2 = 8$

Both examples would require implicit differentiation and the chain rule. The students were given some time to work on the problems. The questions and prompts that were used consistently were “what was the same?” and “what was different?” when referring to the given expressions or in the use of the techniques. In particular, the use of the prompts was to alert students to what was ‘invariant’ in the examples. By using these prompts I was hoping that students would become familiar with these terms and would be able to use them for themselves.

At the end of these sessions, I asked the students to give me one example of implicit differentiation that they could make up for themselves.

From the fifth week onwards, I felt that I had to revert to the more traditional styles of lectures although I still picked out one or two problems for students to work on in class. I maintained the use of the prompts and questions and used other themes in suitable topics. However, most of the problems were mainly dealt with in the tutorial classes.

4.5.3 Review, Reflections and Changes

In this section, I will review several factors that were important in the course such as (1) course organisation, (2) the students; (3) my teaching in terms of my practice and the teaching strategies that were used; and (4) the research methods

(1) **Course organisation** – external factors that were not within my control had some influence over the class

- Students can pre-register for a subject online and register at the Academic Department of the Faculty of Science at the beginning of the semester until the third week. This procedure for students to register for a subject meant that I cannot be sure of the number of the students in my class until about the third or fourth week of semester.
- Due to public holidays and the students' self-extended Chinese New Year holidays, the number of lectures was reduced to 24 out of a possible 30 hours but there was an extra 2 hours make-up lecture.

(2) Students

- The repeat students were very passive with poor class participation. I felt that there was a lack of motivation with very poor record of class attendance and working on tutorial questions, which had to be done in their own time. This was one of the more unexpected factors. I had anticipated some resistance to different teaching methods but not quite in this way. I only found out why, halfway through the semester. Verbal feedback from students indicated that they felt their course was 'irrelevant' and had no career prospects; many wanted to change course but were unsuccessful.
- In addition, the class consisted of students from the various years of study including students who were repeating the subject. However, this created problem in organising the class schedule. To accommodate the students' schedules, tutorials and lectures had to be held in the late afternoon or in the evening.
- Conducting teaching in an atmosphere of interaction was new to the students and surprisingly difficult for me too as, more time was needed to finish a topic and it was sometimes difficult to stop myself '*showing*' a student how to do the mathematics.

(3) My Teaching

(a) My Practice

- I found that changing my practice was difficult as I was still ‘showing’ rather than ‘facilitating’. Even so, I was constantly trying to be aware of my own practice and it was very easy to fall into old habits. I also found it difficult to identify the ‘mathematics structure’ and the ‘mathematical thinking’ parts in the topic of differentiation and how to present the teaching contents differently. I had taught the subject for so many years that it was quite easy just to walk into class and tell them what to do and how to do it, but now, I had to prepare, think out the strategies to carry out and try to find new ways to present the content. I found it hard to find ways to encourage students to ‘see’ for themselves. I tend to fall into the trap of telling them what to see. I did find that some of the students in the class were more interested in participating but I also felt a greater trepidation upon entering the class, as it was not easy to predict what the students would do or ask. I have had to respond ‘spontaneously’ to ‘events’ in the classroom as well as be more sensitive to ‘triggers’ from students. The word ‘triggers’ here is used in the sense of being aware of students’ body language or cues that could help me to know what they are thinking of or what difficulties they are facing. Furthermore and non-trivially, I needed to develop a ‘vocabulary’ in Malay to share my ‘thinking’ with the students.
- How do you ‘display’ thinking? The dilemma of how much to do and how much the students should do for themselves are not easily resolved when you are in front of the class and you only have a 50 minutes teaching period with so much of the syllabus to finish. I become so involved in the teaching and with the students that it was not easy to maintain the objectivity required for the research observations to be recorded.

- In considering the class, my choice was based on the perceived autonomy associated with teaching the group but little did I realise that I would be facing a group with so much background '*problems*'. As described earlier, they had difficulties with their self-esteem. It was very rare to see a whole class of students so lacking in confidence in their choice of course and career. They also had a more varied mathematics background than expected as entry requirements. There were too many repeat students in the group who infected the first year with their negative attitudes towards the course and the subject.
- There were some students who had responded to the teaching strategies and they were active in class, willing to work through the problems that were given and were also more responsive to my queries or request for explanations on their work.
- I managed to develop a relatively good relationship with some of the students but this created new situations. I had students who felt '*gratitude*' at having an '*approachable*' lecturer, someone who cared, that they were always trying to '*please*' me. This was clear from their comments in test papers or verbally. For example, they offer '*apologies*' for not being able to answer questions, thank me profusely for being a '*good*' lecturer while regretting their lack of understanding or ability to answer questions. I was quite at a loss as how to restore their '*self-esteem*' and how to wean them away from this '*overly*' respectful behaviour. I could not distinguish between students' real responses and their responses to '*what I might want*'. I was still committed to teaching the second year Calculus course for the same course group (2 SRI) and I knew that some of these students would be in the group.
- However, this experience strengthened my resolve to continue with the mathematics teaching approach that I had chosen. I felt that there were some of the

students who had gained some confidence from being able to ‘see’ and ‘do’ the mathematics for themselves.

- I still believed that the framework designed by Watson & Mason (1998) was able to deal with the cognitive challenges as well as some of the affective difficulties that the students were facing especially those connected with mathematical anxiety, lack of confidence in problem solving and the uncertainty of how to proceed when they are stuck in a problem. However, I had not shared with the students about the process of Mathematical Thinking as described in Mason, Burton, and Stacey (1982) which I was using and considered doing this for the next class.
- I also believed that if the students become aware that they were using their own powers then they would realize that they were more able than they thought of themselves. I felt that I needed to raise the students’ confidence but through increasing their awareness of their own ability to enhance their mathematical performance.

(b) Teaching Strategies

- After the mid-semester break, there were still 5 weeks of lectures, but I was focused on finishing the syllabus to prepare students for the final examinations. Thus the new strategies were only used for the revision of functions and the topic of Differentiation. I needed more time in implementing an activity based teaching and the two hours slot for lectures was not enough. It was augmented by the tutorial session but due to time-tabling difficulties, not all students could come to the class.
- Another unexpected difficulty was in translating some of the prompts into Malay (the medium of instruction), for instance, the word, “undoing” in English does not have a single Malay word equivalent and a description had to be used instead, which makes the prompt less clear. Thus, there is a need to modify the prompts and questions for future use.

- Throughout the course, there were some students who did not participate actively in the class activities. I found that these students consisted of some individuals who were very competent on tests and some who were very weak. The competent students felt that I was making things more difficult. When I discussed the teaching strategies with some of these students, they claimed that the 'old' way was faster. When I asked them to clarify what they meant, they said, "*you know, give lectures, some examples and answers to the problems*". They said they preferred to do the problems on their own time and said that if they cannot do them then they will come to tutorials. I talked to one of the 'weak' students who was very despondent about not following any of the mathematics taught so far. He said that he felt that he would like to change to a different course, if not in UTM maybe some other institution. He was from the Arts stream when he was in secondary school.
- Many students had forgotten or had faulty recall of a considerable amount of the previous mathematical knowledge learnt at school. They were also weak in linking various mathematical concepts and did not seem to see the need for revision of 'old' mathematical knowledge or to investigate possibly useful connections between current mathematics learnt and the 'old' knowledge. This became more apparent when we had to solve word problems in the Applications of the Derivatives. We had to use concepts about triangles and ratios taught at school in the second and third year of five years of secondary school.
- I also realized the need for students to see the prompts and questions rather than just hear them. Thus for the next class, I planned to put the relevant prompts and questions on my notes and examples slides. Students would also be given copies of my lecture slides

(4) Research Methods

- I started the research with an idea to use students' performance as an indicator of changes in their attitudes, and began with conducting a diagnostic test, collected students entry mathematics results, their foundation mathematics and final results. However, there was Siti and Noor who worked very hard in class but their final results did not show that.
- Examination results do not capture the students' struggle and efforts exerted. Some of the hardest working students did not do well in the examinations.
- It was difficult to carry out observations whilst teaching. The reading of accounts must take into consideration this limitation. Observations usually were made during students' activities. Some exceptional incidents that occur during teaching were noted and written up fully afterwards as soon as it was possible. It was also not easy to record incidents with specific students if that occurred while I was teaching.
- I also needed a more objective view about my own teaching strategies so I will be asking two colleagues to review and evaluate my ideas.
- I will need to ask students and colleagues to evaluate my teaching in the next session.

4.6 The Teaching of Calculus II for Group B02 – semester 1, 2002/03

4.6.1 Students' data – 2 SRI

The number of students in the class was 48 made up of the following students, 9 were from the 4th year, 11 from the 3rd year and 27 were from the 2nd year. 2 students subsequently withdrew from the course. Out of these students, 28 had taken the Basic Calculus course with me. The others had done the Basic Calculus previously with other lecturers. The 28 students came from different years of study and consisted of 21 students from the second year, 4 from the third year and 3 from the fourth year. The others were in my class for the

first time. The subject I was teaching was Calculus II, a 2-credit subject whilst the mainstream students take a 3-credit Engineering Mathematics (Calculus II). Again as described earlier this was a subject to be taught by a single lecturer. The topics in the syllabus were: Conic sections, functions of several variables, Partial differentiation, multiple integrals, and Differential equations (see Table 4.8).

The syllabus started with the conic sections because as described earlier, this particular course accepted students without the additional mathematics background and thus they needed this topic as it was necessary to help them to graph functions of two variables. The syllabus also differed greatly as compared to mainstream engineering students in that firstly it is a 2-credit subject and secondly it did not have topics such as Functions of three variables, Triple integrals, and Vector calculus. Furthermore, mainstream engineering students would have Differential Equations as a separate subject.

Table 4.8: Number of students in 2 SRI

Course: 2 SRI, Industrial Design	
Course year	No. of students
2	27
3*	11
4*	8
Total	46

Table 4.9: Weekly lecture schedule for SSM 2242

WEEK	TOPICS	CONTENTS
1	Revision	Functions: domain, range & graphs
2	Further Geometry:	Conic sections: circle and parabola
3		Ellipse and hyperbola
4		Polar coordinates and parametric curves
5	Functions of two variables:	Graphs, domain and range, level and surfaces curves
6		Partial derivatives
7		Rate of change and multivariable extrema
8	SEMESTER BREAK	
9		Multivariable chain rule

10	Double integrals:	Definition, double integral in Cartesian coordinates
11		Area and volume by double integration
12		Double integration in polar coordinates
13	Ordinary Differential Equations:	Second-order homogenous linear differential equations
14		Second-order non-homogenous linear differential equations, method of undetermined coefficients
15		Method of variation of parameters

4.6.2 Teaching approaches that I used

In the following description, I will pick certain episodes to depict the ambience in the class.

Week 1 – first and second meeting

There were about 44 students in the class for the first meeting with the majority of students in the second year. 4 more repeat students came to class later in the second and third week.

Topic: Functions – domain, range and graphs.

- (a) I started with some revision of finding the domain and of functions of single variables.

Topics: Straight lines, domain and graphs of functions, parabolas.

Lesson focus: recognising a class of functions; specialising and generalising;

prompts and questions used: “*what is the same?*”, “*what is different?*”, “*what can change?*”

This session was conducted as a whole class lesson to about 40 students. Based on my experience with the Basic Calculus group, I decided to present a short summary of ideas about Mathematical Thinking to the class in the first lecture. I had to use the first meeting although I realized that I might not have all the students in class yet, due to the tight syllabus schedule. The students listened to the lectures and there were no questions or comments. Then, I went through the basic concepts about straight lines, starting from the

representation of a point in the rectangular coordinate system. I had prepared some questions in a worksheet and asked the students to do them in pairs. They were asked to hand in their work for some of the questions. The rest of the first session was used by the students to work on the problems. As they worked they talked to each other but the general atmosphere was subdued. Students who had been with me in the basic calculus class appeared to be busy doing the problems and identified their partners quickly. All the students had handed in their work which was a positive sign.

The number of students in class was encouraging although I did not know if there were more students coming but the SRI courses usually have smaller classes so I felt that maybe the majority had turned up.

In the next meeting, we started on the topic of domain, range and graphs of single variable functions. I asked the class if anyone can remember the shape of the graph of $y = x^2$. Many students knew what it looked like and there was a good response. Then, they were asked about the graph $y = x^2 + 1$, what they thought it would look like. A few students responded including Ming. Ming usually sat with Baba. I was happy to see Ming as he was very responsive in my previous class and I felt that I had a supporter in this class. I drew the graph $y = x^2$. With the students input, I drew the graph $y = x^2 + 1$. I gave a third example, $y = 1 - x^2$, the students who had responded earlier were quick with their response without any prompts. However, there were still a small number of students who did not participate. Then, I asked the students to discuss the examples using the prompts and questions: “*what is the same?*” and “*what is different?*” and “*what can change?*”

There was a mixed response from the students. Students who knew me from Basic Calculus appeared to know what I meant and started discussing with students sitting next to them. Some of the new students were not sure of how to respond – they asked “*what do you want?*” or “*what do you mean?*”

When they were invited to interpret the questions as they wished – they were not happy and a couple of students immediately inquired if they will be evaluated on this work. One of them said: “*have to work smart*” – which meant (upon enquiry) – “*do just enough of what is required to get the marks*”.

I answered that since we were working on revision topics that there would be no marks given to the work they were doing or will be handing in. I decided then that we will look at the examples drawn on the board together. We picked out the attributes that were the same and then those that were different but did not manage to answer the question “*what can change?*” Many students were contributing but I did not manage to get more than 10 of the new students to be involved in the discussions.

So, for the next session, I brought along worksheets for the students to work at in groups of twos or threes – with some work to be handed in for evaluation. It appears that if a piece of work will be marked then it has more value. The response was better and there were many pockets of lively discussions. When asked the same questions: “*what is the same*”, “*what is different*”, “*what can change*”, there were attempts now to answer the questions although many still said that they were not sure of “*what I meant*”. However, there were a few students who asked whether the topic was in the syllabus! In fact it is not, as this was a first year topic. It took most of the 50-minute session to go through the worksheet. Examples of the translated worksheets are given in Appendix B.

Week 2 – first and second meeting

Topic: Conic Sections

I presented my lectures on the Conic sections starting with circles. I started with the standard equation $x^2 + y^2 = r^2$ with the centre at the origin (0,0) and radius, r . I showed students how to work out the equation using basic geometry. I presented an example and prepared two more for them to work on. I suggested that students should work in pairs or at most, three in a group. The class was not designed to facilitate group work so most

students chose to work in pairs as that was easier to manage. I believed that most of the students would be able to do the questions as circle was a topic that was also covered in their secondary school syllabus. I then introduced the next part of the lecture. I moved the centre of the circle to a point $(2, 3)$ with a radius of 2. I asked the students to work out the equation. As I walked around, many were using a version of the standard equation of the circle that was taught in school, $x^2 + y^2 + 2gx + 2fy + c = 0$ where the centre is $(-g, -f)$ and the radius, $r = \sqrt{g^2 + f^2 - c}$. I was hoping that they would come to the equation $(x - 2)^2 + (y - 3)^2 = 2^2$, using the same ideas that was used earlier; basic geometry. Of course some students worked out the equation using the second method and gave the answer in an expanded version.

I decided to work through the problem on the board as a whole class session. I work through the problem in two ways, firstly using basic geometry and then again using the formula that they were using. They were busy copying the notes. I then asked them to think about "*what was the same*" and "*what was different*" between finding equations of the circle at the origin and at a different point. After about five minutes, we discussed the answers and a few students contributed the answers. I consolidated the ideas by going through the lectures again about the similarity and differences between the formulas: $x^2 + y^2 = r^2$ and $(x - h)^2 + (y - k)^2 = r^2$.

I also had a tutorial sheet of extra problems for them to work on. The problems were mainly in three categories, 'doing' questions where the students were given information and had to work out the equations of the circles; 'undoing' questions where they were given the equations of the circles and had to deduce information about the centres and radius. They were also word problems for them to work on. The tutorial session was in between the two lectures so we continued on circles during the tutorials.

We worked on parabolas in the second class meeting. I had new students coming to class so I gave them the revision and tutorial sheets for them to work with.

Comments

Generally, the students were participating in the classroom especially in doing the class tasks. However, those who would frequently offer answers when requested were students who were from my Basic Calculus class. Ming was still working with Baba but Baba was more eager to share his responses in the class, although not always giving the correct answers. The general class situation was more encouraging as the students did not appear as despondent as the previous class. A small number of students were not responding to the prompts and questions, nor would they do their class exercises fully. They were the new repeat students from the fourth year.

I was still teaching in the same way, presenting lectures, using some prompts and questions. I used the examples to direct students' attention to the mathematical structures or processes. However, I was aware that I needed to try to make sure that the amount of time I spent for each topic or sub-topic followed the lectures schedule. In the Basic Calculus class, I had to rush through the topics at the end of the semester as I was behind in my lectures.

There is a certain tension in trying to allow students to work on the mathematical concepts taught until they can grasp them as compared to working through the syllabus. Generally, I had chosen to spend some time for students to work on recognising how to use the general formulas and recognise the different variables or constants used. Extra exercises were given for students to work in class as well as for tutorials so that I could see if they had understood. In choosing where in the topic, more time should be spent, I depended on students' difficulties identified from previous experience, my own and that of my colleagues.

Week 3 – ellipses and hyperbolas

We continued with the topics of ellipses and hyperbolas. I am using 1 lecture session for each topic. I worked out the simple standard equation from the definitions with the centre at the origin and spend some time to show how the equation was changed for centres at other points.

However, there were a few students who did not appear to be taking part in class activities. I decided that I would talk to them to find out more about their situation.

Extract from notes:

I am quite happy that majority of the students were participating except for a few who came to class but do not appear to be doing anything. They sit at the back, do not take notes, not doing the tasks. I will need to talk to them!

At the end of the session, I asked for the students to work in their groups, in pairs or groups of three, to hand in examples of questions on the conic sections which they considered as easy and hard. They could pick out the questions from the tutorial sheets, recommended text book or any suitable book on Calculus. They were to hand the work in the next class. They were also asked to pick a question that they thought would be appropriate as an examination question. There would be no marks given for this exercise. 38 students handed in their suggestions. I used those examples to identify what students thought they had understood and which problems they found difficult. I used some of the hard problems as problems for them to work on in tutorials.

Week 4 – Functions of several variables

In one of the classes, some students had brought still life sketches and pictures they painted. These were assignments for one of their classes. Most of the work looked good. The next topic was Functions of several variables so I thought I would use this opportunity to appeal to the students' artistic traits. In trying to picture a 3-D surface, I tried getting

students to come out and create different heights. There were some students who came out but very reluctantly. I thought that it would give them a better idea by what is meant by a surface as well as 3D coordinates. However it did not go down well with the students as they did not like coming out to the front and they had to imagine the axes. Then, we worked on finding the domain and range of the functions as well as drawing the graphs. I started with asking them to draw the plane projection of a drinking bottle, looking down from the top. Most of the students drew the projection correctly. Looking at the circular contour, I asked for the possible function to depict it. They knew it was a circle so that went down well. I asked them to imagine the projection if they were looking at the bottle from the sides. Again, they knew how to describe the projection. I had a few other objects such as a small cone, a pyramid, a cylindrical object and a Japanese cup (no handles). I gave out the objects and asked the students to draw these objects from different perspectives, top looking down and from the sides. They can work in groups or alone. Each group had to pass the objects they had to the next group once they had drawn it. They looked like they enjoyed doing that except for four students (all male). They sat apart. There were some students absent from class as well.

I asked them to imagine their pictures with respect to xyz – axes. I talked about the domains and range of 3D objects based on their diagrams. I moved on to using formulas and how to find domain and range of functions by thinking of the properties of the functions and numbers.

The example that was used in the class:

Find the domain and range for the function, $z = \sqrt{1 - x^2 - y^2}$.

I solved this question and then asked the students to see what could change but to keep the same method of solution. I had prepared the questions on a sheet and handed them out to the students which they could refer to as they worked.

From the sheet:

- 1 (a) Find the domain and range for the function, $z = \sqrt{1 - x^2 - y^2}$.
- (b) Please look at each term in the given function, what can you change but keeping the same method of solution? Please give an example.
- (c) Please give another example. What did you change and what did you keep the same?

The following are samples of answers from the students and their comments (translated from Bahasa Malaysia).

Ming and Baba:

(b) $z = \sqrt{16 - 9x^2 - 9y^2}$ - we change the constants and we kept the form as a circle

$(ax^2 + by^2 = 1)$. Constants for a circle must be the same, a and b are the same.

(c) $z = \sqrt{-12 - 4x^2 + 3y^2}$ - change the constants, positive and negative signs.

Constant x^2 and y^2 are the same, $\sqrt{\quad}$ the same.

$$3y^2 - 4x^2 - 12 \geq 0$$

$$\frac{y^2}{4} - \frac{x^2}{3} \geq \frac{12}{12}$$

$$\frac{y^2}{4} - \frac{x^2}{3} \geq 1 \quad (\text{hyperbola})$$

In this example, I could see that they attempted produce a different example in (c) as compared to the one they gave in (b) although I was not sure of what they meant when they say that, “Constant x^2 and y^2 are the same”. However, from their example, I could infer that they meant that they could have different values of coefficients for x and y rather than constants. Although by doing so, they had changed the function of the domain from a circle to a hyperbola which they recognised. I felt that I would need to ask my colleagues to review the wording of my questions as well to see whether they conveyed what I wanted to ask.

Lee, Wong and Teh:

(b) $z = \sqrt{3 - a^2 - b^2}$ - x term was changed to a ; y was changed to b and the constant to 3. Even though the terms and constant were changed the solution method is still the same.

(c) $z = \sqrt{64 - 4x^2 - y^2}$ - constant changed to 64; x^2 changed to $4x^2$; y^2 still the same. Positive and negative signs are the same as in example 1(a). But the method of solution is different. Variable function in 1(a) is a circle but in example 1 (c) is an ellipse.

Mat and Shah:

(b) $z = \sqrt{64 - 4x^2 - y^2}$ - changing coefficients for x^2 and y^2 and value in the equation.

(c) $z = -\sqrt{x^2 + y^2}$ - changing values to become a different equation.

Comments

When I gave the problems, I was thinking that students might give examples that would be different forms of circles in the general form $z = \sqrt{b^2 - a^2x^2 - a^2y^2}$ but the students' answers were informative. My first thought as I looked through their work was that my 'prompts and questions' were not clear as the students were interpreting them differently from that which I wanted. As I went through their answers, I could also identify the students who did not really understand the questions which would explain why they did things that were different to what I had in mind. What they had chosen to vary showed me something of the scope of their perceived generality, their dimensions of possible variations but raised questions about their appreciation of the structure necessary for a circle. I needed to consider whether the students did not know how to answer the questions or they were not sure what the questions were asking. A review of the prompts and

questions was necessary and I intended to discuss them with my colleagues and also talk to some of the students.

4.6.4 Review, Reflections and Changes

I have a different group of students but 28 students were in my Basic Calculus class. However, the class was livelier and the use of the worksheets seems to help in giving some structure to their working in class. As described earlier, Malaysian students love to work in a structured manner. I am also aware that the main focus of my work was still in bridging the transition from moving from elementary towards advanced mathematical thinking. I am still providing the support for making students aware of how to think about their mathematics. We work on specializing, generalizing, comparing and contrasting, recognizing and characterizing. We do a lot of work on reading and understanding the symbols and notation.

In this semester, I only used the new strategies for the topics of multivariable functions and partial differentiation. There was not enough time to use the methods for the other topics. Using the strategies and breaking up the students to work in groups required more time than the two hour lecture slots available. Although the students appeared to be more active in class, I could categorise them generally into three groups. The first group were students who were quite good with mathematics and were coping well with the Calculus. Some of them were using the prompts and questions but there were students who ignored them unless the prompts and questions were part of the assignment that was graded. The second group were students who found the prompts and questions useful and were using them in their work. This was a mixed group with some who were good with the mathematics and some who were not so good. The third group were those who were not responding to the lessons and rarely turned up for the tutorials. UTM has a policy that allowed only students who had a minimum of 80% attendance record to sit for examinations. I think the students attended class just to fulfil this requirement. They were mostly repeat students and were

quite weak mathematically. I offered some extra revision in any of the earlier mathematics that they needed for this class but they were no takers from this group. This was on top of the revision that I would conduct during the lessons if and when required.

Responses from an evaluation questionnaire handed out at the end of the semester indicated that an important factor in students' enthusiasm for my class was that they liked me. I was described as '*understanding*', '*approachable*' and '*helpful*'. They vaguely described my teaching as '*good*' but gave no indication about what they thought about the teaching strategies I was using. I decided that I would give out my own evaluation questionnaire for any subsequent class.

4.7 The teaching of Basic Calculus – Group C02, Semester 2, 2002/2003

From all of this experience, I decided that I would teach mainstream Engineering students as the students from Industrial Design group were in a category that would be obsolete in the future. I also decided to focus on one subject only, Engineering Mathematics which was an Advanced Calculus course. I needed to negotiate with the Department Head to get the appropriate classes. As mentioned in Section 4.3.1, the new intake of students in UTM would be post-SPM students and the teaching was in English.

However, I forwarded some comments on the students' morale and the syllabus after teaching the SRI students to Head of the Department of Mathematics to be forwarded to the Mechanical Engineering Faculty. In particular, I thought that the students needed to know more of the possible career prospects that would be available to them with the qualifications that they would obtain. I also suggested that the Mathematics requirement should be the same as for other Engineering courses as students who had no Additional Mathematics background found the Mathematics courses very difficult. I also recommended that the Mathematics subjects should be upgraded to three credits.

For the purpose of research, I wanted to work with the mainstream students but I was still assigned to teach the SRI students. Thus, at the beginning of the semester, I found it

difficult to consider this class as part of the research but I decided to use the same teaching strategies on the topic of differentiation, repeating what I had done in the first class and enhancing the teaching with the changes that I did in Calculus II. In particular, I was still using strategies to invoke mathematical thinking. I kept the paired work, maintained classroom tasks that were carried out within the lectures and used the tutorials to work with the students individually. The students were made up of 13 first years, 10 second years, 13 third years and 2 fourth years. 9 students had been in my previous classes and four of them were in both the first year and second year Calculus. I had Baba, Musa and Lily again. The number of students who had applied and accepted the offers from UTM was very small, only 13 and out of those, there were 2 students who had no Additional Mathematics qualifications. Although, to all intents and purposes, the research was temporarily on hold, I still made notes about my observations about interactions with students as well as my personal reflections on my behaviour. These notes were used to reflect upon the research, its direction and shortcomings in my application of the research methods. However, at the end of the semester, I decided to interview five students. They were a mixed group consisting of Liu and Lim (first years); Baba and Musa (second year and third year students respectively), and, Fadil, (a 'direct entry' student and considered as a second year). He had qualifications from a local polytechnic. The interviews were informal but were centered on 4 general issues: (i) how can the lecturer help students to understand? (ii) what would they like to comment on my teaching? (iii) How to help students who are weak in mathematics?, and (iv) what changes have they experienced since coming to UTM?

In the interviews, the students shared their views about how they studied and were generally said that they like the way I taught in class but could not say explicitly which of the teaching acts or strategies appealed to them. However, one student wrote down his responses and said, (translated from Bahasa Malaysia, comments by Liu)

" When teaching a topic, the examples must be changed little by little to show the possible changes to a question. These examples can be copied by the students so that they can read them again & use them as guides for revision for tests. Lecturer should ask students to make their own examples and solve those themselves. This will help them to understand the topics. "

I also gave them some problems to do during the interview and asked them to explain their answers. All the students carried out the exercise and tried to explain their workings which indicated a willingness to communicate mathematically. The responses during the interviews were useful in providing a sample of students' views about their learning and my teaching.

4.7.1 Review, Reflections and Changes

This semester was difficult for me as I could not focus on the research. The interviews were conducted at the end of the semester and were useful in the sense that I had a semi-formal students' feedback about their views on the teaching and learning in my class.

In my teaching, I found that I had a tendency to revert back to my old way of teaching. I would do examples on the board, showed them what I was thinking and then set similar exercises for the students to work on. I was doing a version of their study technique, 'redoing' an example. I was focusing on 'specialising' and not being able to extend the examples to include 'generalising' tasks. In terms of exercises, the problems were concentrating on the procedures and increasing students' understanding on how to use the techniques. I was also focusing on students who appeared to be working hard in the class but not progressing or achieving scores to match. The course work usually consisted of quizzes, assignments and tests. I have to administer these assessments to the students as they were considered as standard assessments in the Department. Students' achievements on these were used as guides of their possible final scores.

However, I had already made up my mind that I needed to change the student groups to study and choose students from the new intake. I intended to use all these observations and notes so to be more systematic in using the research methods with the future groups. I also wanted to take a semester out to prepare the teaching materials and classroom tasks. However, for the academic session, 2003 – 2004, I was asked to teach a special group of students, who were sponsored by an agency called Majlis Amanah Rakyat Malaysia (MARA, Council of Trust for the Bumiputera of Malaysia). These students were selected from various MARA Secondary schools whose trial examination results were excellent and thus they were in the programme before their SPM results were out. I requested to teach Calculus for the new intake but I also ended teaching one more group from the SRI course.

4.8 The teaching of Calculus II – Group D04, Semester 1, 2004/2005

4.8.1 Why I taught the Class

I was assigned a Calculus class of the new curriculum with a subject code SSE 1792. However, two students from the SRI course, Siti and Noor, came to see me to discuss the possibility of my taking a class for Calculus II, SSM 2242. I knew them as they were in my Basic Calculus class of 2001. They said that a group of students wanted to do the subject but were told that it was no longer on offer and that they had to follow an equivalent class from the new curriculum which meant that they had to sit for the new Engineering Mathematics subject with the code SSE 1893. They said that the Head of the Department said that if they could find a lecturer willing to teach and if the number of students was more than 20, then the department would consider allowing the subject to be offered for the semester as a special consideration. They also said that they had about 30 students who would register for the subject and many would prefer if I taught the class as they said that they found that they could understand better. About 13 students would be repeating the subject and had attended either the Basic Calculus or Calculus II of my previous classes. I decided to take the class because I felt sorry for their plight as they would have to take up a

3 credit subject and do it in English. I was teaching both Calculus II and Calculus for the new first years in this semester.

4.8.2 Teaching Strategies

I chose to use the same strategies concentrated on the topics of conic sections, multivariable functions and partial differentiation as before. However, I made some modifications and these will be described as follows.

(i) Spot revisions – I promised the students that I would do revisions on topics needed from Basic Calculus or Algebra if and when needed.

(ii) Using worksheets. I had prepared worksheets to be used during the lessons and for classroom tasks. I had noticed that the previous students did not become familiar with the prompts and questions. Only a few students were using them even though I was using them consistently. I thought that I had to make these prompts and questions explicit and that they needed to be seen. Thus I had chosen the examples, arranging them in a worksheet in the structured manner with the prompts and questions incorporated as part of the classroom tasks. The emphasis was still on specialising, generalising, sorting, characterising and recognising classes of examples. I had also included questions that would require these powers in the worksheets. Similar questions were also included in the tests.

(iii) Group work. I decided that all classroom tasks were to be carried out in groups of 3 – 4 people at all times. I had worksheets only for the topics mentioned above. For the other topics, we used examples from the textbooks or tutorial sheets but the students still had to work in groups.

(iii) Changing the assessment methods. The students were given marks on the classroom tasks that they were doing. They had to sit for two tests but for the first test they were allowed to bring 1 A4 sheet of notes or formulas that they thought could help them do their tests but that they had to hand in the 'crib sheet' with their work. I also allowed the students to decide when the tests would be carried out, letting them decide when they were

ready. I only stipulated that the first test had to be taken within weeks 6 – 8 and that the second tests had to be taken within weeks 11-13.

The students were made up 8 second years, 9 from the third year and 13 fourth year students. Siti and Noor were in the fourth year. They said that they had delayed taking the subject until this semester but wanted to do the subject with me as they had enjoyed their lessons during Basic Calculus. During lessons, the students were very active and participated fully except for three students. It was easier for me to conduct the lessons as they were very responsive. As the class number was small, it was also easier to give personal attention to them during tutorials.

Lily was also in the class but she was still keeping to herself although she was doing her work but I noticed that she was frequently absent. However, a chance meeting and a spontaneous decision on my part to talk to her elicited some interesting information and gave me some insight of the personal problems that she was facing. I offered to help her with the mathematics and she accepted. This small breakthrough gave me a chance to establish a relationship with Lily that brought about changes to her attitude towards working with mathematics. I will write about Lily in Section 6.3.3.3 and present her case as an illustration of a model of change that I would be proposing.

Siti and Noor were very enthusiastic about my teaching. They were working together and were using the prompts and questions, always willing to explain their work and would come to see me often to work on topics that they found difficult. However, they always came prepared with questions and samples of their attempts to solve problems and it was clear from their questions that they have read up the lecture notes.

The whole class atmosphere was much more positive, cheerful and very encouraging to me. Even when I am late for class, I would find them working on some questions while waiting for me.

I had decided that the students could do the first test as a group and that they could bring their own 'crib' sheets which must be one A4 sheet only. The first test was on the topic of Conic Sections and I found that most of the students had copied formulas of the general equations and graphs on their crib sheets. I had included a sorting question where the students had to sort 16 examples of Conic Sections into the various types by inspection only. Surprisingly, even though the students had the formulas for the general equations, many did not manage to sort the equations correctly. The highest correct answer was 11 equations. The last question was a characterising question where students were asked to choose two equations from the previous question and compare those to the equation $Ax^2 + By^2 + Cx + Dy + E = 0$ and discuss the values of A, B, C, D and E and how they are related to the types of Conic Sections. None of them answered the last question.

For the second test, they were asked to sit for it individually but again they were allowed to bring their own crib sheet which must be handed in with their work. The test was on the topics of Multivariable Functions and Partial Differentiation. Going through the crib sheets, I found that all of them had copied worked examples rather than formulas.

4.8.3 Review, Reflections and Changes

I realised that I was also affected by students' behaviour as I found myself looking forward to come to this class because they were very responsive and eager to do the classroom tasks. It made me wonder whether I was similarly affected when the students were not very responsive. I was also more prepared and systematic in carrying out the teaching and in collecting the data. I had given out an evaluation questionnaire (see Appendix C) at the end of the semester about their views and feelings on the teaching, their learning and the teacher. There were several issues that I had identified based on all the experience and these will be discussed in the subsequent section.

4.9 Issues and discussion

I spend four semesters trying out the teaching strategies to two classes of basic Calculus and two classes of Calculus II. I had hoped that the first two classes would have been considered as the settings for the research and that I could have written up the thesis by then. However, my own shortcomings in implementing the research methods, the changes in the intake policies, the special circumstances of the SRI group were very significant and induced me to reconsider the students to be made the research subjects. I needed more typical engineering undergraduates so that results of my study would be more relevant. I knew that by adopting an action research perspective any results would contribute more to local relevance rather than global but relevance was an important consideration. The experience of teaching the SRI students was useful and several issues were identified and these were used as the starting base for the subsequent study with the mainstream engineering undergraduates.

Issue 1: Changing attitudes

My perspective: I found changing difficult but in turns, exciting and disheartening. I had to confront and examine my assumptions about students' difficulties, my own beliefs about mathematics and how to teach it. I had very strong beliefs in the importance of supporting my students' awareness and use of their mathematical thinking powers. I modified and designed strategies and tasks to put my ideas into practice but found that it was not easy to put aside old habits. I had to adopt new ways of teaching, learn how to facilitate, come up with ideas 'on the spot', and deal with students' mathematical and affective difficulties. I do care but I was not very comfortable with the students who felt 'gratitude' and were trying to 'please me'. I wanted them to be independent learners and become more confident with their awareness that they could learn. A few students indicated through their personal or comments in the evaluation questionnaires that my '*friendliness*', '*caring*' and '*understanding*' were important in motivating them to learn.

Students' perspective: The students did not have much say in how a lecturer could teach a subject. When I indicated that I would be teaching the class in a certain manner for some of the topics, the students were told that they could give me their views at any time, about the teaching, the assessment or any other concern that they have. I had to show that I meant what I said and if any student gave their views, it was considered and if necessary brought to the whole class for discussion. Most students indicated their concerns about how these changes would influence their assessment and final examination. Some indicated their willingness to participate and appeared to engage in the class activities and tasks. A few also willingly talked about how they have changed their feelings about learning mathematics after following my classes. It is not easy to measure change and difficult to get firm evidence that the teaching strategies have influenced students but nevertheless some episodes indicated that students thought that the teaching strategies helped them to understand better. However, there were students who were not comfortable and thought the methods were too slow and complicated what was easy. They preferred the 'traditional' ways of teaching. I had acknowledged their concerns but reminded them that the strategies were used for some topics only.

Did the teaching strategies contribute to change in attitudes? My observations indicated that students who were in my classes more than twice appeared more able to communicate their difficulties and the quality of their communication were better. They were able to start off in doing problems but not necessarily getting the correct answers. However, it was not easy to associate this behaviour wholly to the strategies as students' maturity could be a big influence. Rapport was also considered as a factor in the students being more willing to communicate.

Did the strategies contribute to students getting better results? It was not possible to make any conclusions about this aspect as the changes were implemented only in a few topics. However, I can conclude that students who had a poor grasp of previous mathematical

knowledge needed more time as they had to work harder to consolidate their old knowledge and on top of that try to cope with the new concepts.

Issue 2: Factors Influencing Students' Study of Mathematics

Motivation was an important factor in determining students' participation and commitment to the classes. Students who were highly motivated were good in their mathematics, had clear ambitions and good study habits being very consistent in their work for all the subjects, not just the Mathematics. Some students identified lecturer's '*friendliness*' and '*helpfulness*' as factors that could influence their own motivation. Many who were disheartened with the Mathematics said that it was not relevant to their Industrial Design course. However some were even disappointed with the course citing their seniors' difficulties in finding employment as evidence that the course was also irrelevant.

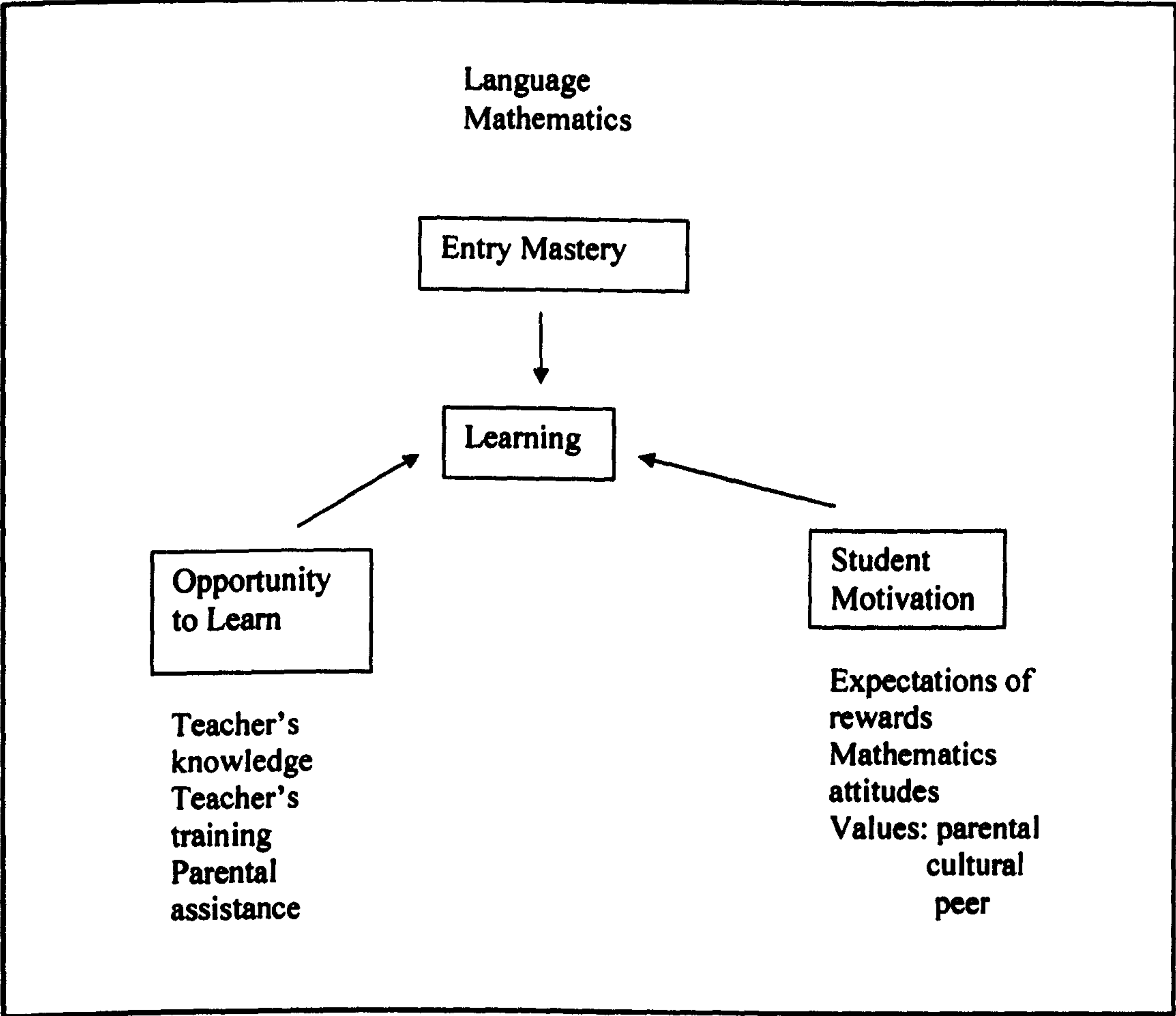
There were students who felt that they were not good in mathematics and expected only to scrape through the course rather than getting good grades. Their beliefs about their mathematical ability influenced their learning behaviour. They did not expect to like nor understand the mathematics taught. Repeating was considered normal.

I had noted that there were students who felt that my '*approachable personality*' was an important factor to motivate and persuade them to participate and make more efforts to deal with the mathematics. In turn, I used this feeling of rapport to persuade them to do more mathematics either during the classroom activities or in extra sessions out of the classroom.

In going through the literature again, I found the work of Cocking and Chipman (1988) who attempted to identify linguistic and cultural variables to explain the poorer performance of language minority students in mathematics when compared to students who spoke English as a primary language. They proposed a model that categorises the factors that influence mathematical learning at school expanded along the lines of Input to the children and Output or Mastery which is child performance. The input consists of (1)

Entry Mastery – the cognitive ability patterns in terms of mathematical concepts, language skills, reading and learning ability, (2) Educational Opportunity – the time on task, quality of instruction, appropriate language and parental or other assistance and (3) Motivation to Engage – cultural/parental values, expectation of awards, and motivational nature of instructional interaction. The output consists of Mastery evaluation looking at Measurement Problems and Language of Test. Although the model was looking at language and children’s Mathematics achievements, in a report about two projects that were looking at the cognitive obstacles in the learning of Calculus, Norman and Prichard (1994) cited and described the model by Cocking and Chipman (1988) as follows in Figure 4. 1.

Figure 4.1: Influences on Learning



In their project, Norman and Prichard were primarily concerned with the Entry Mastery category as they said that this is where many cognitive obstacles originate. I found that the model could also be used as a base to describe the learning situation in my class. In particular, the nature of the change in my teaching was to provide educational opportunities for students to use their mathematical thinking powers and communicate their mathematical knowledge, and hoped that these activities would support changes in students' attitudes towards learning Calculus. More detailed discussion will be presented in Chapter 6.

Issue 3: Working on Mathematical Thinking

I had been focusing on specialising and generalising activities as I believed that the experience would enable students to make comparisons with the old 'rote' learning and learning where they were engaging with 'thinking'. However, I could see that they need to

‘see’ the prompts and questions to be able to appreciate them. I was thinking of providing worksheets for the students and had an idea that a workbook for students could be produced. The strategies were implemented for selected topics and there was a need to use these strategies for all the topics. However, there was the problem of requiring more time to allow for discussion and reflection. The workbook idea became more appropriate but to produce it, I would need to work in a team as it was difficult to design the tasks and prompts and questions that would draw students’ attention the processes of mathematics in topics for Advanced Calculus. I have managed to implement some aspects of ‘invoking thinking’ but had not been able to provide opportunities for students to make linkages, solving non-routine problems and real life problems.

Issue 4: Linking personal theories to public theories

Each learning and instruction theory that was studied highlighted certain aspects and obscured others. Thus, each theory had to be evaluated for what it illuminated about learning and how it could guide the development and design of effective instruction. Pegg & Tall (2005) offered an analysis towards identifying underlying themes in various theories to gain insight into issues concerning the learning of Mathematics. They identified two kinds of theory of cognitive growth; global frameworks of long-term growth and local frameworks of conceptual growth. The theories I reviewed were within their definition of local frameworks. Local cycles of conceptual development relate to specific conceptual areas where a learner attempted to make sense of his new mathematical knowledge and made connections using the overall cognitive structures available to him/her. In restructuring the ideas and issues expounded in the various theories, I had endeavoured to base my efforts on initiating changes on theory, on my experience and on my intuition about how students learn and how to support them to become independent learners. The model that was used to describe the process was given in Section 3.3.1.

Reflective teaching was a necessary occupation to ensure that events noted and observed, personal reflections, colleagues' comments as well as students' comments were fed back into the cycle of developing the teaching and the research.

4.10 Conclusion

Various issues have been identified and discussed in this chapter that would feed into the cycle of research methods and will inform the modifications necessary for future implementation. I tried to maintain the objectivity needed to balance my roles as teacher and researcher, I found that the 'teacher' was more dominant. Thus, I have to exert more effort to focus on the research as well. I will be developing a workbook that will include worksheets for all the topics and to achieve this, I will be working with my colleagues, Dr Tee and Dr Zee. The process of change did not end with the write-up, it still goes on as I tried out my teaching, reflecting and learning from my students' responses.

CHAPTER 5

STUDY OF THE IMPLEMENTATION – PART II

5.1 Introduction

In July 2002, UTM changed their intake requirements policy such that prospective undergraduates had to have post-SPM¹ qualifications and they needed a minimum of 3.3 Cumulative Grade Point Average (CGPA) or its equivalent for entry (see Section 4.3.1). These changes were part of an overall change in higher education policies of the Ministry of Higher Education. Thus, there were changes to the curriculum and the courses. A major change was that the medium of instruction must be English and there should be an emphasis on Soft Skills development in the students. Soft Skills are skills that apply across a variety of jobs and life contexts. They are also known by several other names, including generic skills, life skills, key skills, core skills, essential skills, key competencies, necessary skills, transferable skills and employability skills. In general, generic skills are skills, attitudes and values that facilitate employability. UTM has identified several soft skills such as:

- **Social Skills and Responsibility** – the understanding of the social, cultural, global and environmental responsibilities of a professional engineer, and the need for sustainable development; understanding of the principles of sustainable design and development;
- **Professionalism, Values, Attitudes and Ethics** – the understanding of professional and ethical responsibilities and commitment;
- **Life Long Learning and Information Management** – recognising the need to undertake life-long learning, and possessing/acquiring the capacity to do so;
- **Communication Skills and Team work** – the ability to communicate effectively, not

¹ SPM – Sijil Pelajaran Malaysia (Malaysia Certificate of Education, equivalent to the GCE O-Levels)

only with engineers but also with the community at large;

- **Critical Thinking and Scientific Approach** – the ability to undertake problem identification, formulation and solution;
- **Managerial and Entrepreneurial Skills** – the ability to function effectively as an individual and in a group with the capacity to be a leader or manager as well as an effective team member.

Every lecturer has to choose the soft skills that she can embed in the subjects that she teaches.

With the new entry qualifications, UTM students were now comparable to other undergraduates in most universities in Malaysia and elsewhere in the world. Much of the literature reviewed about the mathematical learning difficulties of undergraduates has been based on students of similar entry qualifications and thus, I presumed, still relevant. However, many of the assumptions and research findings about the mathematical difficulties of engineering undergraduates in UTM had to be re-examined as those data was gathered on students who came into UTM with SPM qualifications. To gain some first hand knowledge of the new batch of students, I taught two groups of students SSE 1792 in the second semester of the academic session 2004/2005. I was part of a team of 9 lecturers teaching several groups of students from different Engineering faculties. I was therefore able to take advantage of the various discussions on students' performance that were held as part of the monitoring and coordination processes to gather information and observations about the students. These general observations will be reported in Section 5.3.

Specifically looking at the mathematics subjects for engineering undergraduates, the post-July Engineering students' first semester mathematics course was a 2-credit Calculus course with the code SSE 1792 and the second semester subject was called Engineering Mathematics, with the code SSE 1893. SSE 1792 was considered a 'bridging subject' and was meant for revision as well as to strengthen students' knowledge and skills in various

differentiation and integration concepts and techniques. SSE 1893 was comparable to the older Advanced Calculus, SSM 2083.

In this chapter, I will describe the teaching of Engineering Mathematics to Block 2 and Block 3 students. Block 2 consists of two groups of students taught in the first semester of the academic session 2005/2006.

Table 5.1: Block 2 students

Group Name	Course/code	Subject code	Faculty	Year/Period	Subject
E05	Electrical Engineering (2 SEM)	SSE 1893	Electrical Engineering	Second year, semester 1 2005/06	Engineering Mathematics
F05	Civil Engineering (2 SAW)	SSE 1893	Civil Engineering	Second year, semester 1 2005/06	Engineering Mathematics

Unfortunately, these groups were not the same as the ones enrolled in SSE 1792 that I taught previously. Despite that, the overall performance and achievements of the students doing SSE 1792 from the various groups were sufficiently comparable that I felt the conclusions made about their mathematical performance (see Section 5.2.2) could be used as background for these students as well.

Block 3 students came from the following classes.

Table 5.2: Block 3 students

Group Name	Course/code	Subject code	Faculty	Year/Period	Lecturer
G07	Electrical Engineering (2 SEM)	SSE 1893	Electrical Engineering	Second year, semester 1 2007/08	Dr. Tee
H07	Electrical Electronic Engineering (2 SEE)	SSE 1893	Electrical Engineering	Second year, semester 1 2007/08	Dr. Zee

During their teaching sessions, I came to observe the classes, collected some of their worksheets and conducted interviews with volunteers. I decided to include these last two

groups as they were using the completed workbook that I had helped to write and the students were taught using the teaching models that were developed as part of the study.

5.2 Review, Reflection and Changes

In Part I, the students who were the subjects of the study were from the Industrial Design Course and were from the pre-July 2002 intake. As changes in intake policies and the curriculum have occurred, at the end of the study, I made a decision to continue the study with groups of students who were from the post-July 2002 intake. A feature of action research is the cycles of '*planning-implementation-review-modify*' whereby all the various factors that influence the teaching and learning situation are examined and re-examined during the progress and development of the research. It is not easy to write in a way that reflects the cycles of research. Thus, in this section, I will present arising issues related to the research methods, new literature reviewed and referred to as well as presenting the review, reflection and changes that I made based on the experiences in Part I.

5.2.1 Research Methods

In Chapter 3, a description of the research methods was given. In Chapter 4, the description of the experience of teaching several classes of Basic Calculus and Calculus II was presented. Various factors and issues were also raised at the end of that chapter and in this section; the changes or modifications made to some of the methods will be presented.

(1) Teaching

How do I maintain objectivity and stamina to carry out the changes to my teaching? What I set out to do was quite daunting to do alone and I needed some avenues to cross check my accounts and my interpretations of class events or interactions with students. Thus, in this semester, I work with two other colleagues, Dr. Tee and Dr. Zee who monitored and provided feedback about my teaching strategies and tasks development. They were also teaching Engineering Mathematics but to other sections. It was not possible to schedule

class observations of my teaching due to time-tabling overlap. However, this gave me two colleagues who could provide independently objective feedback about the tasks that I created, the strategies that I used, and general comments on the direction and progress of the research. At this stage, they were not using the worksheets that I produced. This particular development helped alleviate the feeling of '*changing in isolation*' as well as adding an element of objectivity in evaluating the teaching strategies and research process.

(2) Observations

There were a few difficulties that were identified in making observations from the previous experience. How do you observe 'the teacher teaching' when the teacher is yourself? A common method that has been used in video taping the teaching sessions and unfortunately that choice was not viable due to financial constraints as I would have to hire the equipment and the technician who would operate it. I resolved to be even more critical of my personal reflections but I needed to observe a teacher teaching using the strategies in order to evaluate them. It was also difficult to observe students fully and impartially while I was involved in the teaching and had to make decisions about the direction of the lessons as well as the nature of my intervention with respect to my students' learning.

An alternative that I considered was to observe my colleagues' teaching using the same strategies. It would also give me the opportunity to be a complete observer to the events unfolding in the class and to observe the students. An additional advantage would be the opportunity to check the viability of the teaching strategies being interpreted and used by other lecturers and perhaps find some indications that they were not overly dependent on the individual teacher.

In this cycle of the implementation, I carried out a two part study. Firstly, I taught two groups of Engineering Mathematics students described above. I made observations and recorded data, similar to the stance that I took in the past. Secondly, I undertook to finish the workbook so that the use of the prompts and questions would be explicit for all the

topics in the syllabus. My colleagues were teaching some groups and I observed their teaching. We would also be working together to finish the workbook.

(3) Problem-solving Questionnaire

At the beginning of the course, I gave the students a set of 7 mathematics questions on differentiation and integration that they would have learnt in pre-university courses and in Calculus I (SSE 1792). The questions were to help in giving some ideas of how the students would react to some of the prompts and questions that I would be using as well as give some indication of their understanding of the concepts and techniques in basic differentiation and integration. Each of the questions required students to explain or justify their answers. I wanted to know if students' were already comfortable with giving explanations. Some examples of the questions and details of the students' responses will be given later in Section 5.3.2.

(4) Interviews

I had conducted some interviews with some of the students supported by on-going discussions and conversations. These will be reported in the relevant sections (Section 5.3.3.2)

(5) Students' work

For Block 2 students, the teaching strategies were carried out for the first two topics and some students' work were collected at the end of the teaching sessions in these topics. However, for Block 3 students, the strategies were used throughout the course. They were also using workbooks and thus their work was recorded and already compiled. Some examples of these were collected.

(6) Teaching and Learning Evaluation Questionnaire

I gave out an evaluation questionnaire at the end of the semester. The questionnaire was adapted from Brookfield (1995). Students were asked to evaluate my teaching and their learning experience. The results were presented in Section 5.3.5

(7) Preparation of workbook

In Section 4.9 (Issue 3), the practicability of using a workbook was discussed and the basic idea for using the workbook was to create an environment where students can engage in doing the mathematical tasks on an individual as well as communal levels. The preparation of the workbook was a group undertaking and I worked with Dr. Tee and Dr. Zee to prepare the tasks for all the topics in the Engineering Mathematics syllabus. The workbook was a compilation of mathematical tasks such that students could experience for themselves the mathematical processes such as the process of identifying the general class of problems they were working on. We started working on the workbook in this semester and a draft copy was used with the students in the second semester 2006/2007. However I had to move from the Skudai campus in Johor Bahru to the Kuala Lumpur campus in the same semester and was unable to observe the classes that were using the book. So, I will be reporting on the teaching and the use of the workbook for students in the first semester 2007/2008. The final form and features of the workbook will be reported in Section 5.4.3. In this semester, I was still using worksheets.

5.2.2 Review of the Assumptions about Students' Difficulties and Beliefs

The majority of students who came into UTM post-July 2002 had qualifications from the various Government sponsored Matriculation Colleges and STPM² from Government high schools. A few students had Diploma qualifications from various institutions. With a

² STPM – Sijil Tinggi Pelajaran Malaysia (Malaysia High School Certificate – equivalent to GCE A Levels)

minimum of 3.3 CGPA entry qualifications, the students were considered good students with strong foundation in Mathematics, Physics and Chemistry. Thus, it was generally assumed that these students would not have many of the learning difficulties found in previous generations of undergraduates. It was also assumed that they would have a stronger foundation in basic mathematical concepts that were taught in their pre-university courses.

Nevertheless, in the new curriculum, a subject Calculus I with the code SSE 1792 was offered. It was designed to provide some revision of topics that were thought to be important for the advancement of students to Engineering Mathematics as well as provide a 'bridging' of topics that might not have been covered in depth in their Matriculation or High School curriculum. The Calculus I curriculum design was based on a study of the mathematics syllabus at the pre-university levels as well as observations made by some lecturers in UTM who were involved in the monitoring of standards of examination questions and the marking of examination papers at the Matriculation and High School levels. They found that although most of the topics were similar, the depth of coverage was not the same. In addition, there were some topics that were not covered. Malaysian schools and Matriculation Centres have a national curriculum but they are slightly different from each other.

The SSE 1792 syllabus has the following topics:

Vectors: Scalar and vector, vector notation, equality of two vectors, operations on vectors, vectors in space, dot product, cross product, line equation, angle between two lines, intersection of two lines, distance from a point to a line, distance between two skewed lines, equation of a plane, angle between two planes, angle between a line and a plane, distance from a point to a plane, intersection line of two planes. **Polar Coordinates:** Parametric equations, polar coordinates system, relationship between polar coordinates and Cartesian coordinates, graph sketching of polar curves. **Complex Numbers:** Definition of complex numbers and imaginary number, operations on complex numbers, modulus and argument, Euler's formula, De Moivre's theorem. **Further Transcendental Functions:** Inverse trigonometric functions, hyperbolic functions and inverse hyperbolic functions. **Differentiation:** Differentiation of composite functions involving inverse trigonometric functions, hyperbolic functions, or inverse hyperbolic functions. **Integration:** Integration of

hyperbolic functions, inverse trigonometric functions, and inverse hyperbolic functions. **Partial Derivatives:** definition and techniques of partial differentiation.

It was assumed that other topics in basic Calculus were satisfactorily taught at the Matriculation and High School levels.

To augment the views about the abilities of students' of the new intake, I asked to teach a first year group. A brief remark will be made here on the general observations of the performance and attitudes of students doing the SSE 1792 course in the second semester of the academic session 2004/2005. I was a member of the team of lecturers who taught the students and these observations were based on the various discussions that we had about students' learning and performance as part of the review meetings that were held to manage the course and the teaching. Generally, it was found that the students were highly motivated as judged from the following evidence. Most students had a good class attendance for lectures and tutorials. They were generally quite attentive in class, would do many of the exercises in the textbook and in tutorial sheets. In particular, nearly all the students had very strong recall of previous mathematics knowledge and procedures and they found the current mathematics relatively easy to follow except for a few topics that they had not met in their pre-university mathematics courses. The final examination results of the students were also exceptionally good as most had grades A and B. However, there were a few exceptions and these were students with Diploma entry qualifications from various Polytechnics in Malaysia. They had poor recall of previous mathematics learnt and were more passive in the classroom.

5.2.3 Modifications and adjustment of my beliefs and attitudes

As mentioned earlier in Section 4.2, much of the literature reviewed on UTM's students difficulties were based on students with different entry qualifications. In addition, based on the common general perceptions of the new batch of students who did Calculus I and would be taking Engineering Mathematics, I had to modify my own beliefs and

perceptions. In particular, the students' grasp of their mathematical prior knowledge was stronger and there would be no need for revision. Observation of students' learning behaviour showed that they were diligent workers, well prepared for lectures and were used to working on their own. However, I knew that they were still accustomed to the styles of teaching that depended more on lectures, drills and tests with a strong emphasis on obtaining good examination results. Good results were considered very important, as entry into public universities was very competitive due to the substantially cheaper cost of education and better infrastructure support. The Malaysia Government still subsidised students in public universities.

Thus, my concerns for these students would be to promote the various aspects of mathematical learning that I thought would be important for their advancement and these were, invoking their awareness of their own mathematical thinking powers and to promote communication of mathematical ideas. I anticipated that they would still need support to increase their awareness and use of mathematical thinking.

5.2.4 Changes to my Teaching Semester 1, 2005/06.

The syllabus for the Engineering Mathematics course (SSE 1893) was similar to the older course Advanced Calculus (SSM 2083) but I had to teach this particular group in English. However, there were many modifications that I had to make in the teaching materials that I wanted to use in the classroom.

(1) Lessons learnt from previous experience

Some of the lessons that I learnt from working with the previous groups of students were very useful in helping me plan my teaching for these groups of students. The students wanted and needed tangible materials to work with and something that they could refer to later on. In the previous groups, I thought that if the students made their own notes then that would be more meaningful. However, they preferred the lecturers' notes as that made it more '*authoritative*' and thus the notes represented '*what should be learnt*' and possibly

'what would be examined'. They would also prefer that the work that they did in the classroom were appreciated and this meant that it would contribute to their course marks. The students were also concerned with knowing what the answers to the problems were and would like some of the answers to be given if not all. I also needed a way to make visible the prompts and questions that I used and would like the students to have the questions at hand to help them in their discussion. They also wanted the work to be much more closely relevant to the examinations and the activities should reflect what would be examined. Even so, there were students who appreciated that they could understand the mathematics that they learnt and they could explain it better but they felt that I was too slow in finishing the syllabus. Their concern was still clearly about their ability to perform well in the examinations by virtue of having rehearsed all the possible types of questions.

With due considerations of the students' concerns, my own concerns about students' learning and the observations that were made on the students, I decided to make some changes to my teaching which will be described in the following section. The basic guide for the change is to provide an environment that looked familiar but could also support the changes to the teaching and learning that I wanted to implement. I was trying to bridge the transition from school to university mathematical learning but I realised that I must make the impression of negotiating change from the beginning. One of the lessons that I learnt from the previous experience was that the students needed a way of working that would be different from their old learning experience but with the content presented in such a way that it did not appear to be too different from the textbook.

I was aware that I would be working with different students with seemingly better grasp of prior knowledge and learning styles but I picked out features that were general and that I thought would be the concerns of the new students as well.

(2) Changes made

In response to lessons learnt from the previous groups, there were a few decisions that were made to my teaching and the learning environment.

(i) Team Teaching

In this semester, I had two colleagues to monitor my work as mentioned in Section 5.2.1. This served two main purposes, firstly, adding objectivity to the research process and, secondly, providing the social support that I needed in promoting change.

(ii) Use of Worksheets and Textbook

I had prepared worksheets for the first three chapters in the syllabus. In deciding to use worksheets, I had to consider both the positive and negative aspects. The positive considerations for using the worksheets were as mentioned above. The materials and topics under discussion would be presented in a structured manner and the prompts and questions were made visible and explicit and thus much easier to refer to and used in guiding discussions. The use of prompts and questions were to help make explicit the processes and structures of the mathematics and to provide students with a vocabulary to guide their own queries and thinking. The students also had the worksheets as materials that they could keep and refer to when and if needed. I could also now collect the worksheets for assessing students' learning and give some feedback to each student. Grades could also be given on some of the work but not all. By assigning marks to some of the work, students would feel that it was important to become involved and contribute. The structured work, the common language of working with the mathematics and the explicit contract of learning based on student-centred values created a new social environment which everyone, teacher and students, have agreed to.

However, the main negative aspect of using worksheets was that they made the classroom activity more structured and was more reminiscent of the students' own past experience in

learning. One of the main features in their past learning at SPM and STPM levels was the **large number of workbooks** that they had to work through with their teachers as **preparation for the examinations.** The situation was slightly less daunting for matriculation, **as there were no workbooks** but even so, students had to do a lot of exercises. In this way, **the idea of independent learning can become obscured** and the idea of encouraging **creativity is not supported by class work.**

However, even though the worksheets appeared similar, the nature of the questions asked **was slightly different.** The worksheets contained mathematical tasks that were designed to **focus students' attentions on the mathematical processes and structures in the various topics that they had to learn.** To design the tasks, I sometimes had to select and modify **existing tasks** and in other cases, create new ones. In the main, the tasks required students **to use various powers such as specializing, generalizing, sorting and categorizing, conjecturing and convincing as desirable mathematical behaviour.** Although, the general **presentation of materials was familiar to the students,** the content development would be **different from what they would have experienced in their past learning.** It was definitely a **case of 'something old, something new' as an approach.** Examples of some worksheets are **reproduced in this chapter (see Figure 5.5).** They also used a textbook that was chosen for **this semester, Multivariable Calculus by Smith & Minton, 2002.** Tutorial questions were **taken from this book as there are answers in the book for selected problems.** The textbook **was chosen so that students were encouraged to do extra reading about the topics taught.**

The tasks in the worksheets were created using frameworks provided by Mason & Johnston-Wilder (2004) and *prompts and questions* adapted from Watson & Mason (1998).

I like the fundamental notions of Mason and Johnston-Wilder (2004, pp 8-9) about learners and teachers:

- *learners need to be mentally, emotionally and sometimes, even, physically active;*

- *teachers need to be aware of relevant features of the topic to be learned, of the mathematical structure of the topic, and of ways of 'psychologising the subject matter' so as to make it accessible to students;*
- *tasks become vehicles for learning because of the quality and nature of the interactions and activities arising from them, and the ethos in which the tasks are undertaken;*
- *learning takes place within a setting, that is, an environment of social forces and classroom practices (called a milieu by Brousseau (1997)).*

Their approach made use of insights from activity theory developed by Vygotsky (1978), Leont'ev (1981) and Bruner (1966, 1986). In particular, their approach aims to design tasks so that the students will have,

- *relevant experiences from which to extract, abstract and generalize principles, methods, perspectives and ways of working with mathematics;*
- *stimuli appropriate to the concepts to be worked on;*
- *a supportive and compatible social environment in which to work.*

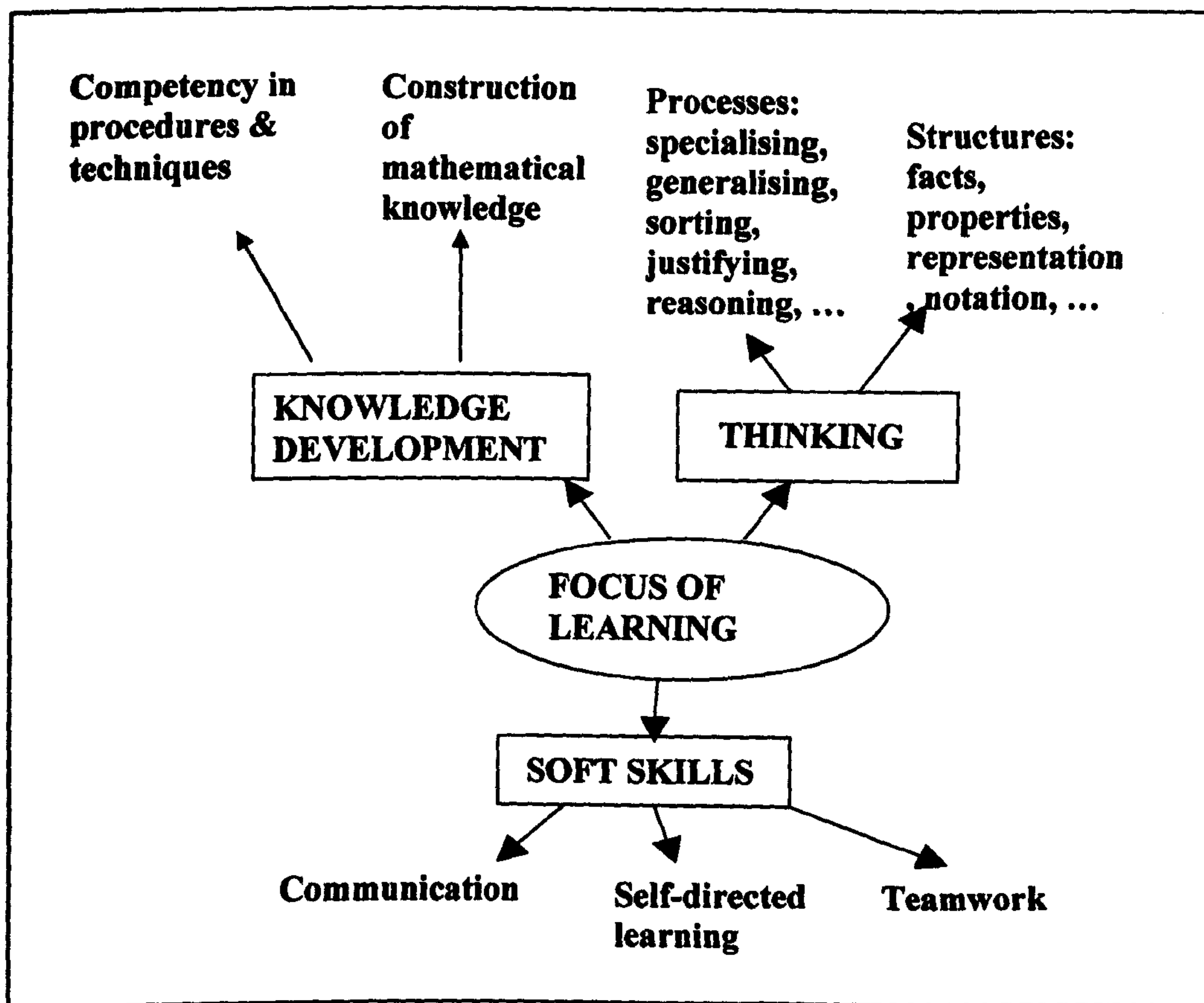
I have used the approaches they recommended extensively because their ideas were practical and could translate theories about learning into classroom activities and tasks.

(iii) The Focus of Learning and Active Learning

Two models were used to add clarity to the teaching situation and these are reproduced below. The first one (Figure 5.1) identifies the focus of learning and the second model (Figure 5.2) identifies the important elements of an active learning environment. The teaching and learning situation needed to contribute to the various concerns such as to enhance students' ability to take charge of their own learning, increase their understanding, communicate their mathematical learning and to increase their awareness of their own mathematical thinking.

The focus of learning identified elements that I thought were important and consistent with the University's philosophy of teaching. These were: Thinking, Knowledge Development and Soft Skills Development, in particular, communication, independent learning and teamwork.

Figure 5.1: Focus of Mathematical Learning



In order to achieve the focus of learning, I made use of active learning theory, as I endeavoured to keep in mind the virtues of students being active in as many senses as possible. The important elements to support effective active learning were talking, listening, reading, writing and reflecting (Meyers & Jones, 1993). In preparing to implement active learning for our classroom, the following aspects were identified.

- (i) **Designing classroom tasks** – in designing the tasks, we had created, selected, and modified existing tasks to focus students' attention on the mathematical processes and structures that we wished them to learn based on the frameworks of Mason & Johnston-Wilder (2004) and Watson & Mason (1998). This was done by changing a bit

of the question, removing some of the information from the question, replacing part of the question and adding a bit to the question that required students to provide explanations and justification for the solutions found (Prestage & Perks, 2001). This was meant to stimulate the use of various powers such as specializing, generalizing, conjecturing and convincing as desirable mathematical behaviour. The tasks were categorized as '*Worked examples*' and '*Questions*' with some prompts and questions by the side. The prompts and questions were intended to direct and guide students' awareness of the fundamentals of *doing* a problem. There was also a section on '*Reflection*', for students to recapture important ideas and concepts.

- (ii) **Determining approaches** – various methods were used towards achieving the teaching goals. It was clear that the approaches were selected by me in the first instance towards achieving goals that I set. Students had their own goals and preferred teaching methods. At the beginning of the class, I made my goals explicit, assured the students that I would listen to their concerns and wherever possible, try to find acceptable solutions to both parties. The students were reminded of the course objectives as well as the university's desired learning outcomes. However, my focus was on the development of their knowledge and understanding of the techniques as well as the identified soft skills.

Teamwork – to achieve this, students were asked to work in a group from the beginning when working with the worksheets. The number of students in the group varied between 2 and 4 people so as to cater for the different styles of learners, the complexity of the problems, to elicit quick feedback or the suitability of the physical environment. I decided that there should be, from the beginning, an 'impression' that some change would take place, thus, all students must work in a team. However, for those students who much prefer to work alone, at least they can make the change by working with one other person rather than a large group. The requirement that they had to work in groups, to discuss and share was a major change for the students. Some

individual work was also required especially in preparing and covering of the lecture notes, in answering tests and the final examination. For example, they were to work as team when doing the structured examples and individually for reflection.

Knowledge Development – the same teaching strategies used previously were continued. I was working on increasing students' awareness of their own mathematical thinking powers.

(iii) Encouraging communication – the use of the prompts and questions was successful in initiating mathematical communication. By working as a team, students were to discuss and share their ideas. Students were also expected to hand in written work and to share their reflection in the classroom.

(iv) Supporting self-directed learning – the structured questions were created to strengthen the students' understanding of mathematical concepts and techniques. The examples and problems would provide the learning experiences where certain mathematical powers and themes were used specifically and explicitly. This was intended to increase students' awareness of their own mathematical thinking powers and the mathematical structures studied. In addition, reading assignments were also given for students to explore certain concepts for themselves.

(v) Identifying types of assessment – I had incorporated both summative and formative types of assessment. The objects of assessment were students' understanding of mathematical facts, standard methods and techniques, standard application of the concepts, problem solving, logical and analytical reasoning. These were: quizzes, tests, quick classroom feedback and written assignments as well as the final examination. The coursework contributed 50% of the total marks. Figure 5.2 below gives a summary of the model for active learning.

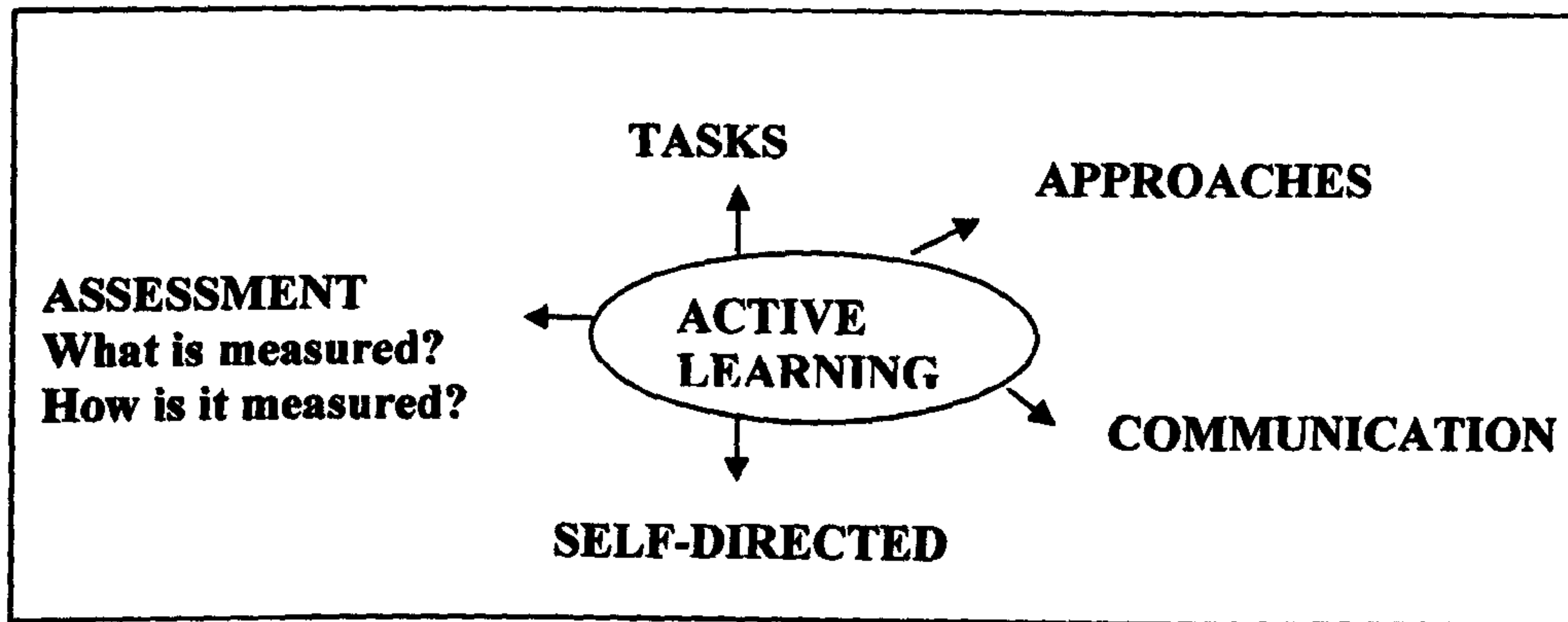


Figure 5.2: Model of Active Learning

These ideas have also been presented as a paper in the Regional Conference of Engineering Education 2007 (Baharun et al, 2007).

5.3 Teaching of Engineering Mathematics

During the first semester of the academic session 2005/2006, I was teaching Engineering Mathematics (subject code SSE 1893) to two groups of students, namely, Section 36 of Electrical Engineering students with the course code of 2 SEM and Section 17 of Civil Engineering students with the course code 2 SAW. I was away on a conference from the 6 – 13 of July and started my classes a week late. This meant that I had to find time to make up for the lost hours. In this section, I will present data about the two groups.

5.3.1 General description

Engineering Mathematics was a three credit subject so that meant that I had three hours lectures and a one hour tutorial session with the students each week. As mentioned earlier, UTM had parallel streams in place for students who followed the old curriculum and for those who followed the new one. This subject was part of the new curriculum for students who entered UTM after July 2002. In Calculus I previously, all the students had passed the course, thus, I did not expect any repeat students. The syllabus for this subject and the class schedule for the semester are as follows:

Table 5.3: Weekly schedule Engineering Mathematics 2004/05

Week 2005	Topics
1 11/7 – 15/7	Chapter 1: Functions of several variables; Domain and range; Level curves, common surfaces, level surfaces.
2 18/7-22/7	Chapter 2: Partial differentiation; Rate of change, the chain rule
3 25/7 – 29/7	Increments and differential ; Extrema of multivariable functions – local and absolute
4 1/8 – 5/8	Chapter 3: Double integrals; Integrals in rectangular coordinates and iterated integrals; Fubini's Theorem; Changing the order of integration.
5 8/8 – 12/8	Double integrals in polar coordinates; Applications of the double integral: Area, volume, mass, centre of gravity, moments.
6 15/8 – 19/8	Triple integrals: Triple integral in rectangular coordinates.
7 22/8 – 26/8	Triple integration in cylindrical coordinates; Triple integration in spherical coordinates; Applications of the triple integral: Mass, centre of gravity, and moments.
8 28/8 – 4/9	Mid-Semester Break
9 5/9 – 9/9	Chapter 4: Vector-valued functions; Graphs of vector functions, differentiation and integration of vectors; Velocity, acceleration, tangents and normals.
10 12/9 – 16/9	Scalar and vector fields, Del operator, gradient, unit tangent and normal vectors, directional derivatives; Divergence and curl.
11 19/9 – 23/9	Chapter 5: Vector Calculus; Line Integrals - Line integrals in scalar and vector fields.
12 26/9 – 30/9	Surface integrals: path independence, potential functions and conservative fields.
13 3/10 – 7/10	Green's Theorem.
14 10/10 – 14/10	Stokes' Theorem.
15 17/10 – 21/10	Gauss Divergence Theorem
22/10 – 30/10	Revision
7/11 – 27/11	Final examinations

In total there were 10 sections of engineering students who were taking this subject and I was one of eight lecturers teaching. As usual, all tests and the final examination were coordinated, the date, time and questions were to be the same for all sections. Each section should have a maximum number of sixty students only. There was some autonomy for the lecturers to decide on the tutorial questions, quizzes and assignments but this would contribute only 10% of the total marks. Tests contributed 40% and the final examination

contributed 50%. Even though I had drawn up the active learning model for possible use by others, I was the only one who used it. I had decided to give students a series of quizzes and/or short assignments, usually from the worksheets and tutorial sheets with the opportunity to improve their marks if required. However, it would only be 10% of the total marks. The first test was 15% and the second test was 25% of the total marks. As for the generic skills, the team of lecturers had agreed that the following would be addressed in this course: communication skills, team work, problem solving and adaptability. I decided that I would focus on mathematical communication skills which I defined as the ability of the students to talk coherently about the mathematics that they had learnt. As for problem solving, my focus was also on mathematical problem solving. Team work would be an essential part of the way the students will be working during the course.

A problem-solving questionnaire was also given to the students during the first class meeting. It was made clear that students' participation in answering the questionnaire was voluntary and would not contribute any mark to their assessment. It consisted of seven mathematics questions from the topics of differentiation and integration that should have been encountered during their pre-university courses as well as revised in the pre-requisite subject SSE 1792. The aim was mainly to find out how much they could remember of what they had learnt and how they would react to the prompts and questions that I would be using in the worksheets. I also wanted to find out if students were able to offer explanations for their solutions. It should be noted that the students had been away for a twelve weeks break before coming back to the university. The students worked on the questionnaire for the whole hour in the first class meeting. Details of responses will be given in Section 5.3.2.

The teaching strategies were similar to the ones I used in the previous classes. The students used the worksheets and I included some of the structured examples as part of the lectures using the appropriate prompts and questions accompanying the problems. I had other

examples that formed part of the lectures and students were sometimes asked to work on these as well in the class. I was hoping that responses to the prompts and questions, either, verbal or written, would further enhance students' communication skills. More specific description of each group of students will be given in the following subsections.

5.3.2 Responses from Problem Solving Investigation

I was away in the first week of the academic session but I had a colleague who went to my classes to give out the problem solving questionnaires and to inform students that my classes would only start in the second week. Thus, the questionnaires were given in the first class meeting of the session for each group. Attendance was very good and all the students who came worked on the questionnaire. There were 62 students in the 2 SEM Section 36 group and 42 students in the 2 SAW Section 17 group. The front cover of the questionnaire had clear explanations about the purpose of the problem solving exercise and the voluntary nature of the students' participation. There were 7 questions in the questionnaire with 1 extra question about the degree of difficulty of the English language used in the questionnaire and some space for students to add any other comment. It should be noted that students from Government institutions of learning had been doing their courses in Bahasa Malaysia (Malay Language) for the SPM and a mix of Bahasa Malaysia and English for the pre-university courses. English was supposed to be used for Mathematics and Science subjects. However, from informal conversations with students, most claimed that the English was only used for the laboratory classes of Physics and Chemistry. I had no means to verify their claims.

Table 5.4 below gives a summary of the topics asked in the questionnaire and what was the focus of each question.

Table 5.4: Summary of Questionnaire Topics

No	Topic	Focus
1	Implicit differentiation: Recognising patterns Recall of implicit differentiation rules and techniques	Same and different Recognising patterns Identify generality Rules of implicit differentiation for trigonometric and exponential function and in general.
2	Product rule and power rule of differentiation: Differentiation of x to the power of a Fractional index and the product rule	Recognising patterns Identify generality Explanations
3	Extending the product rule for 3 and four variables	Explanations
4	Power formula for integration: Recall of formula	Extension of power rule for integration Explanations
5	Integration by substitution: Recall of formula	Same and different Identify generality Explanations
6	Chain rule in differentiation: Students' own examples	Instruction given in words and formula not given Explanations
7	Integration by parts: Students' own examples	Instruction given in words and formula given Explanations

As mentioned above, the questionnaire was given during the first class meeting. A total number of 104 students participated, 62 from 2 SEM and 42 from 2 SAW. However, there were 2 latecomers to my 2 SEM class, thus the final number of students for 2 SEM was 64. However, 1 student decided to withdraw mid-semester so the final number of students was 63. I will be describing in some details the students' responses as it would give some picture of their mastery of Basic Calculus as well as initial reactions to the use of my prompts and questions. Questions from the questionnaire (see Figures 5.3 (1) – (7)) are reproduced below as well as the results of the students' responses (see Tables 5.4 (1) – (7)). Comments for each question will be briefly given with an overall summary at the end of the section.

Figure 5.3 (1) Example of Questions 1 in the Problem Solving Questionnaire

Instructions given to the students at the beginning of the questionnaire:

Please write down what you are thinking of or what you feel as you answer the questions. If you have any difficulty in answering the questions, please state what these difficulties are.

Legend: R – right answer; W – wrong answer

SEM – Electrical students; **SAW** – Civil students

Question	Working for your answer	What you think/comments
<p>1. Given that $y = f(x)$ and</p> $\frac{d}{dx}(y^2) = 2y \frac{dy}{dx}$ $\frac{d}{dx}(y^3) = 3y^2 \frac{dy}{dx}$ $\frac{d}{dx}(y^4) = 4y^3 \frac{dy}{dx}$ <p>Answer the following questions: What is the same between them? What is different between them? Complete the following:</p> $\frac{d}{dx}(y^5) =$ $\frac{d}{dx}(y^{10}) =$ <p>What have you noticed about the technique of differentiation in the above examples, which you can use to answer the following questions?</p> $\frac{d}{dx}(\sin y) =$ $\frac{d}{dx}(e^y) =$ $\frac{d}{dx}(g(y)) =$		

Table 5.5 (1): Results of Question 1

	No	Same	Different	Solving examples	Generality	$\frac{d}{dx}\sin y$	$\frac{d}{dx}e^y$	$\frac{d}{dx}g(y)$
SEM	1	R: 36 W: 0	R: 25 W: 2	R: 62 W: 0	R: 8 W: 1	R: 41 W: 5	R: 29 W: 13	R: 24 W: 11
SAW		R: 30 W: 2	R: 33 W: 1	R: 36 W: 6	R: 6 W: 1	R: 24 W: 10	R: 23 W: 10	R: 6 W: 18
	Total	R: 66 W: 2	R: 58 W: 3	R: 98 W: 6	R: 14 W: 2	R: 65 W: 15	R: 52 W: 23	R: 30 W: 29

All 104 students did the questions $\frac{d}{dx}(y^5) =$ and $\frac{d}{dx}(y^{10}) =$ but only 98 students gave

the correct answers. Meanwhile 80 students attempted question $\frac{d}{dx}(\sin y) =$ with 65

correct answers and 75 students tried question $\frac{d}{dx}(e^y) =$ with 52 correct responses.

However, only 59 students tried to answer $\frac{d}{dx}(g(y)) =$ with 30 right answers. Students

were also able to respond to the given prompts and questions with 66 out of 68 students

being able to identify what was the same between the expressions and 58 out of 61 students

were able to identify what was different correctly. However, only 16 (15.4%) students

attempted to state the generality in the implicit differentiation with 14 (13.5%) correct

responses. Obviously, most of the students were able to answer the specific questions but

did not attempt to answer the prompts and questions nor try to identify the general.

Figure 5.3 (2): Example of Questions 2 in the Problem Solving Questionnaire

Question	Working for your answer	What you think/comments
<p>2. (a) Find $\frac{d}{dx}\left(x^{\frac{3}{2}}\right)$ by writing $\left(x^{\frac{3}{2}}\right)$ as $x \cdot x^{\frac{1}{2}}$ and using the Product Rule. Express your answer as a rational number times a rational power of x.</p> <p>Work parts (b) and (c) by a similar method.</p> <p>(b) Find $\frac{d}{dx}\left(x^{\frac{5}{2}}\right)$.</p> <p>(c) Find $\frac{d}{dx}\left(x^{\frac{7}{2}}\right)$.</p> <p>(d) What patterns do you see in your answers to parts (a), (b), and (c)?</p>		

Table 5.5 (2): Results of Question 2

	2	(a)	(b)	(c)	(d)
SEM		R: 49 W: 3	R: 47 W: 6	R: 42 W: 7	R: 23 W: 8
SAW		R: 25 W: 12	R: 20 W: 12	R: 18 W: 10	R: 6 W: 3
		R: 74 W: 15	R: 67 W: 18	R: 60 W: 17	R: 29 W:11

89 students attempted question 2 (a) with 74 correct responses, meanwhile there were 67 correct responses for 2 (b) out of 85, there were 60 correct responses for 2(c) from a total of 77, only 40 students attempted to describe or state the patterns they saw with 29 correct responses. The main reasons given for not being able to answer were “*I don’t understand the question*” or just left blank.

Figure 5.3 (3): Example of Questions 3 in the Problem Solving Questionnaire

Question	Working for your answer	What you think/comments
<p>3. The Product Rule gives the formula</p> $\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$ <p>for the derivative of the product uv of two differentiable functions of x.</p> <p>(a) Can the product rule above be used to find $\frac{d}{dx}(uvw)$? Explain your answer.</p> <p>(b) Can you extend the product rule to find the derivative of the product $u_1u_2u_3u_4$ of four differentiable functions of x?</p> <p>(c) What happens in general? Can you find the derivative of a product of $u_1u_2u_3 \dots u_n$ of a finite number of n differentiable functions of x?</p>		

Table 5.5 (3): Results of Question 3

	3	(a)	(b)	(c)
SEM		R: 25 W: 18	R: 11 W: 10	R: 4 W: 8
SAW		R: 13 W: 18	R: 9 W: 10	R: 6 W: 7
		R: 38 W: 36	R: 20 W: 20	R: 10 W: 15

74 students responded to question 3(a) with 38 students answering correctly. Meanwhile for 3(b), there were 40 responses with 20 correct ones. As for 3 (c), only 25 students tried the question. There were 10 correct responses or at least they tried to describe a method of solving the problem. Most students wrote down that they have forgotten the topic even as they tried to solve the problem.

Figure 5.3 (4): Example of Questions 4 in the Problem Solving Questionnaire

Question	Working for your answer	What you think/comments
<p>4. Given the power formula for integration:</p> $\int u^n du = \frac{u^{n+1}}{n+1} + C; \quad n \neq -1 \text{ where } u = f(x).$ <p>In each of the following cases, determine whether the given relationship is true or false. State your reasons. If it is true, in what way is the question a special case of the power formula for integration?</p> <p>(a) $\int (2x+1)^2 dx = \frac{(2x+1)^3}{3} + C$</p> <p>(b) $\int 3(2x+1)^2 dx = (2x+1)^3 + C$</p> <p>(c) $\int 6(2x+1)^2 dx = (2x+1)^3 + C$</p>		

Table 5.5 (4): Results of Question 4

	4	(a)	(b)	(c)	Reasons given
SEM		R: 36 W: 20	R: 33 W: 21	R: 29 W: 24	R: 2 W: 1
SAW		R: 20 W: 21	R: 22 W: 19	R: 16 W: 25	R: 1 W: 0
		R: 56 W: 41	R: 55 W: 40	R: 45 W: 49	R: 3 W: 1

97 students attempted 4(a) with 56 correct responses, 95 students attempted 4(b) with 55 correct responses, and 94 students attempted 4(c) with 45 correct responses. However, only 4 students attempted to give reasons for their answers with 3 correct responses.

Figure 5.3 (5): Example of Questions 5 in the Problem Solving Questionnaire

Question	Working for your answer	What you think/comments
<p>5. Consider the following examples. Do not solve.</p> <p>(a) $\int (x^2 + 1)^2 (2x) dx$</p> <p>(b) $\int (x^2 + 1)^3 (2x) dx$</p> <p>(c) $\int (x^2 + 1)^3 x dx$</p> <p>Answer the following questions.</p> <p>What are the same between (a) and (b) and (c)?</p> <p>What is the same about (a) and (b) but different in (c)?</p> <p>What is the same in (b) and (c) but different in (a)?</p> <p>What changes and what stays the same in the integrand as you use integration by substitution?</p> <p>What are the essential features which make these typical examples of integration by substitution?</p>		

Table 5.5 (5): Results of Question 5

	5	(i)	(ii)	(iii)	(iv)	(v)
SEM		R: 36 W: 8	R: 40 W: 3	R: 39 W: 2	R: 16 W: 2	R: 2 W: 8
SAW		R: 36 W: 5	R: 35 W: 4	R: 33 W: 0	R: 8 W: 15	R: 1 W: 13
		R: 72 W: 13	R: 75 W: 7	R: 72 W: 2	R: 24 W: 17	R: 3 W: 21

85 students could describe what was the same between 6 (a), (b) and (c) with 72 correct responses. They also could identify what was the same in 6 (a), (b) but different in (c) with 75 correct responses out 82 total responses. Similarly, they could identify what was the same in 6 (b), (c) but different in (a) with 72 correct responses out of 74 responses. The number of responses declined sharply for the last two parts of the question, with 42 and 24 responses, respectively.

Figure 5.3 (6): Example of Questions 6 in the Problem Solving Questionnaire

Question	Working for your answer	What you think/comments
6. (a) Give an example of a function that can be differentiated by using the Chain Rule. (b) Give another example that can be differentiated by the Chain Rule but it has a function different from that in example (a). (c) Can you describe the characteristics or properties of functions that show it can be differentiated by using the Chain Rule?		

Table 5.5 (6): Results of Question 6

	6	(a)	(b)	(c)
SEM		R: 3 W: 15	R: 1 W: 12	R: 1 W: 6
SAW		R: 4 W: 12	R: 4 W: 7	R: 0 W: 7
		R: 7 W: 27	R: 5 W:19	R: 1 W: 13

Question 6 had a very low number of responses, ranging from 14 to 34 students' attempts. In the main only 15 students filled out the 'what you think/comment' section with the main comments given were, '*I have forgotten*', '*hard to give examples*' and '*not sure of terminology*'. The other students left the question blank with no attempt or explanations.

Figure 5.3 (7): Example of Questions 7 in the Problem Solving Questionnaire

Question	Working for your answer	What you think/comments
7. The formula for integrating by parts is $\int u dv = uv - \int v du$ (a) Give an example of a function that can be solved using the technique of integration by parts. (b) Give another example of a function that can be solved using the technique of integration by parts but it has a function different from that in example (a).		

(c) Can you describe the characteristics or properties of the integrand that shows it can be solved using the technique of integration by parts?		
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Table 5.5 (7): Results of Question 7

	7	(a)	(b)	(c)
SEM		R: 26 W: 9	R: 19 W: 7	R: 6 W: 14
SAW		R: 15 W: 17	R: 12 W: 12	R: 3 W: 11
		R: 41 W: 26	R: 31 W: 19	R: 9 W: 25

The responses for Question 7 were slightly better with a range of 34 to 67 responses. For 7 (c), only 9 responses were correct out of 34 attempts.

Some students did write some comments in the section where students were supposed to write what they thought or any comments. 42 students wrote that they had forgotten or needed revision for questions that they did not answer, 17 students wrote that they did not understand the questions, 8 students wrote '*hard to explain*', 3 students commented that question 6 and 7 were hard and 2 commented that they needed numbers not words. I also asked the students to rate the difficulty of the level of English used in the questionnaire, on a range from Very Hard, Hard, Fair to Easy. There were 6 students who rated the English as, 'Very Hard', 28 students who rated it 'Hard', 62 stated that it was 'Fair' and 3 students rated it 'Easy'. I also had 6 requests for the teaching to be carried out in Bahasa Malaysia in the section for Other Comments/Suggestions.

Overall Comments

Parts of Questions 1, 2, 4 and 5, elicited the most responses. Students were able to solve the problems when given in specific terms. Question 6 had the least number of responses. As there were a big number of statements by the students that they had forgotten, I found out that they had taken Calculus I in their first semester, Differential Equations in their second semester and that they were in their third semester for Engineering Mathematics. I had anticipated that students might find the questions different from what they were used to so it was actually a pleasant surprise that many attempted to answer the prompts and

questions. It was very clear that the students could do the Basic Calculus. However, some tried to answer the prompts and questions and they were a few students who did not attempt them at all, preferring to leave them blank or with the comments, '*forgotten*' or '*I don't understand*'. Using Question 4 as an illustration, the students were able to answer the questions (a) with 57.7% correct answers out of 97 responses, (b) with 57.9% correct answers out of 95 responses and finally (c) with 47.9% correct answers out of 94 responses. However, only 4 students tried to state reasons for their answers. Looking at the responses for all the questions, many students opted not to answer parts of the questionnaire which required reasons for their working or when some explanations were requested.

5.3.3 Classroom Episodes

In this presentation, I will describe the classroom episodes in two ways. Firstly, the first two weeks period will be described in chronological order so as to give some ideas how the experience in one class influenced my actions on the other as well as how the time-table influenced the pace of my teaching. Secondly, I will select some episodes that I thought were significant in some ways and contributed to the uncovering of various issues that will be picked up in the discussion. I was given two two-hour sessions for 2SAW 17, on Tuesday and Wednesday but for the 2SEM 36 group, I had them for a lecture at 12.00 noon. Their tutorial slot was on Thursday with two lectures slot on Wednesday and Friday. I have reproduced my original time-table here as it gives an indication of the workload I carried as well as how the classes for the two groups were distributed. I had another class to teach during the semester but I only taught it for the first 7 weeks with another colleague taking over for the last 7 weeks. This class was made up of a group of students who were repeating the Foundation Mathematics course from the old syllabus. I also had two students to supervise for their final year undergraduate projects. I had missed the first week of the session because I was away on a conference so I had three hours of lectures to make

up for the classes, 2 SEM and 2 SAW but a colleague had given the questionnaires to the students on my behalf.

Figure 5.4: Teaching Time-table

Time/Day	8.00-8.50	9.00-9.50	10.00-10.50	11.00-11.50	12.00-12.50		14.00-14.50	15.00-15.50	16.00-16.50	17.00-17.50
MONDAY					SSE 1893 L FKE-36 P08 115A		Foundation Maths (L) C13-412-02			
TUESDAY		SSE 1893 L 2 SAW 17 C07 415								
WEDNESDAY			SSE 1893 L 2SAW 17 C07 415	SSE 1893 T 2SAW 17 C07 415	SSE 1893 L FKE-36 P07 411B		DEPT SEMINAR		Foundation Maths (L/T) C18-213	
THURSDAY			U/G Project Teoh		SSE 1893 T FKE-36 P03 220				Foundation Maths (L) C13-211	
FRIDAY			SSE 1893 L FKE-36 P03 221	U/G Project Kartini						

5.3.3.1 Description of the first two weeks, 18/07/05 – 29/07/05

The description covers the first two weeks of my teaching. However, I had missed the first week so I was actually a week behind. There were 10 sections in total and students were only from the Civil and Electrical Engineering faculties. I had arranged for a colleague to meet the students in their first class meeting so as to explain my absence as well as to hand out the problem solving questionnaires. The class scenes are only briefly described highlighting certain events that I had noted in my own class notes.

Week 2: 18/07/05 – 22/07/05

Monday, 18/07/05, 12.00 – 12.50 pm, 2 SEM 36

This was the first class session for 2SEM 36 students in the 12.00 noon slot. I had 64 students in the class, 2 more than those who answered the questionnaires. I gave them an explanation about the details of the course, the assessment and the teaching strategies that I would be using. I also shared information about the syllabus, the teaching and learning objectives for the subject. I also explained about how I would conduct my classes and the way of working that I had chosen. I explained that the students would have to work in groups of 2 – 4 people, and that I would be using the worksheets for the first three topics. I also said that for the first few weeks, I would be using the tutorial hour as part of the lectures as well and that in each class we would be having a mix of lectures and activities as well as opportunities for students to review any work that they would have done from the tutorial sheets. I requested that the students followed my way of working for four weeks and promised them that we would review the whole process then. They could make any suggestions to change the way we worked but after the four weeks period. I gave them the general outline for my teaching; that I would start off with a 15-20 minutes lecture session then students were asked to work on the worksheets for 15 minutes and we would end the session with 15-20 minutes of discussion, review or addressing questions from students. Students were to work on the tutorial questions in their own time and in the first four weeks; they could discuss these questions in any of the class session. Tutorial questions will be taken from the textbook.

I gave a lecture on two variables functions, its domain and range with two examples. I handed out Worksheet 1. The SEM classroom was big with individual chairs so the students had some space to break out into groups. The students were asked to work in groups of 2-4 people, thus, there was different groupings in the class, with some students working in pairs, groups of threes and a few 4-persons group. They are in the second year

and looked as if they knew what to do. The group members were mixed, boys and girls as well as different races. They were working on the worksheet but I could see that many were ignoring the prompts and questions. So, I decided to put an example on the board and went through the problem, asking the students to respond to the prompts and questions. There was some response from the students but we ran out of time and I ended the class.

Tuesday, 19/07/05, 9.00 – 10.50 am, 2 SAW 17

For the 2 SAW 17 class, I was given two two-hour slots on consecutive days in the timetable. There were 42 students in the class. I gave them the same explanations about the organisation of the lectures, tutorials and class activities. Although I had given explanations about group work, I saw that the class furniture was not suitable. We were given an older classroom where the seats were in rows but bonded in groups of eight. I promised the students that I would try to find a different class. Due to the bonded chairs and the size of the classroom which was small, I asked students to work mainly in pairs. It was not easy to move around the room. I found it difficult to reach students who sat in the middle rows. However, since the number of students was small, I asked them to spread out but that did not go down too well as they liked to sit in the front rows. They said that they could see the board better so I did not insist. Thus they worked in pairs and I allowed them to choose their own partners but most students just turn to their nearest partner.

I gave a short introduction on Multivariable functions, focusing on two variable functions, the definitions of domain and range and gave the class two examples. I then asked them to work on Worksheet 1. The students worked on the questions but as I walked around, a student asked me a question, '*How to find the domain?*' I pointed out the first example where a function was given and she had to evaluate the example for given values of (x, y) . I asked her, '*which variables give the values of the domain?*' She said, ' x, y '. Then I asked, '*for the given function, what were the values of x and y that would define the function?*' She gave the right answer. There were three parts to Question 1 and the students worked

on the questions for about 20 minutes. Question 1(a) started with the function

$f(x, y) = x^2 + y^2$ with some variations introduced such as 1(b) $f(x, y) = \sqrt{x^2 + y^2}$ and

1(c) $f(x, y) = \frac{1}{\sqrt{x^2 + y^2}}$. Some prompts and questions were given to direct students'

attention to the similarities and differences in the functions. The students were involved in working out the answers. Nevertheless, I insisted that they should take a 10 minutes break.

In Question 2, the students worked on drawing the domain. Worksheet 1 is reproduced below but the format has been changed slightly. The students' worksheet had spaces where they could write down their answers which have been removed in the sample below to save space. For the second question, I had more requests for attention as a number of students wanted to know if they had drawn the domain correctly. We did not finish Question 2 and in addition, I did not finish the class on time. Nevertheless, I felt quite cheerful as the students were very cooperative.

Wednesday, 20/07/05, 10.00 – 11.50 am, 2 SAW 17

I started the class with more examples on finding domain and range of functions. The students were asked to continue work on Worksheet 1. After about 20 minutes, I asked students if they had any questions or comments about worksheet 1 as I wanted to start the lecture on the next topic. A question was asked about how to draw the domain. I put up an example on the board and worked through the example on how to draw the domain. Students were reminded to do extra problems from the textbook and I then continued with lectures on how to draw graphs of two variables functions and handed out Worksheet 2. We were still in the same class as I did not have time to arrange with the administrative section for a new class. The students were working in pairs but I could not be sure if they were working with the same partners. I could see that some of the girls were with the same partners. As I walked around, there were many students who wanted me to check their working and graphs. They were not happy that there were no answers for the questions on

the worksheets. The last hour was designated as a tutorial. The students' continued working on worksheet 2. They were quite engrossed in their work. I requested Examples 1 (c) and 5 to be handed in as assignments that will be marked. In Worksheet 1, students were asked to find domains and ranges and the example was extended in worksheet 2 where students were asked to draw the graphs as well. Example 5 was a question on generalising the ideas of functions to more than three variables. Some examples of selected students' work will be given in Section 5.3.4.

Figure 5.5: Example 1 (c) from Worksheet 1, Example 1 (c) from Worksheet 2 and Example 5 from Worksheet 2

Worksheet 1: Functions of Two Variables

- Evaluating functions
- Identifying the domain and range
- Describing the domain

Question 1:

(a) Let $f(x, y) = x^2 + y^2$

- Evaluate $f(2, 1)$; $f(-4, 3)$; $f(0, -5)$ and $f(u, v)$.
- Find the domain and range of f .

Reflection:

- What did you do to find the value of the function?
- Which variables give the values of the domain?

(b) Let $f(x, y) = \sqrt{x^2 + y^2}$

- Find the domain and range
- Suggest at least three possible values for (x, y) .

Compare 1(a) and 1 (b).

- What remains the same?
- What has changed?
- What have you noticed about the properties of the functions?

(c) Let $f(x, y) = \frac{1}{\sqrt{x^2 + y^2}}$

- Find the domain and range
- Suggest at least three possible values for $f(x, y)$.

Compare 1(b) and 1 (c).

- What remains the same?
- What has changed?
- What have you noticed about the properties of the functions?

Reflection

- Which variables did you look at when you want to find the domain and range?
- What did you do when you wanted to find the domain and range of the functions?
- What can you change in 1 (a), 1(b) and 1(c) but the solution methods stay the same?

Worksheet 2: Graphs of Functions of Two Variables

- Sketching surfaces in 3-D.
- Sketching level curves
- Matching equations with surfaces
- Identifying surfaces and level curves

Example 1: Sketching surfaces in 3-D.

(a) Let $f(x,y) = x^2 + y^2$

- What is the domain and range?
 - What is the trace of the function on the xy – plane?
 - What is the value of z on the xy – plane?
 - Draw the trace.
-
- What is the trace of the function on the yz – plane?
 - What is the value of x on the yz – plane?
 - Draw the trace.
-
- What is the trace of the function on the xz – plane?
 - What is the value of y on the xz – plane?
 - Draw the trace.
 - Can you put together all the different traces to form a graph?

(b) Let $f(x,y) = \sqrt{x^2 + y^2}$
Sketch the graph.

- Compare 1(a) and 1 (b).
- What remains the same?
 - What has changed?
 - What have you noticed about the techniques of graphing?
 - What have you noticed about the method of solving the question?

(c) Let $f(x,y) = \frac{1}{\sqrt{x^2 + y^2}}$
Sketch the graph.

- Compare 1(a), 1(b) and 1 (c).
- What remains the same?
 - What has changed?
 - What connects the different examples?

Reflection

- Which variables did you look at when you want to find the domain and range?
- What did you do when you wanted to find the domain and range of the functions?
- What did you do when you want to draw the traces on the different planes?
- What did you do when you had to put together the different traces of the functions?

<p>Example 5: Extending</p> <p>As you know, the graph of a real-valued function of a single variable is a set in a 2-coordinate space. The graph of a real-valued function of a two independent real variables is a set in a 3-coordinate space. The graph of a real-valued function of a three independent real variables is a set in 4-coordinate space.</p> <ul style="list-style-type: none"> • How can you define the graph of a real-valued function $f(x_1, x_2, x_3, x_4)$ of four independent real variables? • How would you define the graph of a real-valued function $f(x_1, x_2, x_3, x_4, \dots, x_n)$ of n independent real variables? 	<p>Compare 3, 4 and 5.</p> <ul style="list-style-type: none"> • What ideas are similar? • How do you determine the values of f?
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Wednesday, 20/07/05, 12.00 – 12.50 pm, 2 SEM 36

I started off with a lecture on the graphs of functions and how to draw them using level curves and traces. I had a few examples to give, starting with graphs of planes, parabolic and circular cylinders. I asked the students to sit in their groups and gave out worksheet 2. The students were working on the first problem 1 (a) and I could see that they knew how to answer the question. However, they were also providing answers to the prompts and questions. Many were also doing question 1(b). I put up the question on the board. I went through the prompts and questions. Some students responded. I was using the example to end the day's lecture to go through the basic ideas about graphing functions as there would be not enough time to do question 1(c). However, there was a tutorial class for the students on the next day.

Thursday, 21/07/05, 12.00 – 12.50 pm, 2 SEM 36

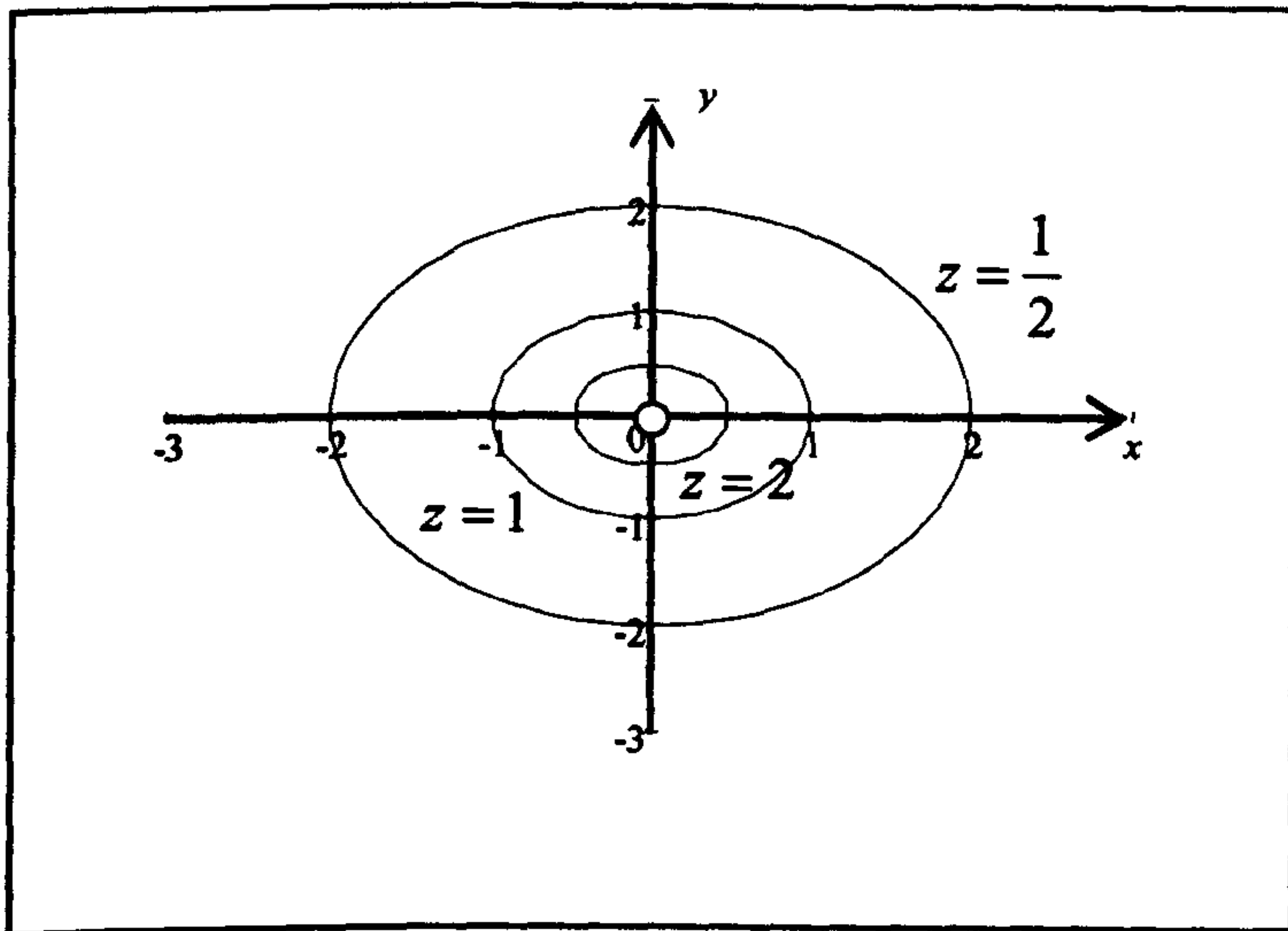
The Faculty of Electrical Engineering has decided that the students would be divided into two groups for tutorials so each group would have a tutorial once in two weeks. I had arbitrarily designated the first half from the list of students to be the first group but students could make voluntary exchanges between the groups. Not all the students have bought the textbook so I had the tutorial questions photocopied and gave them out to the students. A student had question about finding the domain and range of a function, $f(x, y) = \frac{1}{x+y}$.

This was a question from the tutorial exercise in the textbook. I saw that she had the

textbook so I supposed that she must have looked through the problems before coming to class. I asked if she minded that I put the question to the whole class which she did not. I put the function on the board. I asked the class to work on the problem. I went back to the student who asked the problem and sat next to her to work on the problem. I used a question, '*What property of $f(x,y)$ will affect the values of the domain?*' She answered that $(x+y) \neq 0$, but did not write the answer using the set notation. However, she was not sure how to draw the domain. I decided then to address the whole class, and asked the others if they found the domain. Many knew the answer but they were not sure about its graph. I decided to involve the whole class in drawing the graph. I knew that Inequalities was a topic in school mathematics as well as at matriculation or sixth form and there were students who managed to draw the graph. I invited them to share their answer.

Another student asked about the graph of the function, $f(x,y) = \frac{1}{\sqrt{x^2 + y^2}}$. This was question 1 (c) from the worksheet. I had collected this question from the 2 SAW 17 class as an assignment to be marked. Some students wanted the question to be solved together as well. They managed to determine the domain but wondered how to depict $(x,y) \neq (0,0)$. I asked them to draw the level curves and see the picture. They chose $c = 1$ and $c = 2$. I suggested $c = \frac{1}{2}$.

Figure 5.6: Graphs of level curves for $f(x, y) = \frac{1}{\sqrt{x^2 + y^2}}$



However, it was not enough just to look at the level curves; traces were also required to draw the graph. I asked the students to look at traces when $x = 0$, and when $y = 0$. I looked at some students' work quickly and saw that they had drawn the level curves. I drew the level curves on the board. There was some confusion about the representation used, the level curves were concentric circles which became bigger as the values of z became smaller but "small circle" at $(0,0)$ indicated a discontinuity.

There was another discussion about the trace when $x = 0$, then $z = \frac{1}{\sqrt{y^2}}$. Was it $z = \frac{1}{y}$ or

$z = \frac{1}{|y|}$? There was a lot of discussion going round the room about what the graph would

look like. Students were trying to put the level curves and traces together. They realised

that $z = \frac{1}{|y|}$. A student asked if it was possible to find traces for values of x and y other than

zero, i.e, traces parallel to the yz -plane and the xz -plane? I said that it was a good

question for all the students to think about. I asked them to finish the exercise and to hand

them in the next day. I also asked for Example 5 to be handed in. We ran out of time so I

decided to end the session. I was not sure if they minded that I did not answer the question.

However, because of the question, I realised that in my examples, I had been focusing on traces when $x = 0$ and $y = 0$. I noted that I will need to talk about such traces in the next class as well as for the 2 SAW class. Still, I was quite happy as it appeared that the students were all engrossed in doing the problems. I made a mental note to find some questions on graph sketching for the students as extra questions which could be found in Chapter 10 in the textbook. For this course, students were only required to refer to Chapters 11 – 14 of the book. I had to make a note to myself to remember that the other half of the class has not had their tutorial and thus had not handed in any assignment yet.

Friday, 22/07/05, 10.00 – 10.50 am, 2 SEM 36

I used the session to talk about sketching graphs using level curves and traces. I looked at traces on planes parallel to the yz – plane and the xz – plane. I also showed examples of graphs of functions with elliptical, hyperbolic and straight lines as level curves. We continued working on worksheet 2. There were Examples 2(a) and (b) as well as 3 (a), (b) and (c). As there was not enough time to work on all the examples, I asked the students to work on any two questions in the class. Students' responses to the prompts and questions during class were quite good, although there some wrong answers. I was walking around quickly to look at their work.

I needed to move on with the syllabus as I had missed about three hours' lectures and I was behind. I had to ensure that they would all cover the topics that would be examined in the first test. I had to plan for make-up classes. This was the drawback when working in multi-sectioned courses as the assessment had to be coordinated and comply with the decisions that were agreed on by the majority of lecturers during coordination meetings. For this course there would be two tests and the final examination. Other coursework was left to the discretion of the individual lecturer.

I introduced functions of three variables to the class. I used the ideas about what was similar about the notation of the functions for one variable, two and three variables and a

simplistic definition of the domain of the functions to connect the definitions of these functions to the notation used.

$y = f(x)$ – domain: subset of the real line

$z = f(x, y)$ – domain: subset of xy – plane

$w = f(x, y, z)$ – domain: subset of a solid

I gave an example of how to draw level surfaces. In my example, the values of k for $f(x, y, z) = k$ were specified. There was only 1 question on how to draw level surfaces in the worksheet but students were asked to draw a typical level surface thus they had to determine a value for k .

Figure 5.7: Example 4 from worksheet 2.

<p>Example 4: Functions of 3 variables and Sketching Level Surfaces</p> <p>Let $f(x, y, z) = x^2 + y^2 + z^2$. Sketch a typical level surface for the function.</p>	<p>Compare 3 and 4.</p> <ul style="list-style-type: none">• What are level surfaces?• How do you determine the values of f?• What are the similarities between 3 and 4?
---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------	----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------

Students were able to answer the question but I was not sure if they referred to the prompts and questions provided. The question was quite easy. At the end of the class, students from yesterday’s tutorial class came up to hand in their work. Students from the other group became worried but I assured that they will be given assignments too.

Reflection: I had to remember that only a half of the SEM students were in the tutorial class and the other half will need to send in their assignments in the next tutorial class. At the end of this week, I was worried about the lecture slots especially for the 2 SAW class as they had two-hour slots on consecutive days. The 2 hour slots were good as that meant they will have more time on class tasks but the downside was that students did not have time to deliberate over the notes as the classes were too close to each other. I found that I could not finish all the topics that I have assigned for the periods especially when I could see that the students were tired. I was also ahead in my lectures for the 2 SEM class when

compared to 2 SAW. I wanted 2+1+1 (hours) sessions for them rather than 2+2 (hours) sessions and a more conducive class environment physically for them but attempts to change classes and their time-table were not successful.

I was also not happy about the split tutorial sessions for the 2 SEM although I like the idea of smaller tutorial groups. On top of that, I was worried about the Wednesday class for the 2 SEM group as it followed immediately the two hour session with 2 SAW. The students were in different parts of the campus and I needed the car to get to the 2 SEM class. I felt a bit tired the last time. I am thinking of changing the time or change the class to a tutorial but I will have to discuss with the students first.

Teaching was easier due to several reasons; students were active in the class, doing problems and asking questions but some were still reluctant to give explanations about their working verbally although they attempted to do this in written form. Students appeared to have good recall of their prior mathematical knowledge and very competent with the relevant mathematical techniques thus there was less need for revision as well. However, if they do need revision they would let me know.

Week 3: 25/07/05 – 29/07/05

Monday, 25/07/05, 12.00 – 12.50 pm, 2 SEM 36

The topic for today was Partial Differentiation. I asked students to work on worksheet 3. The first part was an investigation on the meaning of Partial Differentiation (see Figure 5.8 (a)) for the special case. Students broke up into their groups and I could see that they were attempting to do the questions, drawing the graphs and generally trying to find the equation of the trace when $y=1$. A lot of discussion was taking place. I decided to collect the worksheets so that I could look through their work.

Figure 5.8(a): Extract from Worksheet 3

Worksheet 3: Partial derivatives of functions of two variables

Partial Differentiation and Definition of Partial Derivatives

The special case: Let $z = f(x, y) = \sqrt{4 - x^2 - y^2}$. Draw the graph of the function.

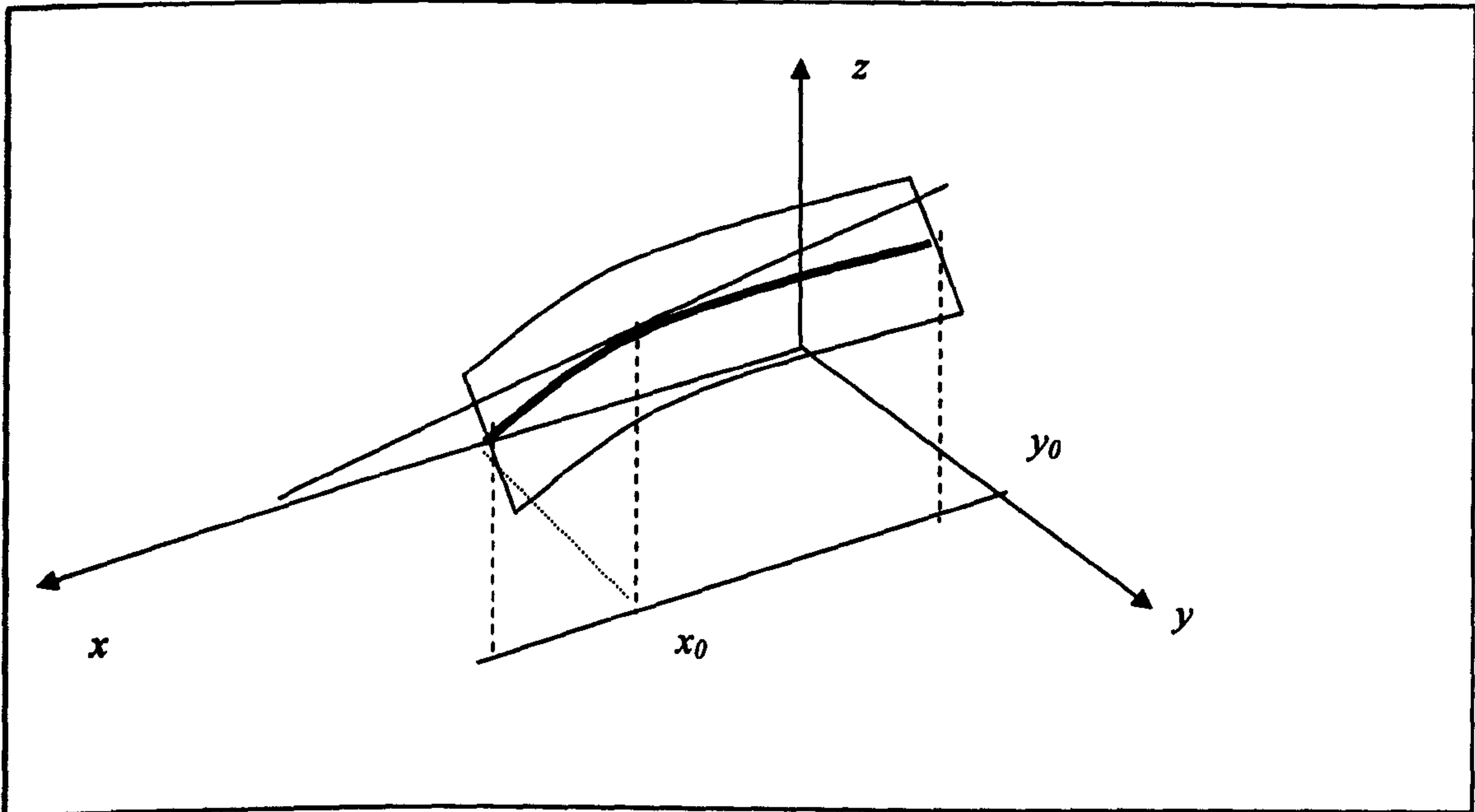
Imagine a plane $y = 1$, parallel to the xz – plane, intersecting the graph. Draw the trace of the curve at the intersection plane and determine its equation.

- What do you see in your picture? Describe your picture.
 - If $y = 1$, what are the possible values of x ?
 - How can you evaluate $z = f(x, y)$?
 - Determine (at least) 3 values for z .
 - On the plane $y = 1$, what can you say about the behaviour of z , x and y ?
 - How would you find the derivative to the trace of the curve?
-
- Try to draw a picture of a slice parallel to the yz – plane, through a particular value of x . Which value of x did you choose and why?
 - Give an interpretation of the tangent to the curve in that direction.

I continued with a lecture on finding partial derivatives for the general case. I had the lecture on transparencies but the students had the same contents in the worksheet. There were some accompanying prompts and questions which I was using. Students were invited to contribute answers. Using the two examples, I invited students to suggest definitions for the partial derivatives. However, some of the students already knew the definition and were actually doing the subsequent problems. They have had a brief introduction to Partial Differentiation in Basic Calculus, SSE 1792. I did not take that into consideration when I did the questions in the worksheet. So we moved on, I gave them the definitions and allowed them to do the problems set in worksheet. They were already quite proficient with the techniques of finding partial derivatives.

Figure 5.8 (b): Extract from Worksheet 3

The General Case: Let $z = f(x, y)$ with the vertical plane, $y = y_0$ intersecting its surface. The graphs are given below.



Referring to the graph, please answer the following questions.

- If $x = x_0$, what is the value z ?
- Identify another point in the same plane, say $x = x_1$, what is the value of z ?
- Find another point in the same plane, say $x = x_2$, what is the value of z ?
- What can be said about the values of x, y and z are changing?

Tuesday, 26/07/05, 9.00 – 10.50 am, 2 SAW 17

I wanted to start with a lecture on the drawing of level surfaces of functions of two variables. I felt that I had to ensure that this group was at the same content stage as my other group. However, they requested a short revision on ellipse and hyperbolas. I gave them a quick lecture about these topics. I gave the lecture on level surfaces after that. I gave two examples, using spheres and hyperboloids. Then, I asked them to do the last question in worksheet 2. I asked the students if there were any questions that they would like to ask about the topics we had covered. Some had questions about the problems but I requested that we kept that for the Wednesday class as the last hour was to be a tutorial

class. I thought that I'd rather have a lecture as they were still fresh. I handed out **Worksheet 3** and asked students to work in their groups to answer the first part of the **exercise.** They students were working on the problem and appeared absorbed in their **discussions.** As I walked around I could see that they were drawing the graph of the given **function.** I could only get close to a few groups because of the seating arrangement. I had **chosen two ways of monitoring students' work** in the class, firstly, just observing what **they were doing** and only get involved if they requested it and secondly, to intercede by **requesting students to explain or share their working** with me or to the whole class. These **particular batches, both 2 SEM and 2 SAW,** seemed to prefer this way of working as they **were quite self-reliant** and only requested for help if they needed it. We had a short break.

After the break, I addressed the whole class and asked some groups to share their answers, **a few volunteered** and I chose three groups. They gave similar answers and the other **groups said that they had the same answers as well.** I continued the lecture by presenting **the general case** in the same way as I had done for the 2 SEM class. We spent the rest of **the session, practising on finding partial derivatives.** The students already knew the basic **techniques of partial differentiation** and were quite good at it so I focused more on revising **differentiation techniques.**

Wednesday, 27/07/05, 10.00 – 11.50 am, 2 SAW 17

I started with a lecture on implicit differentiation and went through two examples on the **board.** Students were asked to work on some questions in **Worksheet 4.** However, **Worksheet 4** was more like a tutorial sheet as I had compiled various questions on the **basic concepts of partial differentiation.** There were some prompts and questions but they **were included in a general manner only.** Figure 5.9 shows the focus of the worksheet and **the problems that were included.**

Figure 5. 9: Focus of Worksheet 4

Worksheet 4: Partial derivatives of functions of two variables

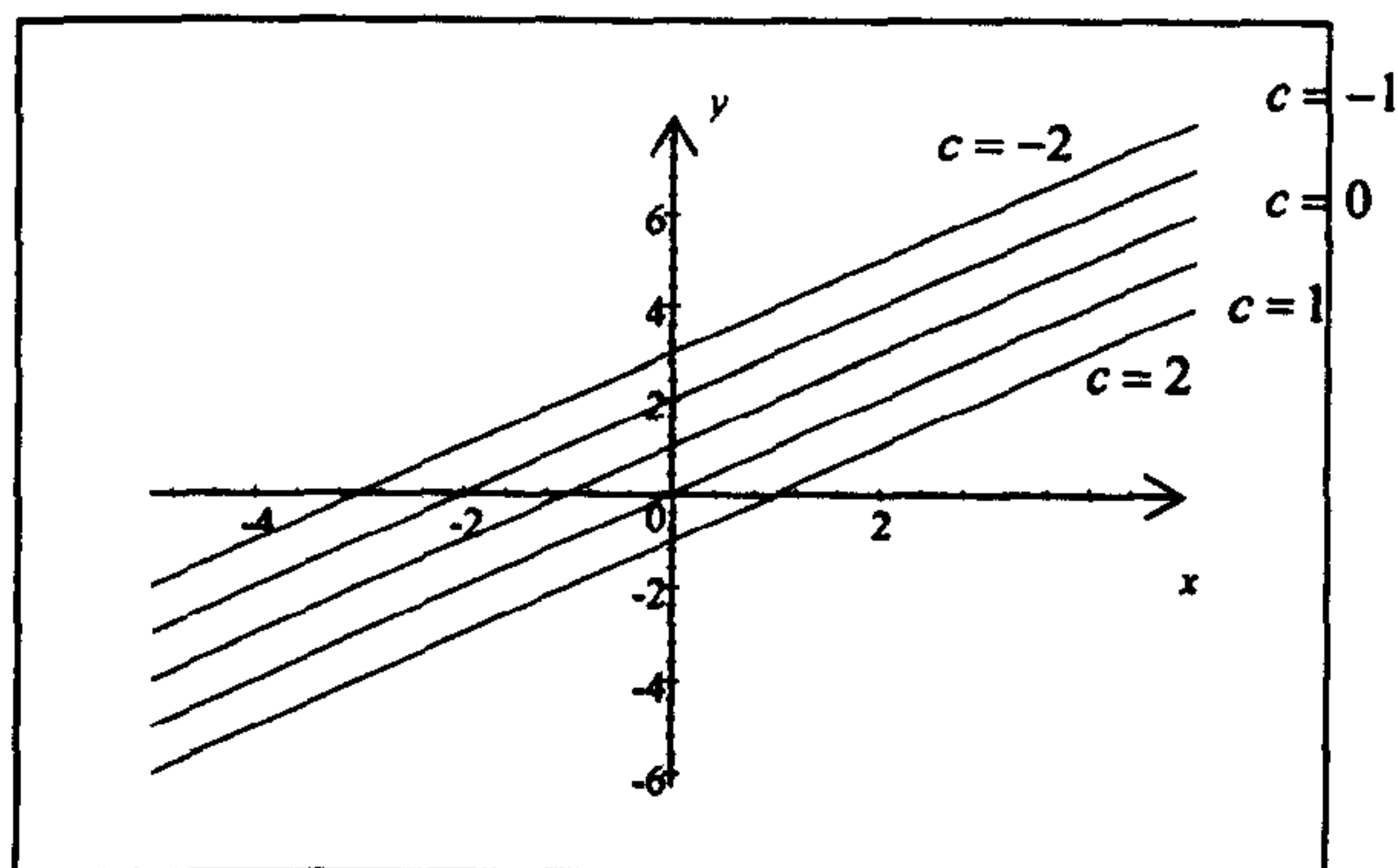
- Finding partial derivatives
- Implicit differentiation
- Partial Derivatives as a Slope of a Tangent and Rate of Change
- Calculating Second-Order Partial Derivatives
- Calculating Mixed Partial Derivatives

Even though I had assigned the hour for Implicit Differentiation, when the students were asked to work in their groups, they were picking and choosing to do problems from the other sections as well. I asked if they had any questions on Implicit Differentiation. There were no questions. I decided to have the break then as the next hour was to be a tutorial session.

I started the session by asking the students to put forward any questions they had. A few had their hands raised. Other students were working away on the worksheets. I approached the students, one person at a time. Their questions were on finding domains and how to visualise and draw the 3-dimension graphs from the level curves or the traces.

An example that was put forward by a student was how to draw the graph from the level curves of $f(x, y) = x - y + 1$. He had the drawing of the level curves (see Figure 5.10) and was not sure of what the solid looked like. I asked him and his partner to look at the equation of the function again and to write it as $z = x - y + 1$ and asked if it reminded him of something similar he has learnt in Calculus I. I knew that they have learned how to draw planes in vector form. I also suggested that they use other ways to sketch the graph and see what they could get.

Figure 5.10: Graph of the level curves of $z = x - y + 1$



Note to myself: I had introduced level curves and then traces as ways to draw the 3-D graphs. Some students were using both methods, in the same order, drawing level curves and then traces, to draw their graphs. They regarded the lecture notes as procedures that needed to be followed when drawing graphs. I was presenting different ways to draw the graphs and that there was choice but obviously some students got a different message.

As I walked over to the next group, they also had questions about drawing graphs. I attended to them and since there were other students who needed attention, I tried to assign about 5 minutes per group. I had the textbook so I wrote extra questions on finding domain, range and drawing graphs on the board. For this morning, I used the tutorial session to attend to the groups individually. However, I also went to groups that did not request any help.

Wednesday, 27/07/05, 12.00 – 12.50 pm, 2 SEM 36

I wanted to discuss with the students whether I could change this class to a tutorial session or change the time slot as I had two hours of class before this. In addition, the previous class was in a different part of the campus which was quite far away so I had to come by car. The students too appeared tired as they had classes before this hour. Class often started about 10 minutes late, waiting for students to settle down. I gave a lecture on Implicit Differentiation and worked through two examples. The students were asked to work on a couple of examples for themselves. The students could do the exercises easily. I moved on

to the next topic, Partial Differentiation as a Slope. Discussion at the end of the class revealed that it was not possible to change the session to a tutorial as one student could not attend the Thursday session as she had another class at the same time. It was the first time that I knew about it. I told her that I would like to talk to her after class to find out more about her situation.

Thursday, 28/07/05, 12.00 – 12.50 pm, 2 SEM 36

This was the tutorial session for the second group. I used the slot to check whether students had any questions on Worksheet 1 and Worksheet 2 as well as Tutorial 1 and Tutorial 2. I planned to spend the first 20 minutes to respond to general queries. A student asked about level curves. We spent some time working on examples on the board together. Students were asked to contribute to the solution of examples that I had put up on the board. The discussion was extended to surface curves. I then put up the extra questions on graphs of functions (used in the other classes) as the students were concerned about how to visualise 3 dimensional graphs. However, we spent more than 20 minutes on the various discussions. I then requested that they hand in the same exercises as their colleagues which would contribute to their coursework. They were to hand in the work on the next day (Friday) in class.

Friday, 29/07/05, 10.00 – 10.50 am, 2 SEM 36

I continued with a lecture on partial derivatives as rate of change and we used problems from Worksheet 4. However, Worksheet 4 was more like a tutorial sheet as I had collected various questions on the different topics already taught. The prompts and questions were also more general in nature and were placed at the beginning of every sub-topic rather than closely linked to each question. The idea was to gradually remove the 'detailed support' and allowed students to work on the problems themselves. I was hoping that they would use the ideas about mathematical thinking from earlier worksheets to guide them.

I had decided to select two questions from this worksheet as an assignment for students in both my classes to submit. I still had the prompts and questions on the worksheet for students to use if they wanted to. The questions are reproduced below.

Figure 5.11: Assignment 2 from Worksheet 4

Assignment 2: from Worksheet 4

Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ from the following functions:

- What does it mean to find the partial derivative of $f(x, y)$ with respect to x ?
- What does it mean to find the partial derivative of $f(x, y)$ with respect to y ?
 - Which variable is changing?
 - Which variable is considered constant?
- What other rules or techniques of differentiation will you need to answer the following?
- What is the relationship between the definition of differentiation and integration?

(9) $f(x, y) = \int_x^y g(t) dt$ (g continuous for all t)

- Try to answer for specific values of n , say, $n = 1, 2, 3, \dots$
- Can you see any pattern in your answer?
- What happens in general?
- What happens when $n \rightarrow \infty$?

(10) $f(x, y) = \sum_{n=0}^{\infty} (xy)^n$ ($|xy| < 1$)

Selected students' responses to the above worksheets will be discussed in Section 5.3.4.

5.3.3.2 Selected Classroom Episodes

In this presentation, I will adopt a slightly different style of narration. After two weeks of teaching, various issues were identified and in this section, those issues such as class organisation and scheduling, assessment, students' feedback and my reflection on the teaching and learning situation will be described supported by the classroom episodes.

However as the semester progressed, there were other issues that became apparent and these included a review of students' learning behaviour and my own responses to their perceived concerns as well as to the demands of the teaching situation.

(1) Class organisation and scheduling

Excerpt from my notes:

Tuesday, 02/08/05, 2 SAW 17: Students came in late. Some were about 10 minutes late. The room was very hot as well as the air conditioning was down. When I asked why they were late, it seemed that there was a change in their time-table as a lecturer had moved his class from this block to the M38 building which was a long walk from where this class was. I was annoyed, not at the students but at the thoughtlessness of my colleague, whoever he was! Students had to walk down one hill and up another to come to this class! We started the class about 15 minutes later as I had to give them some time to take a breather.

Wednesday, 03/08/05, 2 SAW 17: The students were late again – just my luck, it would seem that my class on both days were after that colleague's sessions. Students looked tired so I decided to have the tutorial in the first hour to give them time to relax a bit and to prepare for my lectures. I also decided that I will need to find a classroom in block M38 so as to help the students and myself. I cannot drum interest out of tired students!!! I'm worried too as I am now slightly behind in my lectures as compared to the SEM group and the fact that they will have their first test on 25/08/05. Topics from week 1 – 5 will be tested. This meant make-up classes have to be arranged.

The autonomy given to lecturers to rearrange the time-tables and change lecture rooms in this particular instance had seriously unsettled my own teaching time. My attempt to find a new class was not successful so I decided to deal with the situation 'as it arose'. In the following weeks, I would take my cue from the students, if they felt that they were alright, I would start off the class with a lecture otherwise I would start with a review of the previous lessons or just gave them a 5 minute rest period. However, that meant that my class was always about 10-15 minutes late. The Wednesday slot was already quite tight as I

had a class with 2 SEM at 12 noon. I decided that I would organise extra classes if required. The scheduling of the teaching slots later showed to have influenced the students' learning. Having two consecutive days of two hour slots had shown that the students did not have enough time to read up or review the topics. I could only review the topics in the lecture of the following week. However, always starting late also meant that the 2 SAW students were always slightly behind and I had to schedule some extra classes for the students. These particular students' spirits were quite challenged at the end of the semester but although they were unhappy they were still attentive. They continued to be very worried about getting good grades as they found the latter topics more difficult. They welcomed the extra classes and requested sessions for revision to prepare for the final examination.

(2) Assessment

The Department of Mathematics had a standard guide for the number and kinds of assessment that had to be carried out (see Table 5.6). I only had some freedom in determining the assignments and quizzes to be carried out and that contributed only 10% to the coursework. The tests and final examination were coordinated and all students had to sit for the same papers.

Table 5.6: Information about Assessment given to Students

Item	Topics	Marks	Duration/date
Test 1	From weeks 1 - 5	15%	1 hr – 25/08/05, 8.30pm
Test 2	From weeks 6 - 10	25%	1.5 hrs – 08/09/05, 9.00 am
Assignments/Quiz		10%	continuous
Final examinations	Comprehensive	50%	3 hrs / Exam week

For both my classes, I had used the responses to the worksheets as the written assignments. Quiz questions were also taken from the worksheets or the tutorial questions sheets. I had encouraged the students to work in groups, 2 – 4 people per group so their written assignments were done in groups but quiz questions were answered individually. However, I had used other assessment techniques that did not contribute to the coursework

marks such as *'one line summary'* and *'identify the muddiest point after each chapter'*, so as to gather students' feedback on the learning. I had also handed out an evaluation questionnaire at the end of the semester to gather more students' feedback on the teaching and their learning experience. I had collected at least 4 written assignments and given out 2 quizzes that contributed to the 10% marks. It was a measure of the students' compliant behaviour that they accepted the demands of the assessment without any objection.

(3) Teaching and Learning Issues

I had promised a review of the teaching styles after the 4th week, thus a discussion with the students about what had been carried out was held in each class. The students had similar reactions; although not enthusiastic they thought that I did not need to change what I did although there were requests for more worked examples. However, the teaching became more lecture oriented with some class activities and most tasks kept to the tutorial sessions to ensure that the syllabus was covered. I still had worksheets but these were not written out in the same format as Worksheet 1 – 3. The latter ones were like tutorial questions sheets with some general prompts and questions for students to refer to. I also had to organise some extra lectures and revision classes to prepare for the tests and the final examination.

(4) Students' Learning Behaviour

Although, I have only described what was happening in the first two weeks of class, it was obvious that these particular groups of students were generally well behaved, most of them worked and read up on their own time and were very independent learners but motivated with the specific goals of doing well in the tests and examination. In conversations about their concerns, a majority were worried about their examination performance and felt that they needed more practice for better results. They worked on tests and examination papers from previous years to prepare for the tests and examination. These preparations were helpful as the Test and Examination questions were prepared by lecturers on the team and were quite similar to previous years' papers. To illustrate, I will only concentrate on the

first three topics of real multivariable functions, partial derivatives and multiple integrals. Below (Table 5.7), is a list of the learning objectives, knowledge and skills expected of students' learning for the given topics and these were examined in the tests and examinations.

Table 5.7: Learning Objectives, Skills and Knowledge

Learning Objectives	Skills and knowledge
Determine the domain and range of functions Sketching of graphs	<ul style="list-style-type: none"> • Know the behaviour and characteristics of standard functions • Able to sketch graphs of standard functions • Able to extend the above facts and skills to more complex functions
Find partial derivatives	<ul style="list-style-type: none"> • Know the techniques of differentiation • Able to extend the above techniques to multivariable functions
Applications of partial derivatives: error and extrema of functions	<ul style="list-style-type: none"> • Able to identify problem situations • Able to choose the correct procedures • Able to make linkages and connections of several mathematical ideas and representation
Evaluate double integrals	<ul style="list-style-type: none"> • Know the techniques of integration • Setting up the integral • Evaluating the integral in Cartesian and Polar coordinates systems
Evaluate triple integrals	<ul style="list-style-type: none"> • Know the techniques of integration • Setting up the integral • Evaluating the integral in Cartesian, Cylindrical and Spherical coordinates systems
Applications: area, volume, moment, moment of inertia, centre of mass	<ul style="list-style-type: none"> • Able to identify problem situations • Able to choose the correct procedures • Able to make linkages and connections of several mathematical ideas and representation

Thus, the way the tests and examinations questions were written further encouraged students' '*drill and practice*' learning. As a member of a teaching team, it was not possible to bring about changes to the major assessment items such as tests and examination without consensus. Therefore, the assessment of students' mathematical thinking, their ability to solve non-routine problems and their recognition and awareness of mathematical processes were only carried out in the class tasks and assignments. In terms of students' learning behaviour, they participated in the class activities but I felt that it was more in the

nature of accommodating the lecturer's wishes or idiosyncrasies as they see them whilst they got on with their own styles of learning at the same time. This conclusion was supported by feedback from students that was gathered during the interviews which will be presented in the following section.

5.3.3.3 Students' Interviews

I conducted interviews with two groups of students from each section. Participation in the interviews was voluntary. The number of students who volunteered was 12 from 2 SAW and 15 from 2 SEM. I arranged for a convenient time to conduct the interviews and in the event, only 6 students turned up from 2 SAW and 9 students from 2 SEM. In reporting the interviews, I will focus on selected individual students but at the same time, will present the responses and issues that all had presented. The interviews were unstructured but I had prepared a list of general themes for them to talk about. These were (i) their own evaluation of their mathematical abilities, (ii) how they study and how they deal with difficult concepts found in the course, (iii) their comments on desirable teaching styles and, (iv) their comments on my teaching in particular. The interviews were conducted on 19th, 21st and 24th October 2005. The first two dates were at the end of the last week of the teaching session and the last date was during the study week before the final examination. The conversation was conducted in Bahasa Malaysia which was the students' choice. As additional background information, Table 5.8 below gives the students' performance in the various assessments that was conducted. The results will serve as the typical indicators of the students' ability to grasp the concepts taught and ability to answer the questions that were set in the tests and examinations. All names are pseudonyms.

Table 5.8: List of interviewees and their mathematics results

Date of interview	Course	Name	Coursework Marks 50%				Examination Marks 50%	Final grade
			Assignments 10%	Test 1 15%	Test 2 25%	Total 50%		
19/10/05	2 SAW	Fizah	10	11	19	40	37	B+
		Ila	9	12.5	12	34	31	B-
21/10/05	2 SEM	Chong	10	11.5	17	39	43	A-
		Idah	9.5	11.5	21.5	43	44	A
		Din	9	15	22	46	39	A
		Zafar	9.5	12.5	21.5	44	42	A
		Irah	10	10	13	33	34	B-
		Ezam	10	13	17	40	36	B+
21/10/05	2 SEM	Wang	10	14.5	18.5	43	47	A
		Fara	9	12.5	19.5	41	35	B+
		Lim	10	8.5	18.5	37	34	B
24/10/05	2 SAW	Haz	9.5	11.5	21.5	43	45	A
		Wan	9.5	11.5	20	41	39	A-
		Adam	9.5	8	13.5	31	29	C+
		Saif	10	12.5	20.5	43	34	B+

(i) How they rated their own mathematical ability

Surprisingly, all the students said that they had average or above average ability and had to work hard in this course. They said that mathematics was easier at school and that they were much better at mathematics then. During the interviews, I only had their coursework marks and could see that 10 students had marks in the 40 – 50% bracket and 6 students had marks in the 30-39% bracket. Understandably, as they had just finished the topics of Line and Surface Integrals, there was a strong feeling that the mathematics was difficult. Two students in particular, Adam and Saif, said that they usually make many careless mistakes in their workings. Ila and Irah also said that they found Differential Equations and Engineering Mathematics difficult especially when trying to read up for themselves. My observations had shown that these students seemed to cope very well during class work and were able to answer the tests questions reasonably well except for Irah and Adam. Lim

did not do too well in the first test but was better in the second test. Although the test questions were not set by me, students were familiar by the type of questions asked. I had made sure that examples of typical test questions were given in the worksheets or the tutorial sheets for students to work on.

(ii) How they study and how they deal with difficult concepts found in the course

Their learning strategies that they claimed were used most often were quite similar to each other. Among the most common responses were:

- To pay attention in class and try to understand while a topic was taught;
- If there was any difficulty, they would discuss or study with their friends;
- Read up the notes and examples and tried out the same examples for themselves
- Tried to do other examples;
- Talk to the lecturer.

However, there were slight differences amongst the strategies used by the boys and girls. All the boys said that they would go to the lecturer as a last resort. They ranked the order of preference in their study methods as 'read for themselves, study with friends, their seniors and last, if and only if necessary, their lecturers'. Meanwhile, the girls, Ida, Irah, Fizah, Ila and Fara, also identified the lecturer as the last person they would go to but she was considered an important resource person that they would definitely seek out. Adam said that he did not do a lot of examples but read up the examples I gave in class. In trying out other problems, most said that they only did the ones that had answers at the back. Saif said that if he had difficulties with any problem, he would just leave it and do others. All worked on past year tests and examination questions as preparation for the tests and final examination. When I queried Haz why he did not go to the lecturer for help, he said that it was easier to find and talk to his friends. He said that you had to dress up proper, speak proper and also find the time that the lecturer was free to see you and that was inconvenient.

(iii) Comments on desirable teaching styles

They were asked to describe the teaching methods that they liked to see more of. All said that they would have liked more examples, especially examination questions that were solved and the solution methods clearly identified. They did not like too much theory, wanted more questions to practise on and answers for the problems. They would prefer that harder examples were given and worked on together during class. When I asked why the answers were important to them, they said that the answers gave them some indication if they had answered correctly.

(iv) Comments on my teaching in particular

They had participated and responded very well to the prompts and questions in class sessions which made me think that they liked them and found them useful. However interviews with students and from conversations during the latter part of the semester when I was not using the worksheets gave a different picture. They only use the prompts and questions because I used them. Wan said that, *"have to trust the lecturer, she knows best."* Most did not use them for themselves, only a small number thought they were helpful. Three students, Chong, Wang and Idah, were most enthusiastic and said that they found them very helpful and guided them in their work.

One student, Lim, said that I was too long winded. When I asked what he meant, he said that I gave too much explanation during my teaching whereas he found that the problems asked in examinations were quite straight forward and they could do it without the need for too much explanation. A few students liked that they could work out the problems from the basics and that they could understand the techniques in a general way. They found the teaching methods helpful. Adam said that when he studied before, he did not talk a lot but that during this course, he had discussed the tasks more. Haz said that he found the examples in the worksheets very helpful but did not use the prompts and questions if

working on his own. He thought that I should increase activities that would attract the students' interests. He also said that although he has no problem with the 2 + 2 hours teaching slots, many of his classmates were not able to keep up with the lectures. Fizah and Ila (2 SAW) said that for the 2 hours lecture sessions, 1 hour of lecture was okay but the next hour should be for class activities, otherwise it was difficult to absorb too much information. Idah and Ira said that they could see how the prompts and questions helped them *'to see and think'* about the problems especially on topics that they have just studied. Chong said that the way I taught made the topics looked easy and easily understood. Lim, however, thought that I was making the mathematics harder as he found the mathematics was easier to understand when he read the text book. An interesting word that the students used to described when they have understood was that they could *see* the mathematics and that they could do other problems.

5.3.3.4 Colleagues' support

In this semester, I had two colleagues who were involved in the evaluation, review and rewriting the worksheets into a workbook format. Their involvement served a two-fold purpose, firstly, they examined the teaching strategies, worksheets format and questions more objectively, and secondly, they provided the support that I needed such as reviewing and checking my teaching experiences, interpretations and reflections. In particular, they helped to check the suitability of the tasks assigned in the worksheets and my interpretations as well as suggestions for further interaction with the students. Another important aspect of their participation was the emotional and intellectual support that they gave me.

5.3.4 Working on Mathematical Thinking

I had adopted several strategies to support increasing students' awareness of mathematical thinking and these have been explained in Section 4.2.2. I had prepared the lecture notes and the worksheets to follow as much as possible the basic ideas that I had described.

However, these were used as the basic materials and there were times during the lectures or class activities, when I had to make decisions on the direction that the lessons were to take or include other materials that were needed. In the description of the class episodes, I had included some examples of such incidences.

1. Working on the powers specialising and generalising and the theme invariance amidst change

In every task there were several powers being used but I had decided that the main focus of the early worksheets was on the powers of *specialising* and *generalising* and the theme *invariance amidst change*. Students were already using these powers but I wanted them to recognise and be aware of these powers when they were using them. The explicit use of mathematical themes was also thought to provide linkages between mathematical ideas and to expose the structures of the mathematics, prompts and questions were used to draw the students' attention to these powers. In Figure 5.12 below, Questions 1(a), 1(b), 1(c) and Example 5 from Worksheet 2 have been extracted. In these questions, I had identified the focus as the following:

Figure 5.12: Mathematical Thinking Powers

Sub-theme: Range of Change

Activities: Specialising and Generalising, Characterising, Expressing

Problem: Finding the domain of a square root function, defining and drawing the graph of a function

Focus of Attention: property of function, possible range of change for the radicand, using traces and level curves in drawing functions, extending the idea of a graph for multivariable functions of 3 or more variables.

However, the students were only exposed to the worksheet. The focus of attention was explained to the students during the lectures and class activities, through whole class, small

group or individual communication. The first part, 1(a) was very structured and students worked through the question as a whole class exercise (Figure 5.13).

Figure 5.13: Question 1 (a) and (b) from Worksheet 2

<p>Example 1: Sketching surfaces in 3-D.</p> <p>(a) Let $f(x, y) = x^2 + y^2$</p> <ul style="list-style-type: none"> • What is the domain and range? • What is the trace of the function on the xy – plane? • What is the value of z on the xy – plane? • Draw the trace. <hr/> <ul style="list-style-type: none"> • What is the trace of the function on the yz – plane? • What is the value of x on the yz – plane? • Draw the trace. <hr/> <ul style="list-style-type: none"> • What is the trace of the function on the xz – plane? • What is the value of y on the xz – plane? • Draw the trace. • Can you put together all the different traces to form a graph? 	
<p>(b) Let $f(x, y) = \sqrt{x^2 + y^2}$, Sketch the graph.</p>	<p>Compare 1(a) and 1 (b).</p> <ul style="list-style-type: none"> • What remains the same? • What has changed? • What have you noticed about the techniques of graphing? • What have you noticed about the method of solving the question?
<p>(c) Let $f(x, y) = \frac{1}{\sqrt{x^2 + y^2}}$, Sketch the graph.</p>	<p>Compare 1(a), 1(b) and 1 (c).</p> <ul style="list-style-type: none"> • What remains the same? • What has changed? • What connects the different examples?

<p>Example 5: Extending</p> <p>As you know, the graph of a real-valued function of a single variable is a set in a 2-coordinate space. The graph of a real-valued function of a two independent real variables is a set in a 3-coordinate space. The graph of a real-valued function of a three independent real variables is a set in 4-coordinate space.</p> <ul style="list-style-type: none"> • How can you define the graph of a real-valued function $f(x_1, x_2, x_3, x_4)$ of four independent real variables? • How would you define the graph of a real-valued function $f(x_1, x_2, x_3, x_4, \dots, x_n)$ of n independent real variables? 	<p>Compare 3, 4 and 5.</p> <ul style="list-style-type: none"> • What ideas are similar? • How do you determine the values of f?
--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------	----------------------------------------------------------------------------------------------------------------------------------------------------------------------

They were asked to do Question 1(b) and 1(c) in their groups but they were to submit 1(c) and Example 5 individually. The exercises were carried out by students on different days.

There was also some miscommunication as there were students who decided to do Examples 1(c) and 5 on their own rather than in a group. Here, I will present an example of the work submitted by Chong. He did his work on his own. He had managed to answer the question and even had all the workings written out but then he cancelled them out. When I asked him why he cancelled out the working, he said that he thought that he should only show the correct answer. He was also not very sure how the domain was to be represented although he drew the correct level curves, traces and finally the graph of the function.

20/7/2005

Let $f(x,y) = \frac{1}{\sqrt{x^2+y^2}}$. Sketch the graph.

$f(x,y)$ is real number if $\sqrt{x^2+y^2} > 0$ and $\sqrt{x^2+y^2} \neq 0$

$\therefore x^2+y^2 > 0$ domain: $-\infty < x < \infty, x \neq 0$ range: $0 < z < \infty$
 $-\infty < y < \infty, y \neq 0$

$z = f(x,y)$, let $z = k$

if $x=0, k = \frac{1}{\sqrt{0^2+y^2}}$

$$= \frac{1}{|y|}$$

if $y=0, k = \frac{1}{\sqrt{x^2+0^2}}$

$$= \frac{1}{|x|}$$

if $k \neq 0, 0 = \frac{1}{\sqrt{x^2+y^2}}$

$\therefore \sqrt{x^2+y^2} = \infty$ on the xy -plane.

if $k=1, 1 = \frac{1}{\sqrt{x^2+y^2}}$

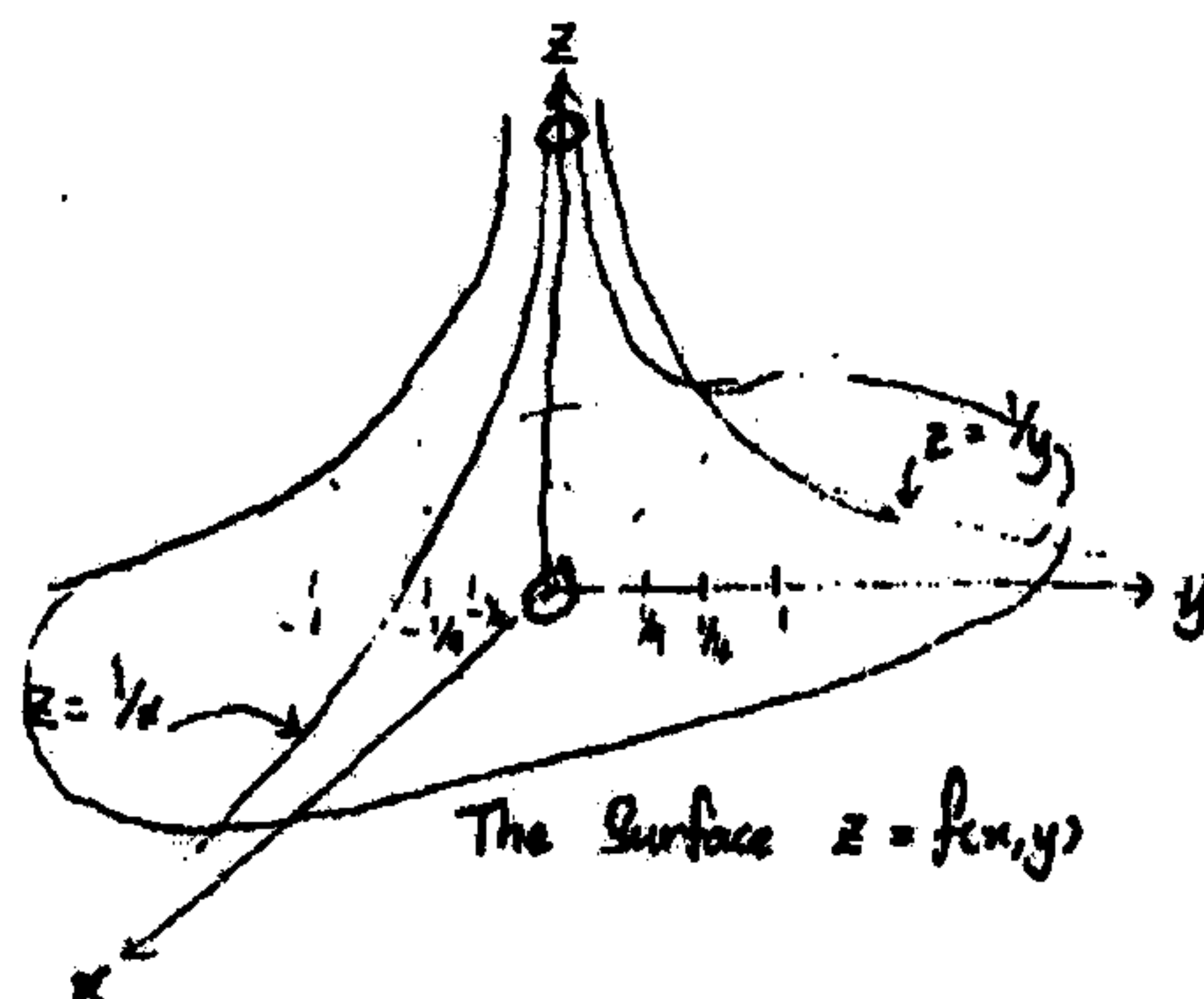
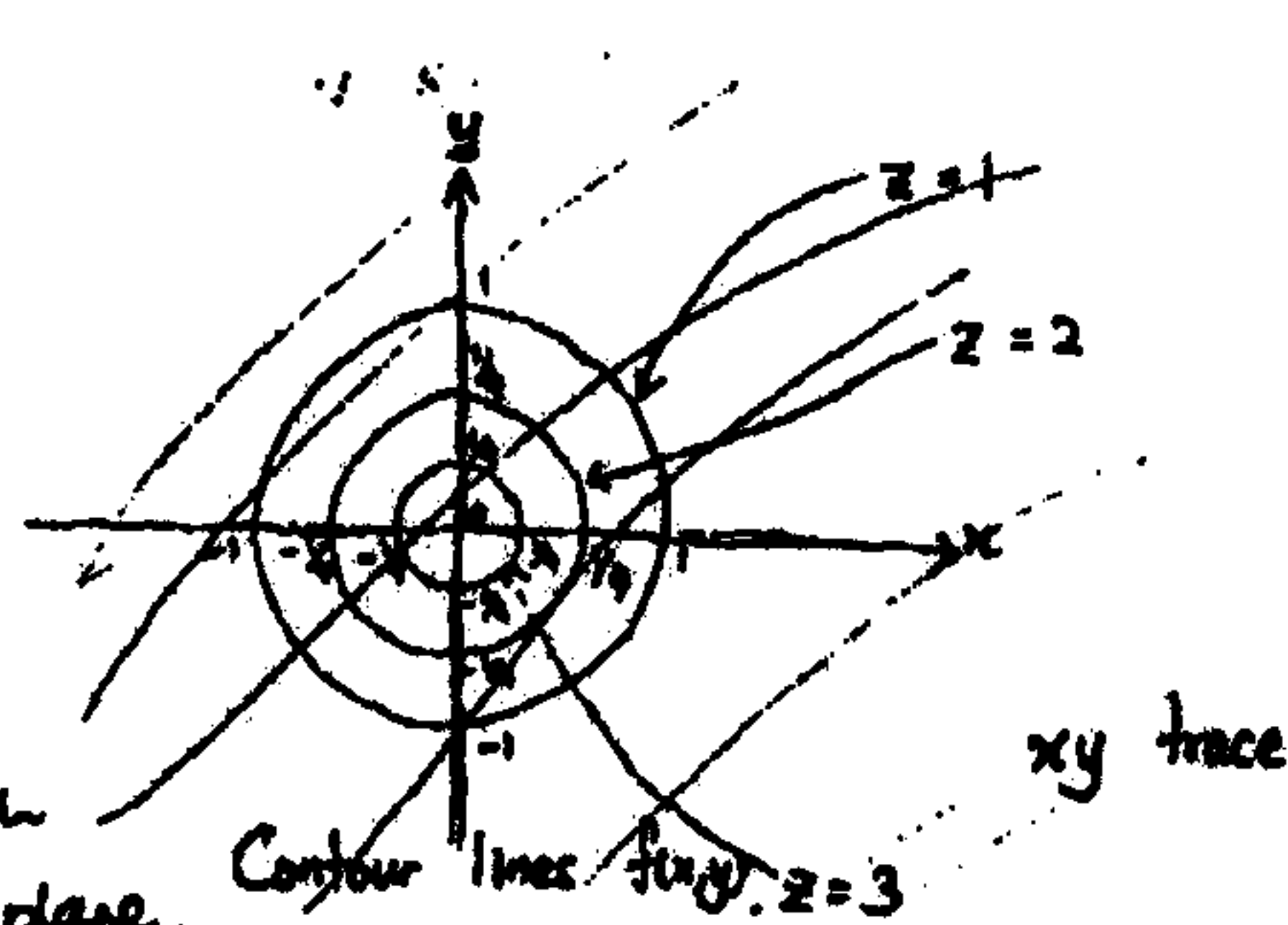
$$x^2+y^2=1$$

if $k=2, 2 = \frac{1}{\sqrt{x^2+y^2}}$

$$x^2+y^2=\frac{1}{4}$$

if $k=3, 3 = \frac{1}{\sqrt{x^2+y^2}}$

$$x^2+y^2=\frac{1}{9}$$



I had included Example 5 which was a question on generalising the ideas of the graph of three or more variables of a multivariable functions. Chong had answered the question correctly and had indicated that he had followed on from the arguments that were set in the question.

Example 5: Extending

As you know, the graph of a real-valued function of a single variable is a set in a 2-coordinate space. The graph of a real-valued function of a two independent real variables is a set in a 3-coordinate space. The graph of a real-valued function of three independent real variables is a set in 4-coordinate space.

• How can you define the graph of a real-valued function $f(x_1, x_2, x_3, x_4)$ of four independent real variables?

• How ~~ex~~^{would} you define the graph of a real-valued function $f(x_1, x_2, x_3, x_4, \dots, x_n)$ of n independent real variables?

(1) According the given information, number of domain have a link with number of range in each function number of range always exist one more than number of domain. Therefore, 4 independent real variables ~~with~~ is a set in a 5-coordinate space.

(2) The real-valued of a n independent real variable is a set in a $(n+1)$ -coordinate space.

In contrast, Fara worked with her group for example 1(c) and submitted a 'sanitised' version of her work. There were three people in her group. She had indicated very briefly what she did without any details. In answering Example 5, she had offered some explanations without actually following the argument that was given as an introduction to the question. I was not sure whether she did this with her group but in going through the students' work, I did not find any other that had similar explanations which made me concluded that she did Example 5 by herself.

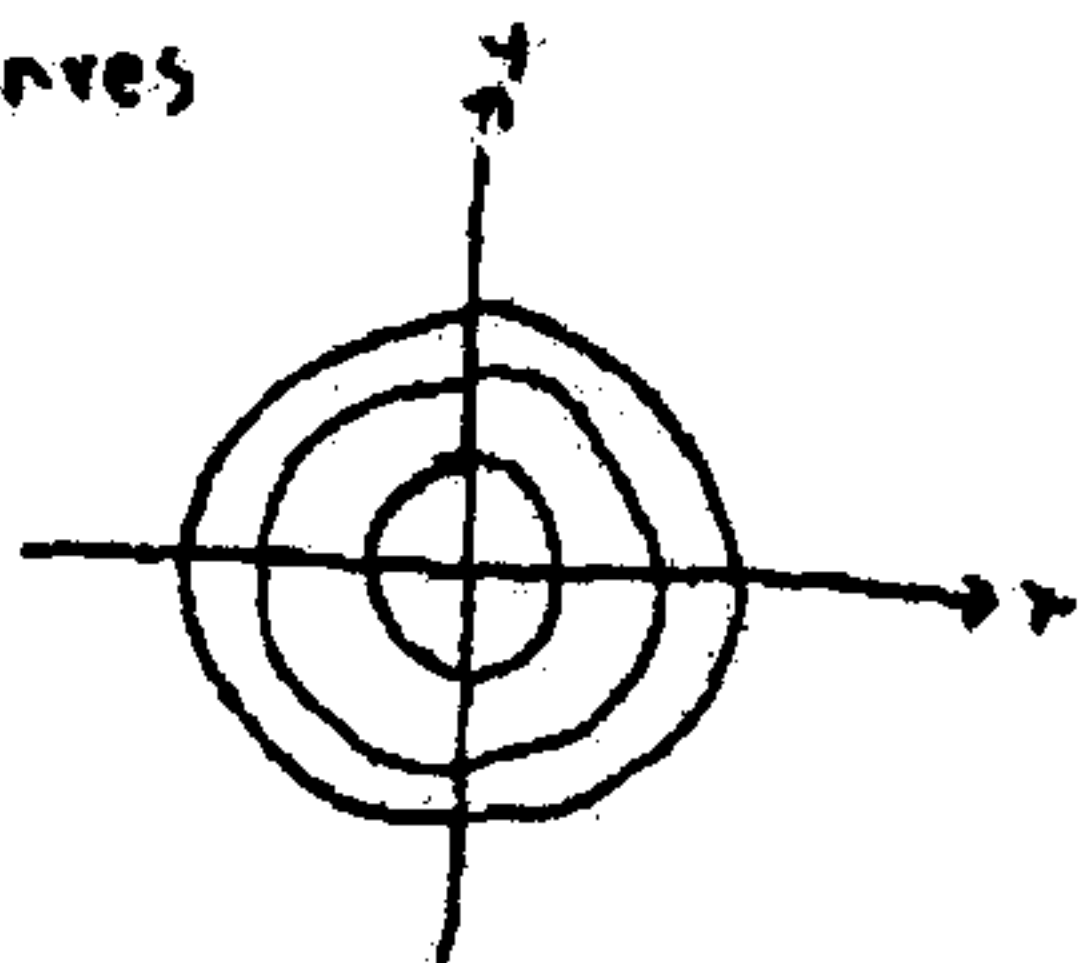
Fara's worksheet

$$f(x,y) = \frac{1}{\sqrt{x^2 + y^2}}$$

Domain : $x^2 + y^2 > 0$

Range : $z > 0 \rightarrow z = f(x,y) = \frac{1}{\sqrt{x^2 + y^2}}$

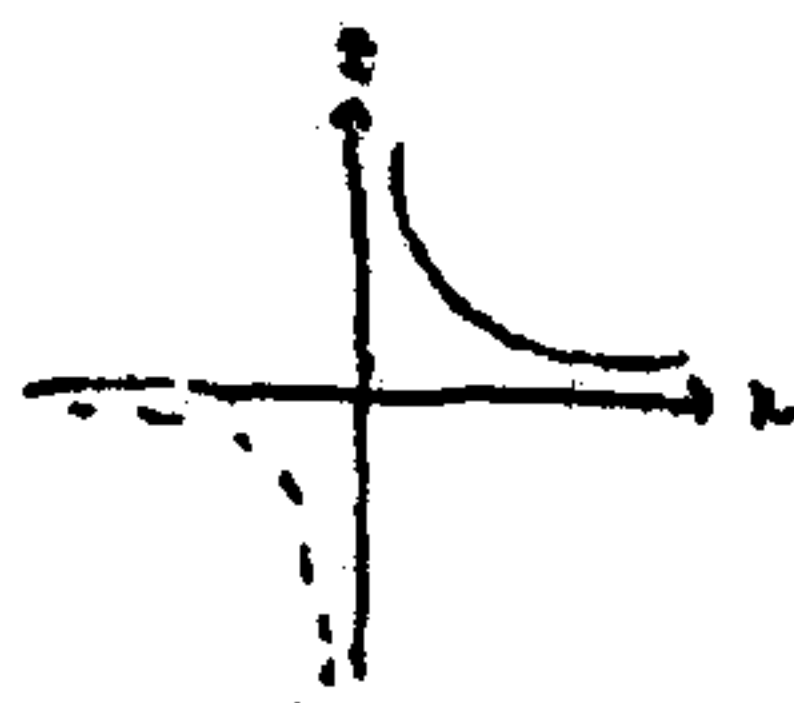
Level curves



xz -plane, $y=0$

$$z = \frac{1}{\sqrt{x^2}}$$

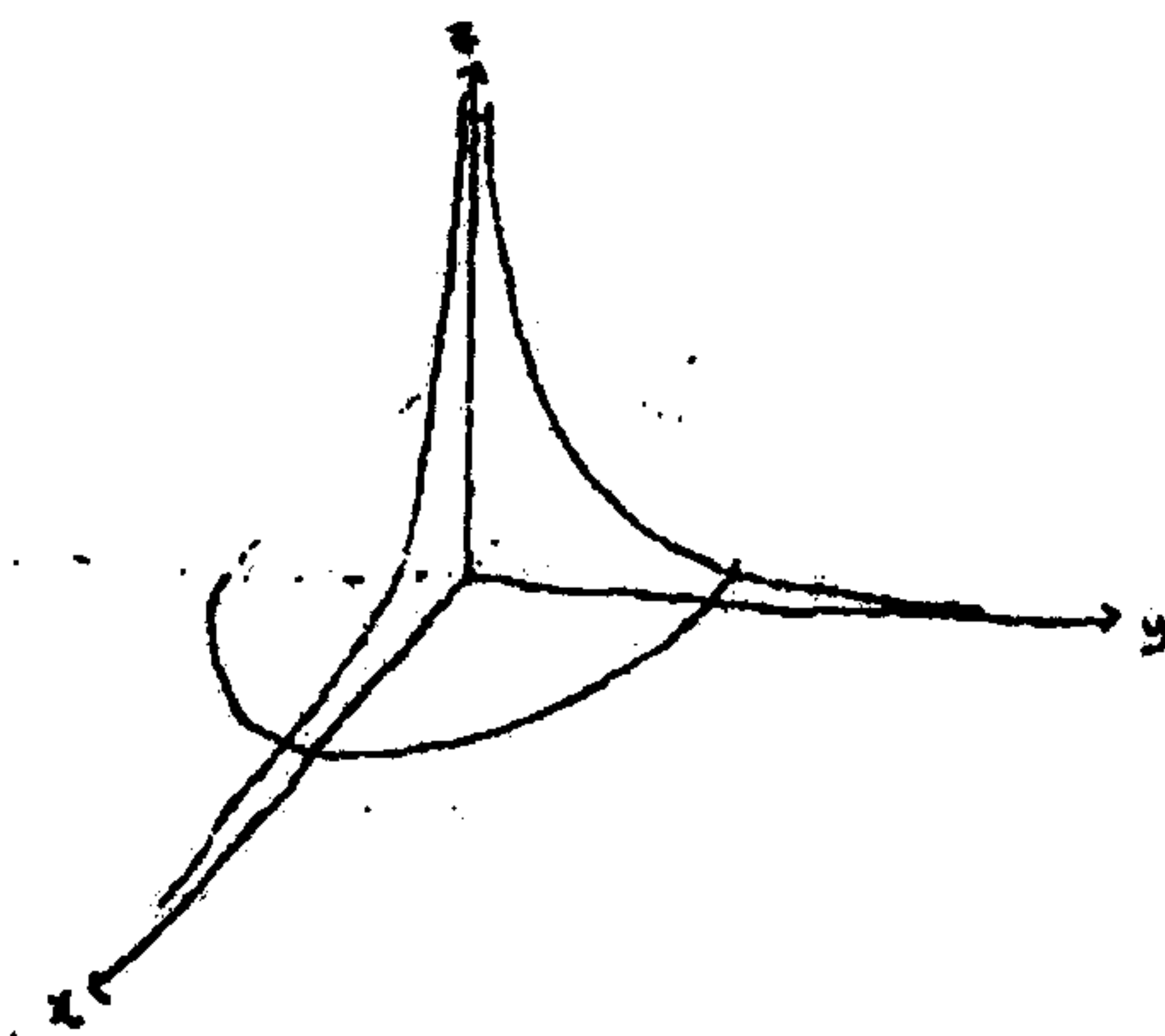
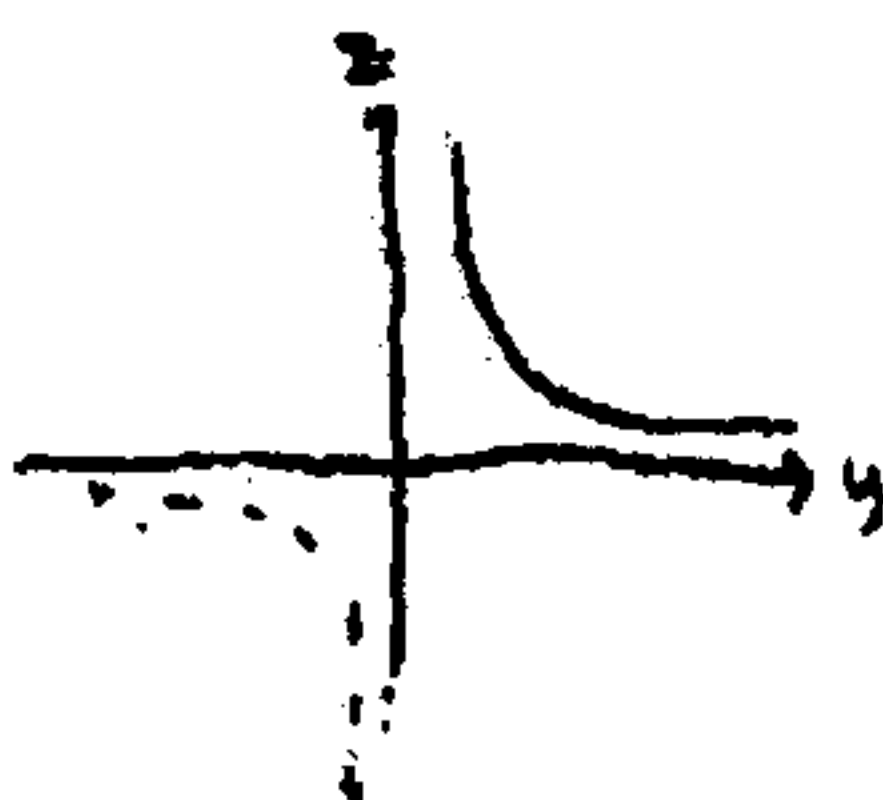
$$z = \frac{1}{|x|}$$



yz -plane, $x=0$

$$z = \frac{1}{\sqrt{y^2}}$$

$$z = \frac{1}{|y|}$$



How can you define the graph of a real-valued function $f(x_1, x_2, x_3, x_4)$ of four independent real variables?

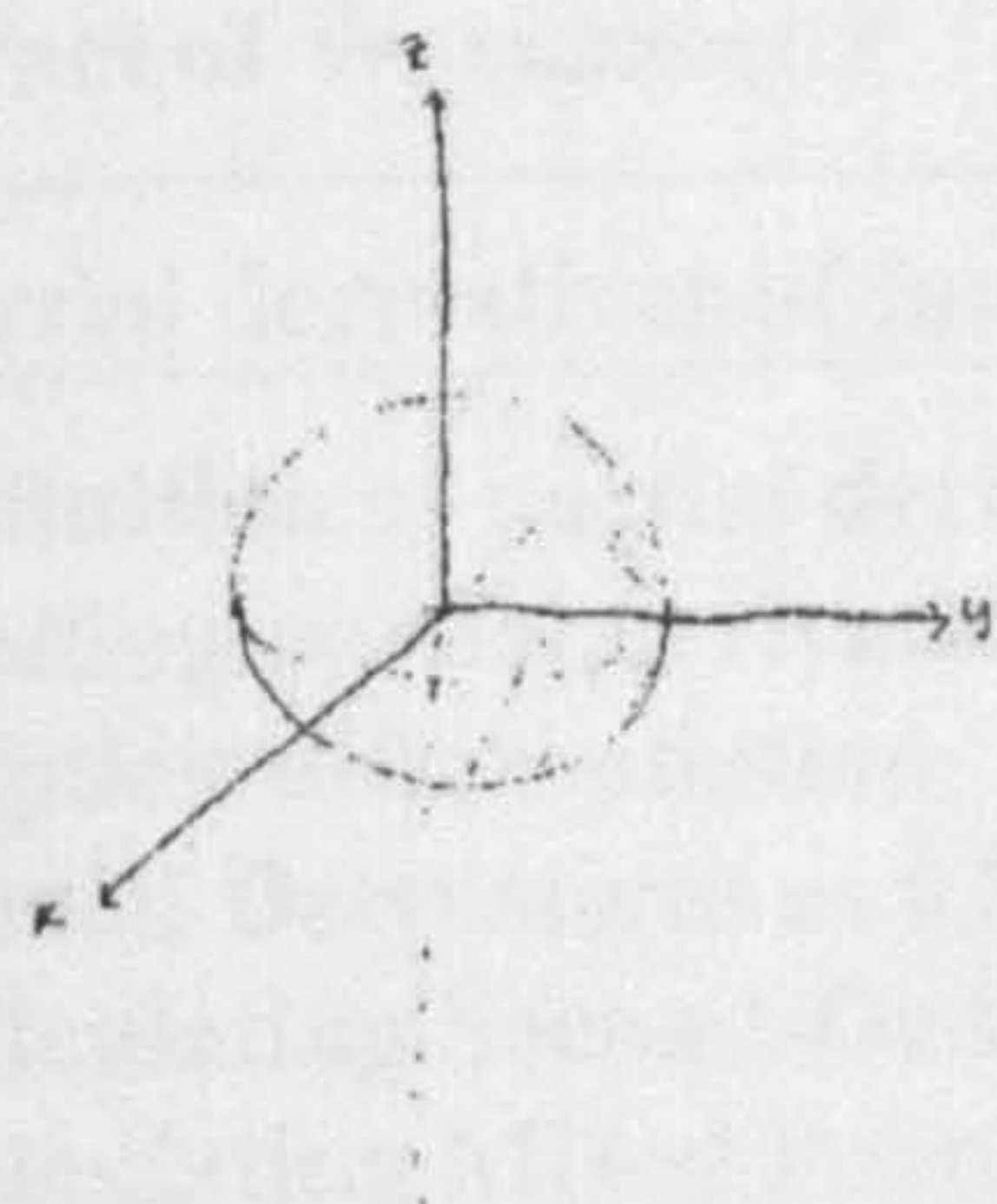
How would you define the graph of a real-valued function $f(x_1, x_2, x_3, x_4, \dots, x_n)$ of n independent real variables?

For a function of $f(x, y, z) = x^2 + y^2 + z^2 = w$

It is known that w cannot be zero. Many surfaces are sketched due to various value of w

Thus for $f(x_1, x_2, x_3, x_4) = c$, different 3-dimensional object with volume can be sketched at various value of c .

As an example for a function of $f(w, x, y, z) = w^2 + x^2 + y^2 + z^2 = c$



The graph of n independent real variables would be very complex.

These worksheets were collected early in the course, in the second week of the lecture sessions. In class, they appeared enthusiastic, answered the prompts and questions in whole class sessions or when I was interacting with them in their groups or individually. However, they did not submit their answers to the prompts and questions in the assignments although I requested that they should also submit them. These worksheets here were representative of the way students thought how their work should be submitted. Even Chong who thought that I would like to see the workings had changed his mind and decided to cancel out all the workings while making sure that they could still be read. They were still attuned to the old and familiar ways of working on mathematics. The right answers were important and only the important workings should be presented. What they

were thinking of or how they thought out a problem were considered as private domains and need not be exhibited.

2. Identifying what are the generalities in a mathematical technique

In Worksheet 3, students were asked to start off working on the special case of finding partial derivatives and a few prompts and questions were given to guide them to be more aware of the process of partial differentiation. Here, I will include two samples, from Haz and Wan. Figure 5.14 gives an extract of the question.

Figure 5.14: Extract of Worksheet 3

Worksheet 3: Partial derivatives of functions of two variables

- Definition of partial derivatives
- Finding partial derivatives
- Implicit differentiation
- Partial Derivatives as a Slope of a Tangent and Rate of Change
- Calculating Second-Order Partial Derivatives
- Calculating Mixed Partial Derivatives
- Increments and Differentials
- The Chain Rule
- Local and Absolute Extrema

Partial Differentiation and Definition of Partial Derivatives

The special case: Let $z = f(x, y) = \sqrt{4 - x^2 - y^2}$. Draw the graph of the function.

Imagine a plane $y = 1$, parallel to the xz – plane , intersecting the graph. Draw the trace of the curve at the intersection plane and determine its equation.

- What do you see in your picture? Describe your picture.
- If $y = 1$, what are the possible values of x ?
- How can you evaluate $z = f(x, y)$?
- Determine (at least) 3 values for z .
- On the plane $y = 1$, what can you say about the behaviour of z , x and y ?
- How would you find the derivative to the trace of the curve?

Wan had drawn the function, going through the same techniques that he had learnt in earlier worksheets. He identified the domain and range and had used traces on the different coordinate planes to draw the graph and in the second part; there were some attempts to answer the prompts and questions. I had collected the work spontaneously and many students had not filled in their worksheets fully. Wan had also attempted to identify the plane $y=1$ and the graph of the intersection but he did not offer an equation for intersecting trace.

Partial derivatives of functions of two variables

- Definition of partial derivatives
- Finding partial derivatives
- Implicit differentiation
- Partial Derivatives as a Slope of a Tangent and Rate of Change
- Calculating Second-Order Partial Derivatives
- Calculating Mixed Partial Derivatives
- Increments and Differentials
- The Chain Rule
- Local and Absolute Extrema

Partial Differentiation and Definition of Partial Derivatives

The special case: Let $z = f(x, y) = \sqrt{4 - x^2 - y^2}$. Draw the graph of the function.

Imagine a plane $y=1$, parallel to the xz -plane, intersecting the graph. Draw the trace of the curve at the intersection plane and determine its equation.

$$z = \sqrt{4 - x^2 - y^2}$$

$$xy\text{-plane } z=0$$

$$0 = \sqrt{4 - x^2 - y^2}$$

$$0 = 4 - x^2 - y^2$$

$$x^2 + y^2 = 4$$

$$x^2 + y^2 = 4$$

$$x^2 + y^2 = 4$$

$$x^2 + y^2 = 4$$

$$x^2 + y^2 = 4$$

$$x^2 + y^2 = 4$$

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$$x^2 + y^2 = 4$$

$$x^2 + y^2 = 4$$

$$\text{Domain: } 4 - x^2 - y^2 \geq 0$$

$$x^2 + y^2 \leq 4$$

$$\text{range: } 0 \leq z \leq 2$$

$$z = \sqrt{4 - x^2 - y^2}$$

$$xz\text{-plane } y=0$$

$$z^2 + x^2 = 4$$

$$z^2 + x^2 = 4$$

$$z^2 + x^2 = 4$$

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$$z^2 + x^2 = 4$$

$$z^2 + x^2 = 4$$

- What do you see in your picture? Describe your picture. *in my picture*
- If $y = 1$, what are the possible values of x ?
- How can you evaluate $z = f(x, y)$?
- Determine (at least) 3 values for z . *1, 2, 3*
- On the plane $y = 1$, what can you say about the behaviour of z , x and y ?
- How would you find the derivative to the trace of the curve?

$$\begin{aligned}
 z^2 &= 4 - 1 - x^2 \\
 z^2 &= x^2 + (\sqrt{3})^2 \\
 \textcircled{2} \quad x &= \pm \sqrt{3}
 \end{aligned}$$

Haz's work was more thorough but he did not need to go through the whole exercise to find the derivatives. He already knew how to find the partial derivatives. The students had actually learnt the techniques of partial differentiation in their Calculus I class. The topic was taught by focusing only on the techniques without an introduction to multivariable functions or any reference to the graphs. In his work, Haz had written out the equation of the trace equation but had also differentiated the function. He had attempted to answer the prompts and questions but very sketchily.

Partial derivatives of functions of two variables

- Definition of partial derivatives
- Finding partial derivatives
- Implicit differentiation
- Partial Derivatives as a Slope of a Tangent and Rate of Change
- Calculating Second-Order Partial Derivatives
- Calculating Mixed Partial Derivatives
- Increments and Differentials
- The Chain Rule
- Local and Absolute Extrema

Partial Differentiation and Definition of Partial Derivatives

$$\text{Domain} = x^2 + y^2 \leq 4$$

$$\text{Range} = z \geq 0$$

The special case: Let $z = f(x, y) = \sqrt{4 - x^2 - y^2}$. Draw the graph of the function.

Imagine a plane $y = 1$, parallel to the xz -plane, intersecting the graph. Draw the trace of the curve at the intersection plane and determine its equation.

$$z^2 = 4 - x^2 - y^2$$

xy trace, $z = 0$
 $x^2 + y^2 = 4$

xz trace, $y = 0$
 $z^2 = 4 - x^2$

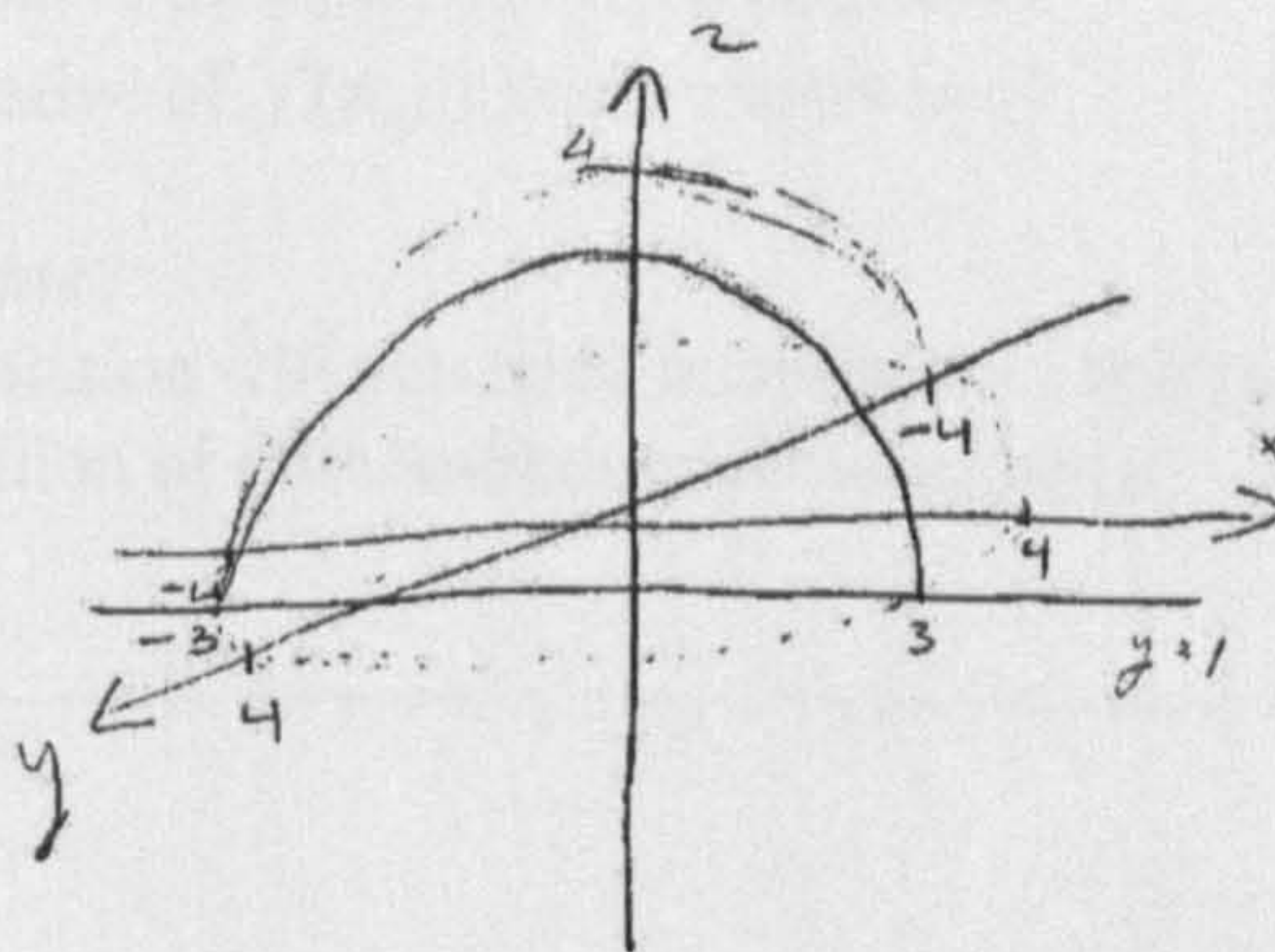
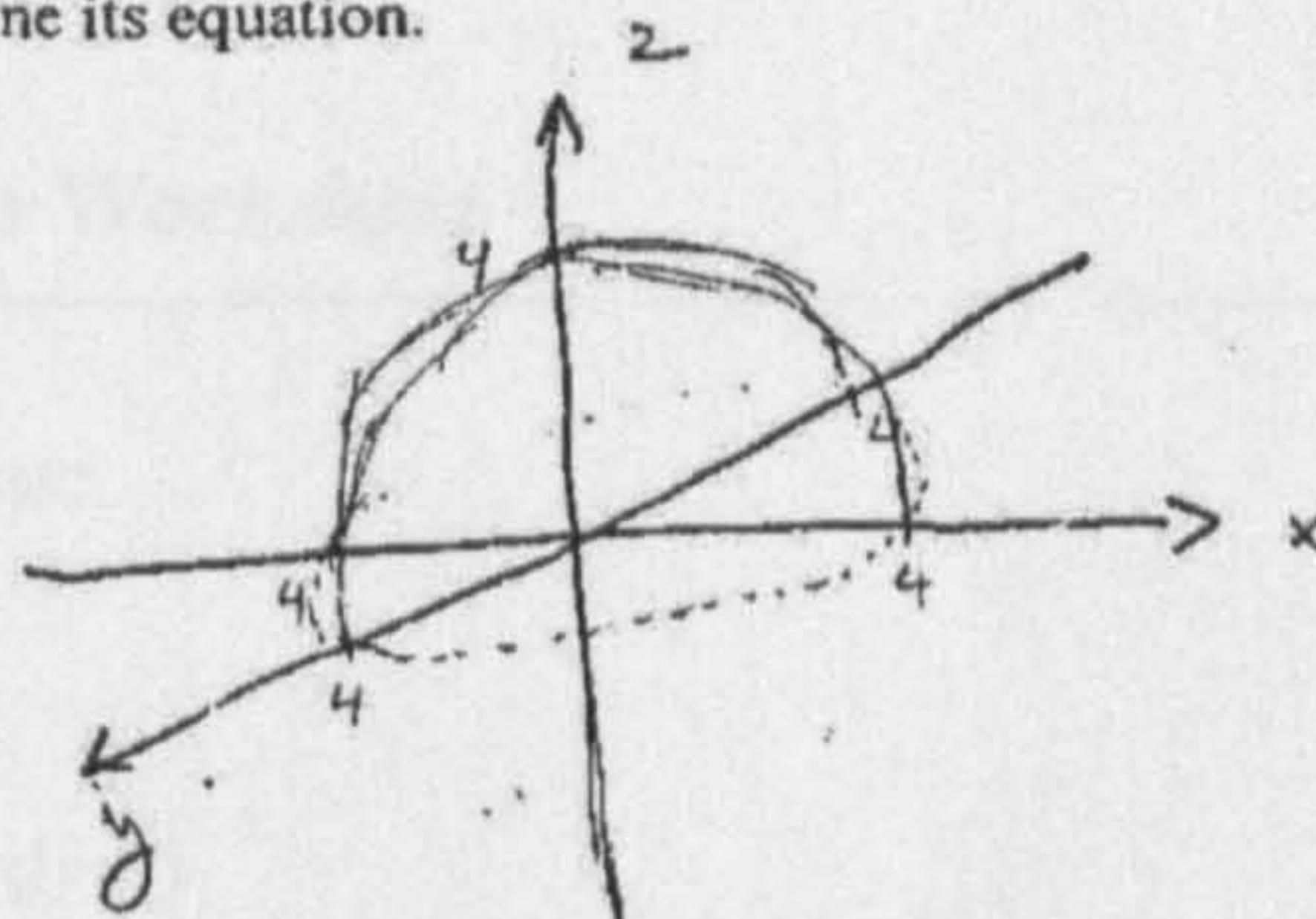
yz trace, $x = 0$
 $z^2 = 4 - y^2$

at xz trace, $y = 1$
 $z^2 = 4 - x^2 - 1$
 $z^2 = 3 - x^2$

$$z^2 = 4 - x^2 - y^2$$

$$z^2 = 3 - x^2 \quad y = 1$$

$$2z \left(\frac{dz}{dx} \right) + 2x + 2y \frac{dy}{dx} = 0$$



- What do you see in your picture? Describe your picture.
- If $y = 1$, what are the possible values of x ? $-3 \leq x \leq 3$
- How can you evaluate $z = f(x, y)$?
- Determine (at least) 3 values for z . $-3, 0, 3$
- On the plane $y = 1$, what can you say about the behaviour of z , x and y ?
- How would you find the derivative to the trace of the curve?

The students were asked to hand in an assignment from Worksheet 4 (see Figure 5.15).

Two questions were given to test students' understanding of the definition of integration and differentiation as well as introducing non-routine questions for them to work on. They could hand in the assignment as group work or individually.

Figure 5.15: Extract of Assignment 2: from Worksheet 4

Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ from the following functions:

(9) $f(x, y) = \int_x^y g(t) dt$ (g continuous for all t)

- What does it mean to find the partial derivative of $f(x, y)$ with respect to x ?
- What does it mean to find the partial derivative of $f(x, y)$ with respect to y ?
 - Which variable is changing?
 - Which variable is considered constant?
- What other rules or techniques of differentiation will you need to answer the following?
- What is the relationship between the definition of differentiation and integration?

(10) $f(x, y) = \sum_{n=0}^{\infty} (xy)^n$ ($|xy| < 1$)

- Try to answer for specific values of n , say, $n = 1, 2, 3, \dots$
- Can you see any pattern in your answer?
- What happens in general?
- What happens when $n \rightarrow \infty$?

$$9. f(x, y) = \int_x^y g(t) dt$$

SEIKS YEN 17

$$\begin{aligned} \frac{\partial f}{\partial x} &= \frac{\partial}{\partial x} (g(y) - g(x)) \\ &= g(y) \frac{dy}{dx} - g'(x) \end{aligned}$$

$$\begin{aligned} \frac{\partial f}{\partial y} &= \frac{\partial}{\partial y} (g(y) - g(x)) \\ &= g'(y) - g(x) \frac{dx}{dy} \end{aligned}$$

$$10. f(n, y) = \sum_{n=0}^y (ny)^n$$

$$\frac{\partial f}{\partial n} = n n^{n-1} y^n \quad \text{and} \quad \frac{\partial f}{\partial y} = n y^{n-1} n^n$$

when $n=1$

$$\frac{\partial f}{\partial n} = y \quad \text{and} \quad \frac{\partial f}{\partial y} = n$$

when $n=2$

$$\frac{\partial f}{\partial n} = 2ny^2$$

$$\frac{\partial f}{\partial y} = 2yn^2$$

there for:

There were four students in Haz's group. I could see that there was some confusion of the meaning of differentiation and its relationship to integration as well as in the notation to represent the concepts.

$$\sum_{n=0}^2 (ny^n) = (ny)^0 + (ny)^1 + (ny)^2$$

$$\sum_{n=0}^{\infty} (ny^n) = (ny)^0 + (ny)^1 + (ny)^2 + \dots \dots (ny)^n$$

$$= 1 + ny + (ny)^2 + (ny)^n$$

In answering the second question, they had found the general partial derivatives but were not able to expand the 'sigma' notation correctly in their answer. They could do the 'general' and managed to identify a few particular cases, when $n = 1$ and $n = 2$, but could not put the ideas together to present the answer.

Wan worked in a group of four and presented the following worksheets. His group was also confused about the First Fundamental Theorem of Calculus and its symbolic representation. They showed a tendency to move straight into the problem and liked to present the answers as quickly as possible. For the second problem, their way of working indicated that they might have followed the prompts and questions provided but they did not manage to follow through until they could identify the general answer. Unsurprisingly, the students were quite upset with the assignment as they did not manage to score high marks for their efforts. However, I managed to calm them by allowing them to redeem any low scoring assignments by doing other assignments and choosing only the best marks. This could be seen in the assignments marks achieved by the students which were in the range 9.5 to 10 for all the interviewees. This performance was also typical of all the other students.

$$9 \quad f(x, y) = \int g(t) \cdot dt$$

$$f(x, y) = g(y) + c - [g(x) + c]$$

$$f(x, y) = g(y) - g(x)$$

$$\frac{\partial f}{\partial x} = -dg(x)$$

$$\frac{\partial f}{\partial y} = dg(y)$$

$$10 \quad f(x, y) = \sum_{n=0}^{\infty} (xy)^n \quad (|xy| < 1)$$

$$= (xy)^0 + (xy)^1 + (xy)^2 + (xy)^3 + \dots$$

$$f(x, y) = (xy)^1$$

$$\frac{\partial f}{\partial x} = y$$

$$\frac{\partial f}{\partial y} = x$$

$$f(x, y) = x^2 y^2$$

$$\frac{\partial f}{\partial x} = y^2 \cdot 2x$$

$$\frac{\partial f}{\partial y} = x^2 (2y)$$

$$f(x, y) = x^3 y^3$$

$$\frac{\partial f}{\partial x} = y^3 (3x^2)$$

$$\frac{\partial f}{\partial y} = x^3 (3y^2)$$

Yes we can see many pattern

	$f(x,y) = (xy)^{-2}$
	we know that $(xy)^{-2} \neq 0$
	so,
	$f(x,y) \neq 0$
	if

3. Introduction to problem solving

The students’ exposure to problem solving was mainly in solving routine problems to support conceptual understanding consolidation. In this section, I will report on an episode where students were asked to answer an example from a previous Examination paper. This exercise was well received by the students and their participation and queries were enthusiastic. This was in the later part of the course. This particular episode occurred on 26/09/05 in the class of 2 SEM.

Figure 5.16 A sample of an examination question for triple integration in spherical coordinates.

2 (c)

Use spherical coordinates to evaluate the integral

$$\iiint_G z \, dV$$

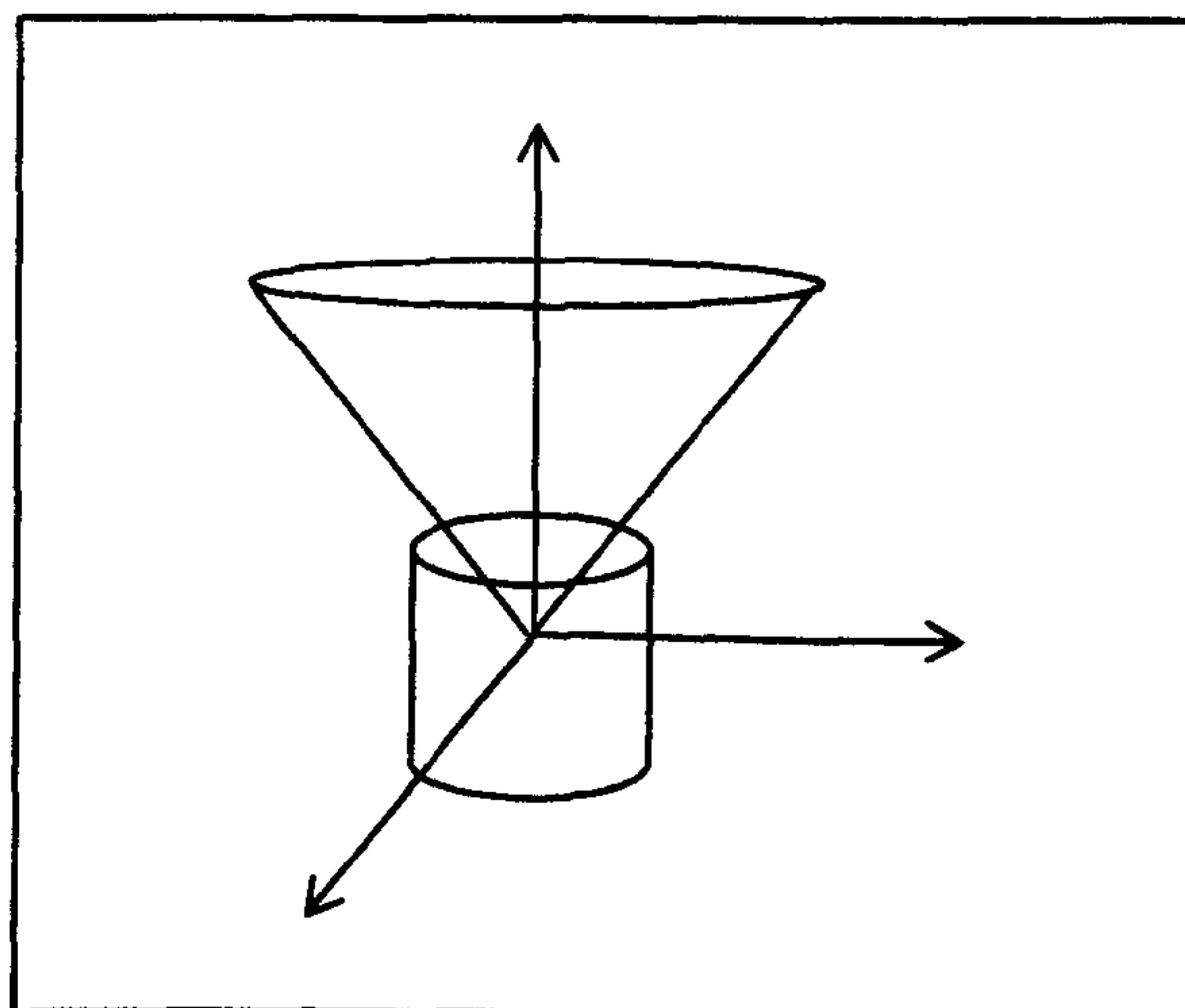
over the solid G bounded below by the cylinder $x^2 + y^2 = 1$ and above by the cone $z = \sqrt{x^2 + y^2}$.

[6 marks]

I had been using a past year examination paper and choosing relevant questions as part of the class exercise especially at the end of each presentation of the corresponding topic. In this class, we had just finished the topic of Triple Integrals. This question was on evaluating a triple integral using spherical coordinates. The students were asked to work in their groups and I walked around the class to monitor their progress. A particular group called me over. There were one girl and two boys in the group. They were having problem with drawing the solid of integration. They had a diagram drawn but could not determine the limits of integration.

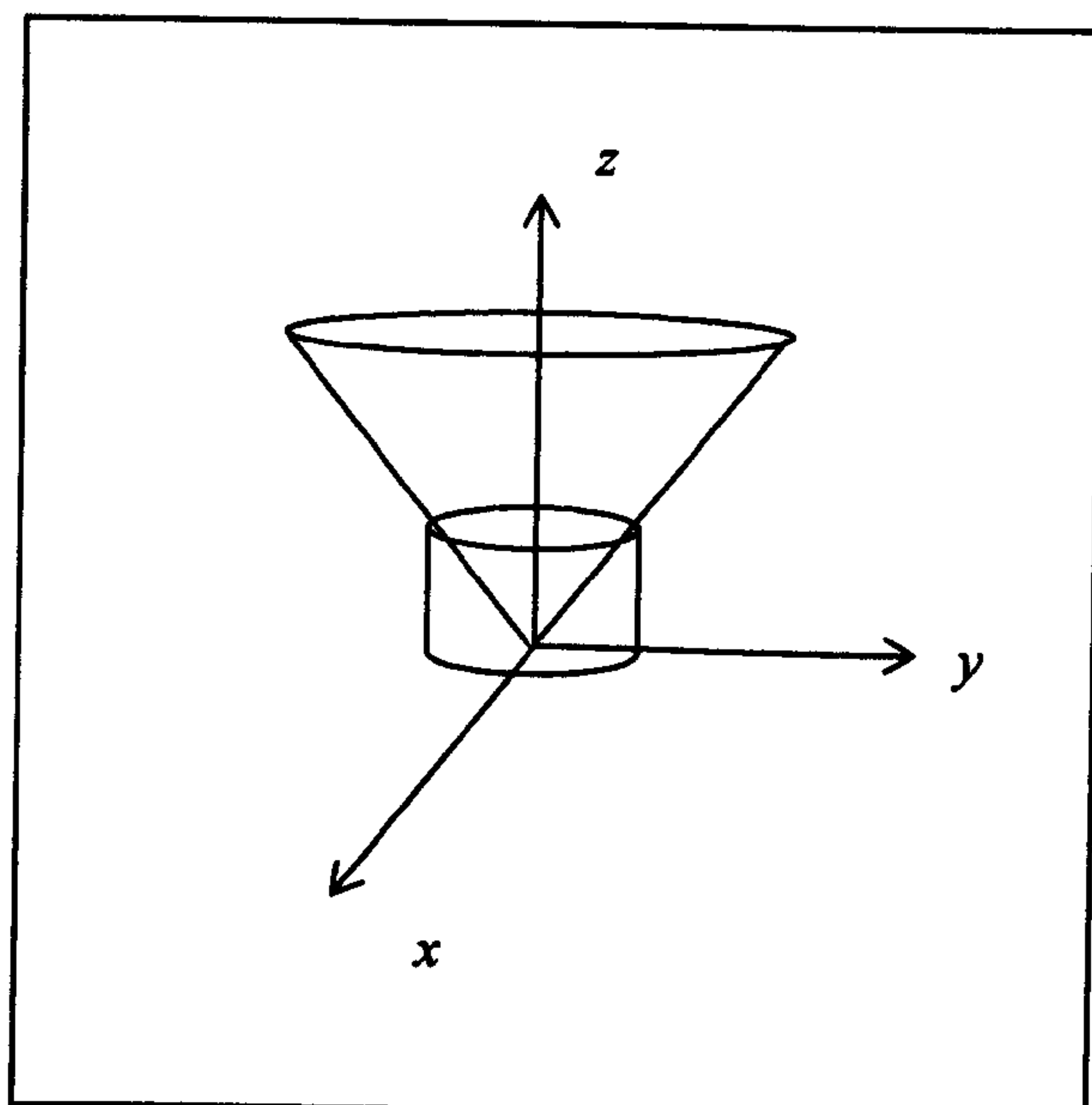
Excerpts from class notes, 26/09/05, 2 SEM:

Episode 1: A group calls me over. I can't remember their names except for one of the boys, Shidi. I asked the others for their names, Hani and Wang. They've drawn the graph of the solid and the picture looks like this:



Shidi asks, 'which part of the diagram is the region referred to?' and Wang also asks, 'how to identify the limits?'

I said, 'have you checked your graph against the information in the question?' They began reassessing their graph. I think they were taking the cue from my question and realised that the graph was not right. They managed to draw the right graph.



I said, 'to find the limits, you have to remember how you measure ρ and ϕ in spherical coordinates?'

They started looking through their notes. Shidi said, ' ρ is from origin to cylinder, $x^2 + y^2 = 1$ '. He adds, 'is $\rho = 1$?

RAR (me): 'why do you think it's 1?'

Shidi: 'cos $x^2 + y^2 = 1$ '

RAR: 'what does $\rho = 1$ means in terms of spherical coords?'

Wang: 'a sphere of radius 1.'

RAR: 'but you have a cylinder, how do you determine ρ ?'

Shidi indicated ρ on the diagram, entry at origin, exit at the side of the cylinder.

RAR: 'what are x and y in terms of spherical cords?'

They started writing down the definition:

$$\begin{aligned} x &= r \cos \theta & y &= r \sin \theta \\ x &= \rho \sin \phi \cos \theta & y &= \rho \sin \phi \sin \theta \end{aligned}$$

$$x^2 + y^2 = 1 \Rightarrow \rho^2 \sin^2 \phi \cos^2 \theta + \rho^2 \sin^2 \phi \sin^2 \theta = 1$$

$$\rho^2 \sin^2 \phi = 1$$

$$\rho^2 = \csc^2 \phi$$

$$\rho = \csc \phi$$

They appeared more at ease when doing the mathematics and are more confident. They started working out the value of ϕ .

$$\tan \phi = 1 \Rightarrow \phi = \frac{\pi}{4}.$$

I moved on as it appeared that they can finish the question easily now.

Episode 2: *A group of five girls called me over. I can't remember all their names. They were working on a different problem. It was a problem that they had taken for a different past year paper. The question is reproduced here.*

2 (b) Evaluate $\iiint_G (6 + 4y) dV$ where G is the solid in the first octant bounded above by

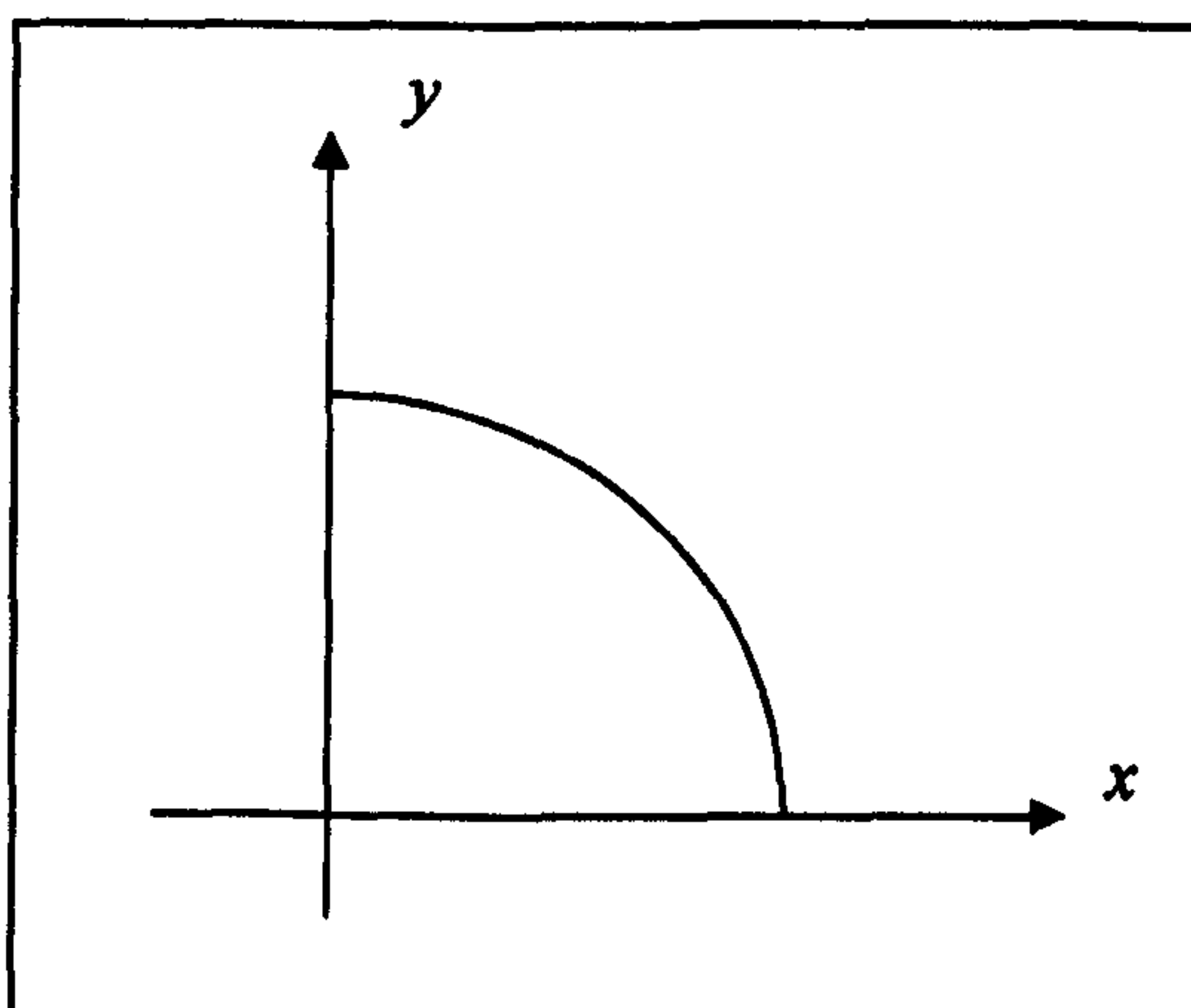
the cone $z = \sqrt{x^2 + y^2}$, below by the xy -plane and on the side by the cylinder $x^2 + y^2 = 1$.

I looked at the question and the work that has been done. I can see that they had the solid drawn correctly and had identified G .

Girl 1: 'how do I know the limits of y and x ?'

RAR: 'from the diagram, identify R , the region of integration.'

Girl 1: 'here's my R .' shows me the picture of R .



She also shows me the integral as it was written out in her notebook:

$$\iiint_0^{\sqrt{x^2+y^2}} (6+4y) dz dy dx.$$

She has identified the z-limits correctly.

RAR: 'how do you determine the y limits?'

They said that they were not sure. I drew a line parallel to the y-axis. 'Can you read the limits?'

Girl 2: Yes, $y = 0$ to $y = \sqrt{1-x^2}$.

RAR: 'what are the x-limits?'

Girl 3: ' $x = 0$ to $x = \sqrt{1-y^2}$ ', reading from a line parallel to the x-axis.

RAR: 'no, $x = 0$ to $x = 1$ '

Girls 2 and 3: 'why is it not the way we read it?'

I gave an explanation and then ask them to set up $\iiint (6+4y) dz dx dy$, using the same solid. The other two girls were listening but did not contribute to the discussion.

For the topic of triple integration, the students had difficulty in drawing the graphs, choosing the order of integration and setting up the limits of integration. They used worked examples as their reference and usually tried to look for the connections from examples to procedures that they should memorise. In the words of Midah, 'give a question, show the method, the formula. This question, this is the technique. This will help me remember the technique'. At this period in the course, the students were discussing more, using some prompts and questions, they asked questions more with each other, with me and some were quite keen to know and understand the techniques that were used rather than just finishing the questions.

5.3.5 Results of the Teaching and Learning Evaluation

An evaluation questionnaire was given to the students at the end of the semester. The questionnaire was adapted from Brookfield (1995) in which students answered a six part questionnaire. The questionnaire is given in Appendix C. The different parts were divided as the following.

- (A) consisted of 6 questions for students to evaluate the lecturer's command of the subject and her effectiveness as a teacher.**
- (B) students were asked to describe how the course has affected their learning in 6 items such as, knowledge of content of the discipline, mathematical thinking, critical thinking, writing, speaking, team work and 1 final item for students to make other comments.**
- (C) several statements for students to complete that involve various aspects of their learning.**
- (D) several statements about the teaching and the lecturer that students must rate.**
- (E) two statements that asked students to identify their most exciting and boring moments during the course.**
- (F) a section where students could put down their suggestions to improve the teaching and learning.**

The questionnaire was given in the last class session and many students did not turn up. I had 35 respondents out of a total of 64 students from the 2 SEM class and 24 respondents out of a total of 43 students from 2 SAW. In this section, I will only briefly present the results of the evaluation exercise.

Table 5.9: Summary of students' responses

Students' Response	Class	Number of students	Percentage
Very strong to positive responses	2 SEM	23	
	2 SAW	22	
	Total	47	43.9%
Average to negative responses	2 SEM	12	
	2 SAW	2	
	Total	14	13.1%

Out of the 6 sections, section B had the least number of responses. There were 22 who gave some responses for the section. 20 respondents were very positive which implied that the students who felt that the course had helped them were motivated to share their thoughts. The other 2 said that the course only supported average growth in them in the items given. With only about 58% respondents, the only conclusion that I could make it's that some students liked the course but they were others who did not and the rest whose opinions were not known.

5.3.6 Conclusion

There were six topics in this course and I had I had only chosen Multi-variable Functions and Partial Differentiation as topics where the teaching strategies and worksheets with the prompts and questions specially prepared. For the other topics, I had used similar strategies but the prompts and questions were displayed on the lecture notes or in my explanations but the students did not have them explicitly stated in the tutorial sheets. I had noticed that the students' needed the prompts and questions to be explicit and visible before they used it. They also needed to see that these strategies were integral to the course as then they will engage with them. In general, the students only considered actions that would contribute to achieving good results in their course.

5.4 Teaching of G07 and H07 – First Semester, 2007/2008

The idea of using a workbook has been discussed in previous sections (See Sections 4.9 (3) and 5.2 (7)). The basic idea for using the workbook was to create an environment where

students can engage in doing the mathematical tasks on an individual as well as communal level consistently throughout the course. A draft copy was ready for students' use in the second semester 2006/2007. However, I was no longer teaching in Johor Bahru then, but my colleagues, Dr Tee and Dr Zee, conducted their classes using the same ideas. I was involved in the preparation of the workbook, its review and editing. I was also involved in occasional discussions that we held to discuss the students' development. We had collected feedback from the first group of students using the workbook, corrected and improved the book. The new version was used with the students in the first semester 2007/2008.

5.4.1 Research methods

In this last cycle of the implementation, my colleagues taught the subject and I observed some their teaching. I had scheduled 2 sessions of observations, early in the semester and one at the end of it, and the students' interviews were arranged to coincide with my visits. However, there was frequent sharing of information and discussions about the development of the classes and students. Being a complete observer has given me an opportunity to observe the teaching and learning situation with a more objective perspective. It also gave me an opportunity to evaluate the viability of the teaching strategies being used by other lecturers. Both my colleagues had prepared a questionnaire requesting comments on the teaching approach that this was administered to the students at the beginning and again at the end of the semester. The results of this questionnaire were shared with me. However, most of the data was collected mainly through students' self-reflection, classroom observations, students' written work, discussion and interviews.

5.4.1.1 Observations

I had observed two classes, one each of Dr Tee and Dr Zee. The first observation was conducted just before the semester break. This was slightly later than was planned but there were unavoidable circumstances as I was also teaching in Kuala Lumpur and arrangements had to be made so that I could stay in Johor Bahru for the observation

exercises. I stayed for a week and attended two of the lectures of each lecturer but the total number of hours was three as each had one two-hour session for their classes.

5.4.1.2 Group Discussion and Interviews

In addition to observing the classes, I had also decided to talk to the students. The form of communication chosen was group discussion, individual conversations as well as interviews with volunteers. Due to the circumstances during the lectures, I had group and individual discussions with the students only during the first visit. In the second visit, I had arranged for interviews with students who volunteered.

5.4.1.3 Students' work

Additional data was collected in the form of students' work which were copies of their working in their workbooks. In particular, copies of the students' attempts in solving problems and their reflections were taken. I had also given students some problems for them to solve during the interviews as I wanted to observe their spontaneous attempts at problem solving. Copies of these attempts were also collected.

5.4.1.4 Questionnaires

A questionnaire to elicit students' views and comments on the teaching and learning of Engineering Mathematics and the use of the workbook was developed by my colleagues and given out to their students at the beginning and the end of the semester. The results was shared with me and will be presented in Section 5.4.6.4 .

5.4.2 Teaching Strategies

My colleagues carried out teaching strategies that we had agreed on and conformed to those that have been discussed in Chapter 4.

5.4.3 Use of the Workbook

As mentioned earlier, the workbook was prepared together with two colleagues; Dr. Tee and Dr. Zee. My contribution was mainly for the first three chapters, Multivariable

Functions, Partial Differentiation and Multiple Integrals. The last two chapters, Vector Valued Functions and Vector Calculus were prepared by my colleagues and my role was mainly as an evaluator and editor.

The first draft was used in the second semester of 2006/2007 but I was not able to observe the classes taught because I had moved to Kuala Lumpur. We were working on the book as the semester progressed. The first draft was printed in individual booklets, chapters 1 to 5. In this semester, we had the students' comments from the first draft to help edit and upgrade the second version. We managed to publish the workbook through a publisher's fast print programme to provide modules or text "as it is" without being edited. This meant that the workbook looked like a proper textbook.

To ensure that the other aims of learning were given due consideration, the workbook had five distinctive features. The five features were different sections in the book such as *Illustrations, Structured Examples, Reflections, Review Exercises* and *Further Exercises*. Under *Illustrations*, examples with complete solutions and explanations were given to draw students' attention to misconceptions and to introduce some applications. In the section of *Structured Examples*, the problems were adapted and modified as well as structured in a manner to build concepts and ideas as well as revisit important issues highlighted in *Illustrations*. In both these sections, the prompts and questions were explicitly stated so as to make them more visible to the students and increase their awareness of the prompts and questions that could be used. Students were expected to write out answers that would be connected to their understanding of important concepts and mathematical techniques under the section of *Reflections*. The *Review Exercises* provided students with more examples to work on meanwhile *Further Exercises* provided harder problems to solve.

5.4.4 Observations

The first class that I observed was Dr Tee's, H07, 2 SEE. She introduced me to the students and explained the reason for my presence. I noticed that the students were already seated in their working groups when we arrived in the class. Dr. Tee has explained that she usually conducted her class in that manner. Her students were seated in groups throughout as her lectures alternated with group work. However, in this particular week, students were reviewing topics as a test was going to be held in the next week. They all had their workbooks and were working away on the various topics to be examined in the test. I had a walk about and sat with several groups, listening in to their discussion and also conducted a group interview if the students were willing. Some excerpts will be given in the next section. In her second class, she had started off teaching a topic on the Applications of the triple integral for about 20 minutes and then asked students to work on the relevant questions in the workbook. The topics had been structured and presented in the manner that we had prepared.

In contrast, Dr. Zee's first class (G07, 2 SEM) was seated as in a typical classroom. The furniture was not appropriate for group seating. She had informed me earlier that she had to conduct the class as lectures as she needed to catch up on the topics. However, there was time allocated for working on the exercises as well as for students to put forward any other queries. She also introduced me to the class with similar explanations about the reasons for my presence. I was seated at the back listening to her lectures and only went to talk to some of the students during the class tasks time. Both group of students appeared unperturbed by my presence. In the second class, Dr. Zee had prepared some samples of examination questions for the students to work on and I had more time to talk to her students.

As I had arrived in Week 5, I could see that the students were accustomed to the pace and the way the class was conducted. The lectures were interspersed with mathematical tasks

during which the lecturers function as facilitators. The classes were lively, students were occupied in their discussions and occasionally, a member of a group might ask for help from the lecturer. The lecturers were relaxed and moving around to monitor the progress of the different groups. Lectures, lessons review and working out selected examples were addressed to the whole class.

I also went to sit with a few groups during their activity time and carried out discussions with the whole group. Individual conversations were carried in sessions out of class so as not to disrupt their lessons. The students appeared at ease with my presence and seemed quite comfortable in sharing their views and comments. My colleagues had informed the students quite early in the course that I would be visiting and this helped in fostering students' acceptance of my presence and my questioning.

The second visit was carried out in Week 9. I carried out similar activities, observing the classes, listening to students' discussion and conducting interviews away from class with volunteers. The students were studying the last few sub-topics in Triple Integrals for the lectures but they were doing questions on earlier sub-topics in their tutorials.

5.4.5 Group discussions and Interviews

First visit

As mentioned above, I had spent most of my time in the class by sitting in selected groups during the class activity session, listening in to students as they do their work and participating in their discussion or initiating a discussion about the mathematics. I had also collected students' views on the workbook during conversations held at sessions away from class. Here, I will first report the group discussions held with two groups as I tried to explore students' way of working with the mathematics. Both groups were from Dr Tee's class.

Episode 1: I sat with a group of 5 students, 2 girls and 3 boys. I had asked if I could discuss with them their views about the course and the workbook. The conversations

centred on how they used the workbook. I had noted down their comments which I will present together with the views of other students. Then, I asked if they could work on a problem for me as a group.

RAR: what did you do to find the domain and range of functions?

Their responses were similar, 'look at the properties of functions and if there are restrictions then determine values of x that defines the function or make it real.'

RAR: could you find the domain of $z = \ln y$ and sketch the graph of the function?

They discussed the problem.

Boy 1: 'log something always positive, so y is greater than zero, the domain is y greater than 0'

RAR: what about the values of x ?

Girl 1: 'infinity'

Girl 2: 'any x '

Boy 2: 'all along the x -axis'

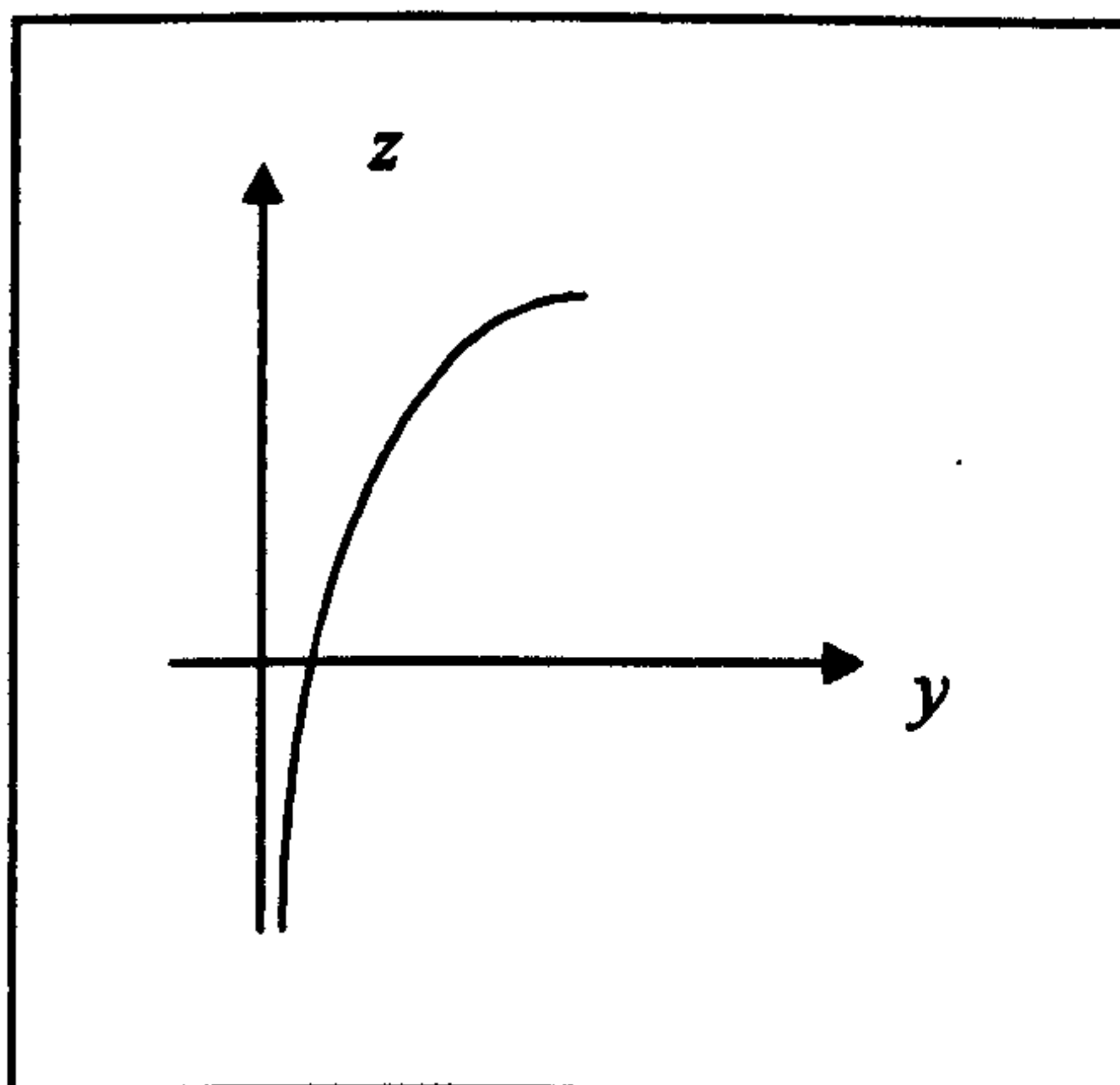
Boy 1: 'all x '

Boy 3: ' $x = 0$ ' and adds, 'how to find x , it's not there'

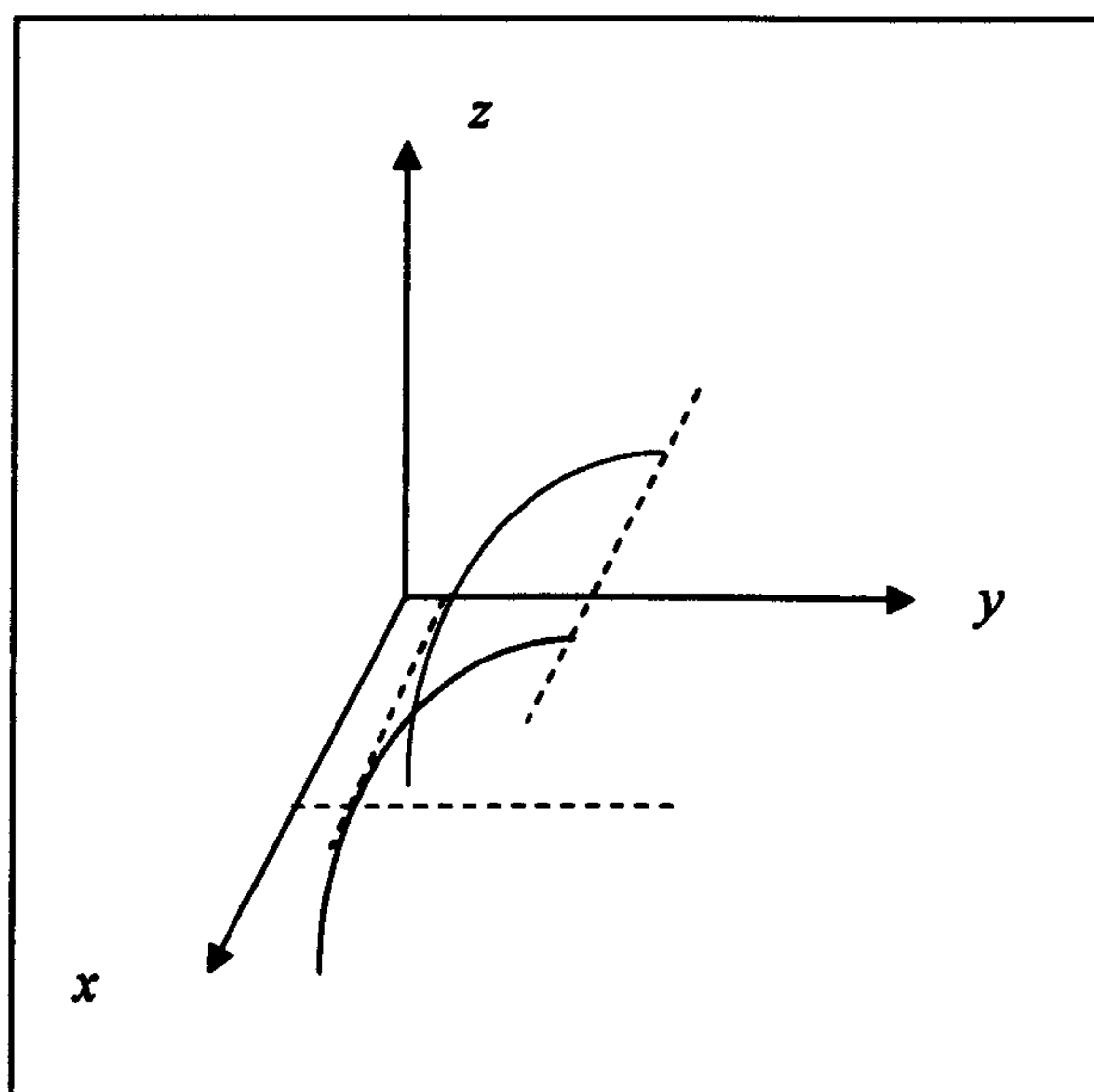
Boy 1 tried to explain to his friend about how to find the values of x .

RAR: 'what about the graph?'

Girl 1 drew a graph on her paper.



Girl 2: Adds to the picture and drew a 3-D graph.



Boy 2 said: 'like that, ah? Don't look right.'

There was laughter and more discussions about the graph of the function.

Episode 2: I sat with another group. There were three boys in the group. They said they had another member but he did not come to class on that day. Similarly, I collected views on the use of the workbook then I asked them to solve a problem.

RAR: 'Given $f(x, y) = x \sin x + ye^{xy}$, find f_y and f_x . Can you describe what you do to find the partial derivatives?' I wrote down the function on a piece of paper.

Boy 4: 'ah..., f_y ... y changing, x constant'

There was agreement but no one tried to solve the problem.

Boy 5: ' f_x , x changing and y constant'

RAR: 'do you need to use any differentiation rules to solve this?'

There was agreement that they would need to use 'uv formula' (product rule).

RAR: 'which part will need it if we want f_y ?'

Boy 4: 'both'

Boy 5: 'no, only ye^{xy} '

Boy 6: 'both also can as $\frac{dx}{dy}$ is zero' He adds, 'just do...use the chain rule, you know, uv'

My impression of the students was that they appeared at ease and relaxed working in their groups. They were willing to answer my questions, worked on problems that I gave and appeared more willing to try out solutions and were not concerned about what was the right answer. In both episodes, none of the students asked me if their answer was correct. I could not say if it was because they were more comfortable with problem solving or because there was no 'gain' in getting the right answers to my questions.

Episode 3: I was in Dr. Zee's class and talked to a couple of students. They were working in pairs as they could not break out into groups in the classroom they had. As before, I asked for their views and comments on using the workbook before I asked them to solve a problem or at least talked about how to solve a problem.

RAR: 'without actually solving the problem, do you think it is possible to evaluate

$$\int_0^4 \int_{\frac{1}{2}}^2 e^{y^2} dy dx \text{ in this order?}'$$

Boy 7: 'can but complicated'

Boy 8: 'not sure, not yet revise'

RAR: 'what information do the limits give?'

Boy 7: 'can sketch the graph, the area of integration'

Boy 8: 'dy – limit along y; dx – limit along x'

RAR: 'if you have to reverse the order of integration, what are the new limits and how do you find them?'

Boy 7: 'from the graph'

Boy 8: 'just reverse'

They were quite friendly when I asked about their views on the workbook but were more stilted in their responses on the problem solving. In all the episodes, I was asking questions

on topics that they had done in their previous lessons as they were working on the topic of the Double Integrals in Polar Coordinates for the current lessons.

Students’ views on the use of the workbook

In total, I had talked to 21 students from 6 groups which consisted of three groups each from the classes G07 and H07. The total number of students in the two classes was 100. I asked them to describe how they use the book and what they thought of the book. Below is a summary of their comments is given in Table 5.10.

Table 5.10: Summary of students’ views

How they use the book?	Number of students (Total 21)
Usually work on the examples in class	21
Will attempt to fill out the part on Reflection	21
The prompts and questions are useful and help to make the concepts clearer	3
Only read the prompts and questions when they cannot do the examples	4
Do not read the prompts and questions, just ignore	10
Read the prompts and questions but found them difficult to answer, no examples and not sure what the lecturers want	2

Comments about the workbook	Number of students (Total 21)
It was useful and very good for team work	6
Find it easier to understand as compared to the textbook	4
Hard to look up the notes for revision as they are all over the place, prefers traditional format, notes then problems	4
Wants full answers to the problems, i.e. with workings shown	16
Wants at least answers to some of the exercises to become more confident of doing the problems right	21
Needs more challenging question	1

It was clear that for these results that the students felt that they had to use the workbook because we were using it. The majority of students preferred to be given worked examples rather than the ones that we gave them where they had to work them out for themselves. They also preferred the 'traditional' way of teaching, i.e. the way that they were accustomed to.

Second visit

The second visit was carried out in Week 9 and I carried out similar activities, class observations and interviews with students. However, the interviews were with a few volunteering students and were carried out away from class. The students had their Mid-semester break in Week 7. The adjustment was made to accommodate the various important holidays that would occur sometime during the semester. In this visit, all the classes that I observed were taught by Dr. Zee as Dr. Tee was away on leave and her classes were covered by Dr. Zee. Both of them were slightly behind and the students were studying the last few parts of Triple Integrals such as Integration in Spherical Coordinates and Applications of the Triple Integral. My general impression was that students in both classes were more involved in using the workbook especially when Dr. Zee was reviewing the *Reflection* section.

Excerpts from class notes: Wednesday, 05/09/07, 9.00 am

I am in Dr. Tee's class but conducted by Dr. Zee. She explains why she is conducting class, acknowledges my presence and reminds them of who I was and what I am doing in their class. She starts off on a question on evaluating a triple integral in spherical coordinates. She is solving her first question as a whole class exercise. Some students responding but others still not ready to participate. Some students straggling in 5 – 10 minutes past the hour. She starts on a second question; the class livelier as students read out their answers. I could see that Dr. Zee using the problems and responses to students' questions as 'review and summary' session for the topic. The class set up is similar to before; the students

seated at round tables in their own groups. She then asks them to do the structured examples on their own. I walk around the class, listening in on students' discussion. Dr. Zee is also making the rounds.

The question students are working on.

<p>Question 2 Convert to spherical coordinates and evaluate:</p> <p>(i) $\int_0^{\sqrt{2}} \int_0^{\sqrt{2-x^2}} \int_0^{\sqrt{2-x^2-y^2}} (x^2 + y^2 + z^2) dz dy dx$</p> <p>(ii) $\int_0^{\sqrt{2}} \int_{-\sqrt{2-z^2}}^{\sqrt{2-z^2}} \int_0^{\sqrt{2-y^2-z^2}} xz(x^2 + y^2 + z^2) dx dy dz$</p>	<p>Prompts/Questions</p> <ul style="list-style-type: none"> • What do the limits of integration tell you about the region? Sketch the region. • How do you change the integral to spherical coordinates? <ul style="list-style-type: none"> ○ What is the relation between the Cartesian and spherical coordinates? • Compare the integrals in rectangular and spherical coordinates. <ul style="list-style-type: none"> ○ When are spherical coordinates convenient to use?
----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------	------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------

I sat with a group of three boys as they talked to each other while solving the problem. Boy1 wants to convert straight off. Boy2 says that they have 'draw the diagram'. Boy1 asks, 'how to find the diagram'. Boy 3 says, 'from the limits'. Boy1 says, "oh, yes" and he write down the limits, copies out the values for z, y and x limits. Boy2 is drawing with his friends giving comments and suggestions. Boy3 does the conversion correctly and evaluated the new integral.

In this visit, I saw that the students were more involved in their group discussion; they did not seem to ask for Dr. Zee's help much and in the groups that I sat in, they did not seem to need the reassurance of the lecturer to check whether their work or answer was correct. They were also helping and supporting each other more. The same situation was also felt in Dr. Zee's own class, she had conducted a lecture for about 20 minutes and then the class was asked to work on the tasks from the book. The students worked in smaller groups, 2 or three people only but they appeared more engrossed in their work. Of course, my observations were compared only to the atmosphere that was observed on my previous visits.

5.4.6 Students' Responses

In this section, I will present the students' own responses that were collected through the interviews and copies of their problem solving and reflections in the workbook.

5.4.6.1 Interviews

I had arranged for students' interviews and I had a session with two female students in the afternoon of 05/09/07 and two sessions in the morning of Friday, 07/09/07. The interviews were conducted with the students in a group (see Table 5.10). The conversations with the students were to be about some topics that I had chosen such as, (i) students' views about the mathematics they were studying, (ii) how they studied, (iii) views about the teaching and the workbook, particularly, about what they liked or disliked as well as comments and suggestions for improvements. I had two groups from Dr. Zee's class and one group from Dr. Tee's class.

Table 5.11: List of interviewees

Names	Time slot	Lecturer's name
Tikah and Fizah	05/09/07, Wednesday, 12.00 – 12.55pm, 2 SEE	Dr. Zee
Khai, Razi, Faiz, Raden,	07/09/07, Friday, 11.15 – 12.10 noon, 2 SEM	Dr. Tee
Nasrul, Ng, Siti, Nur, Idah	07/09/07, Friday, 12.15 – 1.10 noon, 2 SEE	Dr. Zee

In the first part of the interviews, I asked students to talk about their learning experiences in the past and in the Engineering Mathematics course. Generally, the students were articulate and shared what they thought easily. I interviewed 5 girls and 6 boys. 7 students had Matriculation entry qualifications but the four students that I talked to from Dr. Tee's class had Diploma qualifications from UTM's Kuala Lumpur Campus. They had very good entry results and the lowest score amongst them was Faiz's, a CGPA of 3.42. In the second part of the interview, I asked the students to work through some problems that I had chosen, preferably as a group and to talk about what they were doing. In general, the conversations were in Bahasa Malaysia but mixed with some English. In reporting the

interviews, I will summarise the main issues that were obtained from the students' responses and select some individual responses as illustrations.

(i) Students' views about the mathematics they were studying

All the students said that there was more to study with various topics, sub-topics and not much time spent on each. When the interviews were held, they were finishing the topic of Multiple Integration. All the students lamented that they found visualising and sketching the graphs difficult. They said they could cope with choosing the right integration techniques but for multiple integrals, the sketch was important to determine the correct limits before they could integrate.

Tikah (girl) said, "... *integration – I can do that but I have to sketch the graph – that's where the difficulty is...*"

Faiz (boy): "...*when we got to integrals, we had to sketch, sketch..*"

RAR: "*why don't you like to sketch? You were from the diploma course in KL...*"

Khai (boy): "...*we used Autocad ..*"

Ng (boy): "*my problem I know the form but I can't draw it, that's my problem.*"

Siti (girl): "*Engineering maths interesting but needs imagination, 3-D and all that.*"

The students said that they liked the course, found it interesting but they had difficulty with drawing 3 dimensional graphs which started off from the first topic of functions. They had no difficulty in using the techniques of differentiation and integration although sometimes they need to revise earlier concepts and techniques they have learnt.

(ii) Their study strategies

Their study techniques were still similar to the other students that I have talked to; they worked on the exercises from the workbook, tutorial sheets, text book as well as past year tests and examination questions.

RAR: "...*why so many problems?*"

Nasrul: "... if they change the question, will be hard for me so I need to know more."

Razi: "past year questions really helps as the questions don't change much so it really helps"

They also talked about reading the notes and supported that with reading the text book. The girls especially talked about re-doing the worked examples.

Tikah (talking about how she revises): "...read the notes, see how the topics relate, look at the examples, do them again. I don't do new topics until I understand the ones I'm reading.."

Nasrul: "listen in class, do the exercises later. But I follow my mood sometimes I feel lazy, near the Exams, I do more.."

However, Ng, talked about his need to understand the concepts.

Ng (boy): " likes to understand, you know get the point, then it's easier to do the problems."

RAR: "...what do you mean, the point?"

Ng: "...you know, every topic has the point, you get the point then all questions are the same."

RAR: " how do you know what is the important point? "

Ng: " see how a problem is solved, how they change, if you know then can do other problems."

These students seemed to more competent with the mathematical knowledge and techniques previously learnt. Their study methods consisted mainly of using examples to identify and understand the important concepts and techniques. However, as has been stated elsewhere, the fact that the examination questions were routine and similar to each other year in year out, gave them the reassurance that this method of study was effective.

(iii) Views about the teaching and the workbook

In the interview, the students were asked to give their views or comments about three aspects of the book: (a) what they liked, (b) what they disliked and (c) comments or suggestions for improvements.

(a) What they liked

Tikah: *“very good, first there’s the questions, the spaces, not just watching what the lecturer teach, we get to write too. My lecturer, Dr. Zee, gives us time to write.”*

Faiz: *“like the Illustrations, many different kinds of questions, first basic, hard and harder ones. Summaries, theorems are in boxes, easy to read.”*

Raden: *“when I don’t come, I don’t know, reading the book, the notes, very helpful.”*

All the students liked the book because of the examples and the fact that it was a workbook as they had to participate in the solving of the examples. They also appreciated the different kinds of examples that covered the basic questions as well as harder ones. However, when it came to using the *Prompts and Questions*, the results were similar to the previous batches of students. Some (four students) said that they liked them; they were helpful and really useful in trying to understand the concepts and answering the problems. Others (five students) said that they did not use them unless they are stuck and a few (two students) said that they just glance through them and did not use them at all.

In terms of the teaching, the students were very happy that their lecturers were helpful and more approachable.

Tikah: *“the lecturer really helps, if her teaching is effective, we want to do more and more (exercises) ... she doesn’t need to use...you know sophisticated methods, if she’s good in explaining ...that’s enough ...”*

Fizah: *“lecturer can really influence a student, I don't like a lecturer who just reads her notes. I like it when she makes us do the work together in the lectures...”*

They also liked that they had to participate more in the lectures and in doing the mathematical activities.

(b) What they disliked

All responded that they did not like that there were no answers to most of the questions.

When I asked why the answers were necessary to them, the reply was that it was a quick way to know if they were right. Three students said that it was not easy to do the advanced questions by themselves and that they still needed someone to teach them.

(c) Suggestions for improvements

The main suggestion that they gave was that to include more answers for the questions and not just some selected ones as was already provided in the book.

5.4.6.2 Working on Mathematical Thinking

The main idea underlying the teaching strategies was the need to integrate the thinking strategies to help students become more aware of the mathematical processes and empower them to take charge of their own thinking and learning. In the episodes that I had presented, it could be said that the students were more independent and appeared to be able to work out how to solve the problems that they were given. However, in the interviews and discussions, they did not ascribe their awareness directly to the use of the prompts and questions. The lecturers used these consistently to direct students' attention in class during the lectures and worked examples session. When I was discussing with the students, they were using the language that their lecturers used thus there was some indications that they were absorbing the 'culture' of learning and solving problems that were established by their lecturers. They were also more confident with talking about the mathematics even when they were having difficulties with some of the concepts. What strike me most was

that there was more mathematical communication, verbal and written, which were circumstances that gave me great pleasure.

Here, I will reproduce two examples of the work done by Tikah and Fiza; Example (a) is a copy of an earlier assignment that they submitted to Dr. Zee which was answered in a mix of Bahasa Malaysia and English. Example (b) was a copy of the work they did together during the interview.

Example (a): Copy of Tikah and Fizah's assignment

Tikah and Fizah

Construct 2 examples of double integrals

$$\textcircled{1} \iint_{x=y} \frac{\sin y}{y} dy dx$$

$$\text{let } u=y \quad du=dy \Rightarrow \int \left[\int \frac{\sin u}{u} du \right] dx$$

- Double integrals ini tidak boleh di kanitikan walaupun setelah disubstitute (digantikan) dengan menganggap $u=y$
 $du=dy$.

- Oleh itu, persamaan ini perlu di 'reverse' kan

$$\Rightarrow \iint_{y=x} \frac{\sin y}{y} dx dy \quad \text{X}$$

$$\textcircled{2} \iint y^2 \sin x dy dx \Rightarrow \iint u \sin x dy dx$$

$$\text{let } u=y^2$$

$$du=2y dy$$

$$\frac{du}{2} = y dy$$

↳ Variable y tidak boleh diwujudkan dalam persamaan.

- Oleh itu, persamaan perlu di 'reverse' kan dahulu

$$\Rightarrow \iint_{y=x} y^2 \sin x dx dy \quad \text{X}$$

Reflection

→
sebelah

Reflection :

① Evaluate double integrals inner function x change

$$\int \int f(x,y) dx dy = \int \left[\int_{x=a}^{x=b} f(x,y) dx \right] dy \leftarrow y \text{ constant}$$

i) Kamirkan inner function apabila ^{nilai} x -berubah dan y -constant.

$$\int_{y=a}^{y=b} A(x,y) dy \leftarrow \text{hasil kamiran diatas}$$

ii) Hasil kamiran akan dikamirkan sekali lagi terhadap $-y$

② choose the preferable order of integration

$$\iint_R 2-y dA = \int_{y=a}^{y=b} \int_{x=a}^{x=b} x \cos xy dx dy$$

or

$$\int_{x=a}^{x=b} \int_{y=a}^{y=b} x \cos xy dy dx$$

-the choice of order depends on the integrand.

-so, we choose the easier integration, ~~dan~~ sama ada terhadap

(dx) x -dahulu atau y -dahulu (dy)

- untuk mengelakkan integration by parts.

③ limit.

- lukis (plot) graph.

- fixed kan salah satu variable.

$$\text{cth: } \iint f(x,y) dx dy \rightarrow \text{fixed kan variable } y$$

$$\iint f(x,y) dy dx \rightarrow \text{fixed kan variable } x$$

- ambil value of lower and upper value.

④

①-Tgk graph.

- Tgk order, define limit of the region.

It was an assignment to construct two examples of double integrals. They had chosen two examples which they claimed had to be solved by reversing the order of integration. Their examples were not complete nor were they correct but they attempted to explain why it was necessary to reverse the order of integration. This was followed by a summary of how

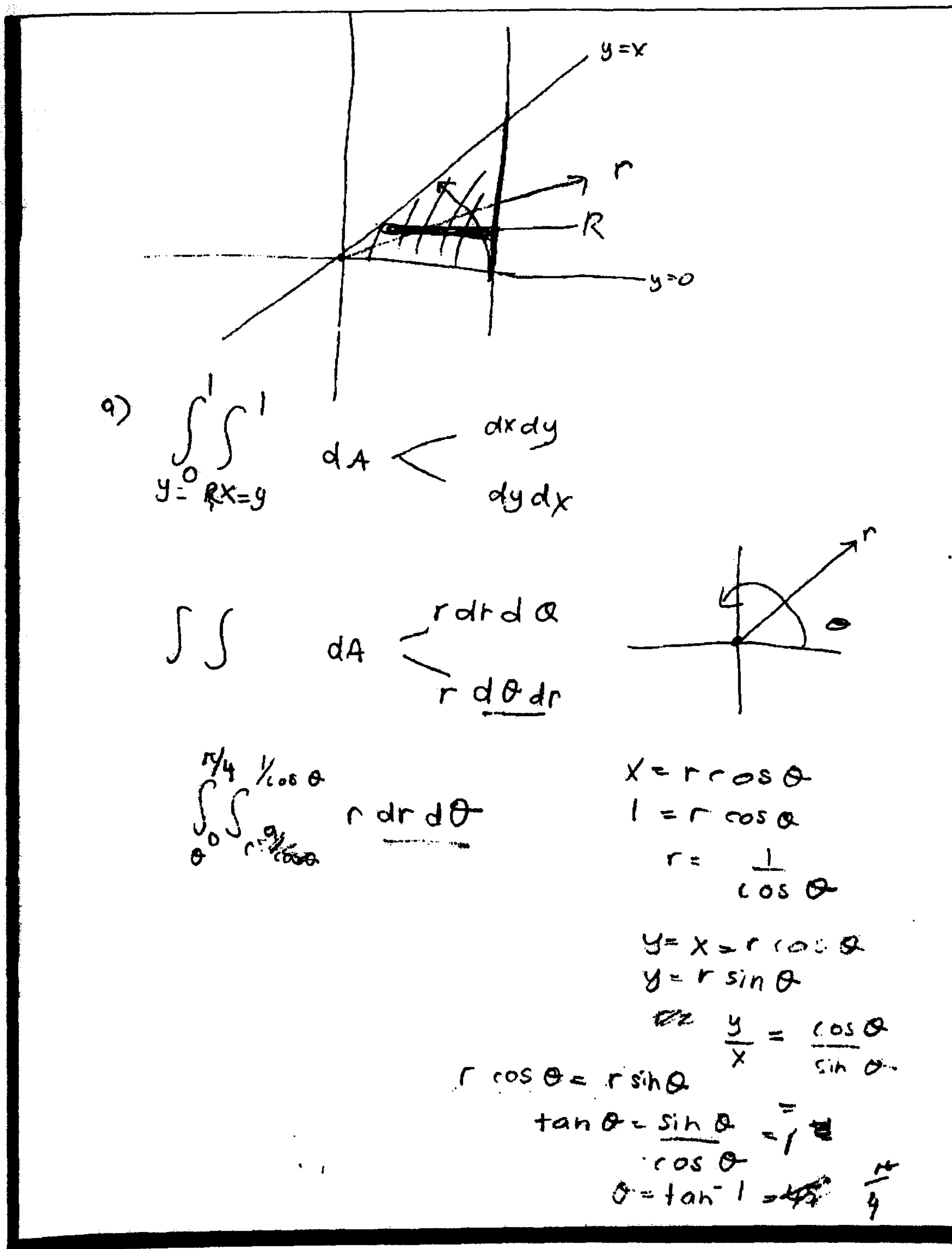
to evaluate double integrals. It can be seen that they had written out a detailed explanations of how to do this.

During the interview, they were asked to solve the following questions:

<p>Question 2</p> <p>Let R be the region enclosed by $y = x, y = 0$ and $x = 1$.</p> <p>(a) Express $\iint_R dA$ as an iterated integral in rectangular coordinates.</p> <p>(b) Express $\iint_R dA$ in terms of polar coordinates.</p>	<p>Prompts/Questions</p> <ul style="list-style-type: none"> • Can you identify and sketch the region of integration? • How do you write the iterated integral? <ul style="list-style-type: none"> ◦ What are the limits of integration in the rectangular coordinates? In polar coordinates?
<p>Question 3</p> <p>Use polar coordinates to evaluate the double integral.</p> <p>(a) $\iint_R y dA$; R is the disk $x^2 + y^2 \leq 4$.</p>	<p>Prompts/Questions</p> <ul style="list-style-type: none"> • How do you change Cartesian to polar form? <ul style="list-style-type: none"> ◦ Integrand ◦ Differential of integration (dA) ◦ Domain of integration • How do you find the limits of integration? • What basic integration techniques do you need?

Below is a copy of their work. As they worked, they were discussing with each other; about what dA means in Cartesian coordinates and which order to take. However, this was not clearly indicated in their working. Looking at the limits and their sketch, it was clear that they had taken the order $dx dy$. There was more discussion on what dA was in polar coordinates and how the limits should be read. They had worked out the conversion and knew the technique of how to read the limits but made a mistake in identifying the lower limit of r .

Example (b): Work done during interview.



The second problem they worked on is given below:

$\int_0^{2\pi} \int_0^2 dA$
 $x^2 + y^2 \leq 4$
 $rad = 2$

$\frac{dx dy}{r dr d\theta}$
 $r dr d\theta$

$\iint r$

$y = r \sin \theta$

$\int_0^{2\pi} \int_0^2 r \sin \theta r dr d\theta$

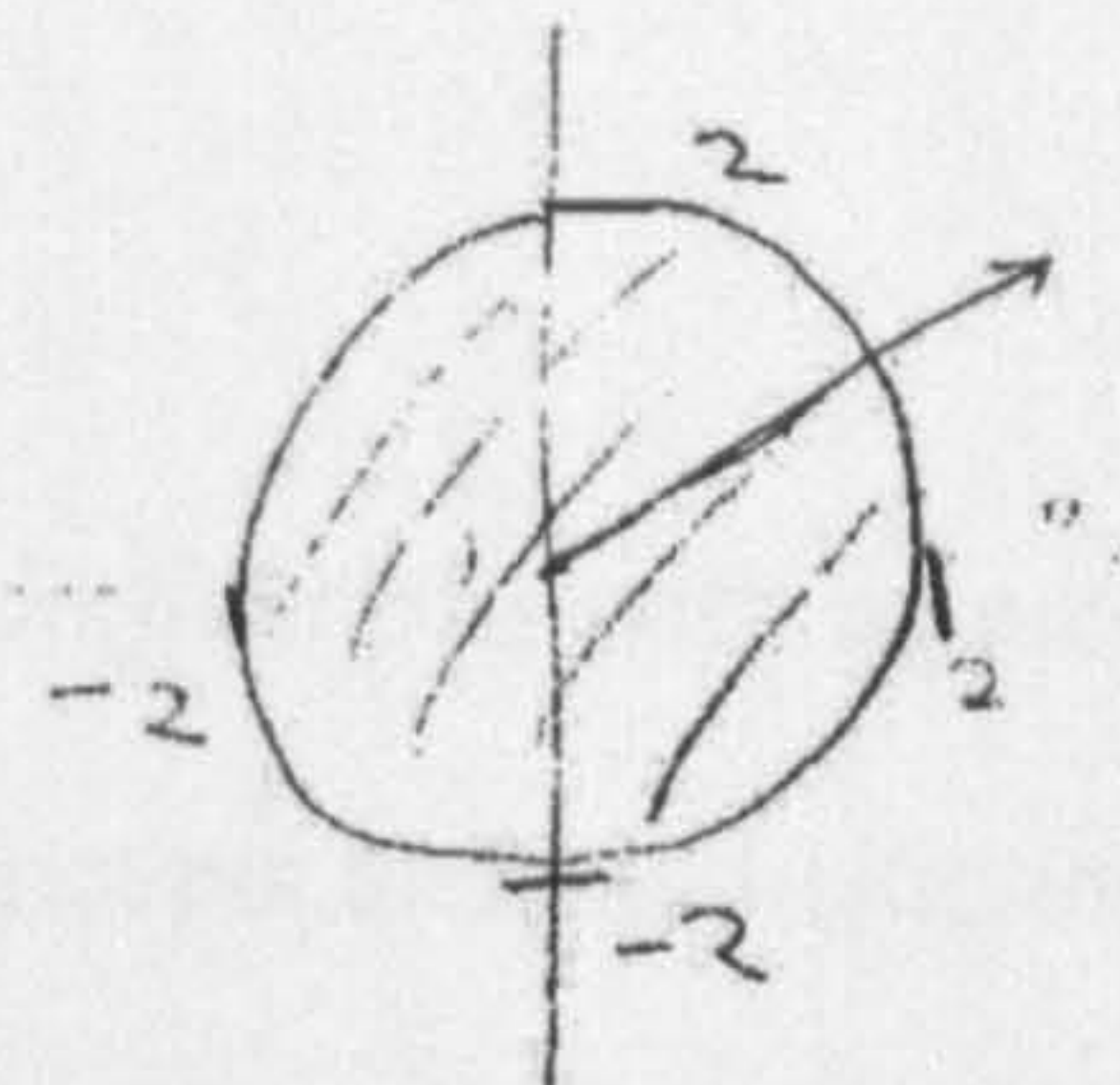
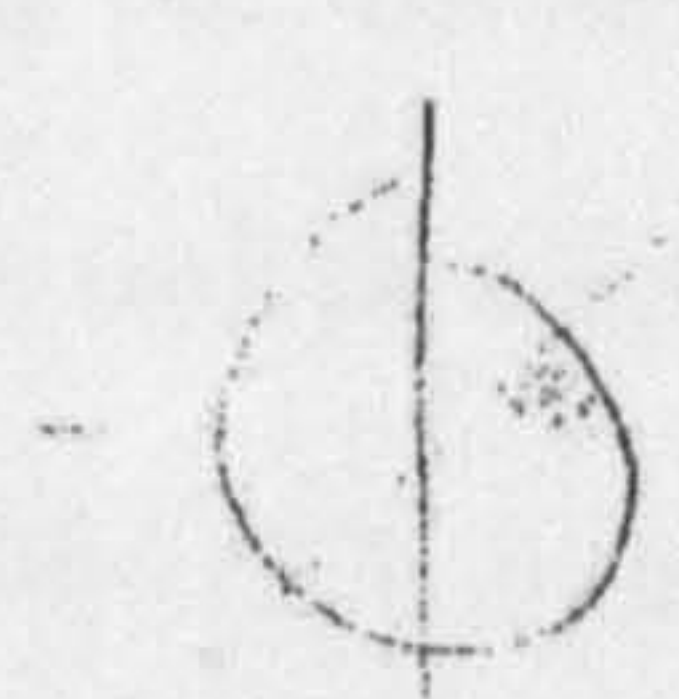
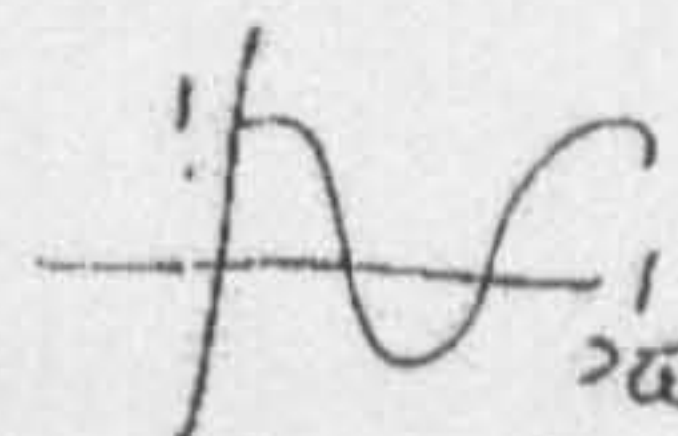
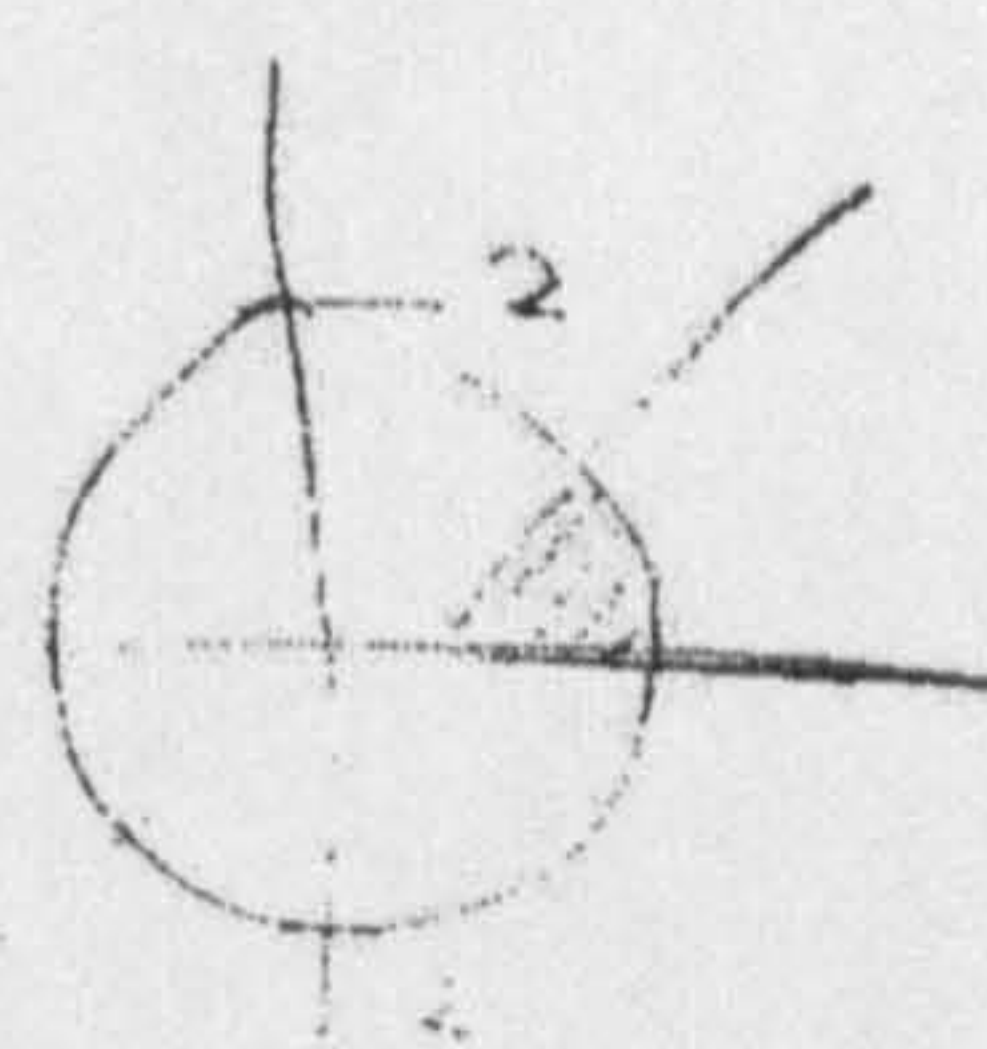
$\int_0^{2\pi} \int_0^2 r^2 \sin \theta dr d\theta$

$\int_0^{2\pi} \left. \frac{r^3 \sin \theta}{3} \right|_0^2 d\theta$

$\int_0^{2\pi} \frac{8}{3} \sin \theta d\theta$

$\left. -\cos \theta \cdot \frac{8}{3} \right|_0^{2\pi}$

$-\frac{8}{3} + \frac{8}{3} = 0$

They got zero as an answer and were quite concerned as they felt that it was wrong. Fizah whispered, "can't be zero, can't it?" They 'talked through' their work again but could not find the mistake and decided to submit what they had written.

In themselves, these are not significant pieces of work but looking at the work with the added element of the on-going discussion, verbalisation of mathematical ideas and techniques, showed a remarkable shift in mathematical behaviour for the students. Although they were just as concerned about the lack of answers in the workbook, they did

not request for my confirmation of the 'correctness' of their work when they finished with it.

5.4.6.3 Students' Reflections

In this workbook, we had included a section on *Reflections* with some questions that were connected to their understanding of important concepts and mathematical techniques of the topics being taught. Students were expected to fill them out and these were inspected periodically. The section functions as a review element as well as a means for students to think out what they knew and to make that knowledge explicit but in written form. As the section was part of the workbook, most of the students would attempt to fill them out although not always correctly.

Figure 5.17 (a) and (b) gives a couple of samples of the reflections written by students. Figure 5.17(a) is an example of another student's (Mohd) submitted assignment of constructing examples of double integrals. Meanwhile Figure 5.17(b) is a reflection submitted by Wahidah, one of the interviewees.

Figure 5.17(a): Sample of reflection handed in by Mohd.

no HD

EX 3.7

Construct two examples of double integrals that are readily evaluated by integrating in one order but not in the reverse order.

(i) $\int_0^2 \int_0^{2y^3} e^{y^4} \ln y \, dx \, dy$

(ii) $\int_1^2 \int_0^{\ln x} \frac{x}{\ln x} \, dy \, dx$

Reflection

- How do you evaluate double integrals?
 - sketch the double integral
 - choose the easiest way to evaluate the double integral (either (horizontal strip, x-integration first) or (vertical strip, y-integration first)).
- How do you choose the preferable order of integration?
 - choose the easiest way to integrate for the case both type can be integrate first.
 - For the case that one type region, choose only the one can be integrate first.
- How are the limits of integration determined?
 - For x-fixed, determine the interval for x and ranges for y.
 - For y-fixed, determine the interval for y and ranges for x.
- What do you do to evaluate double integrals by reversing the order of integration?
 - choose the suitable limit, then integrate.

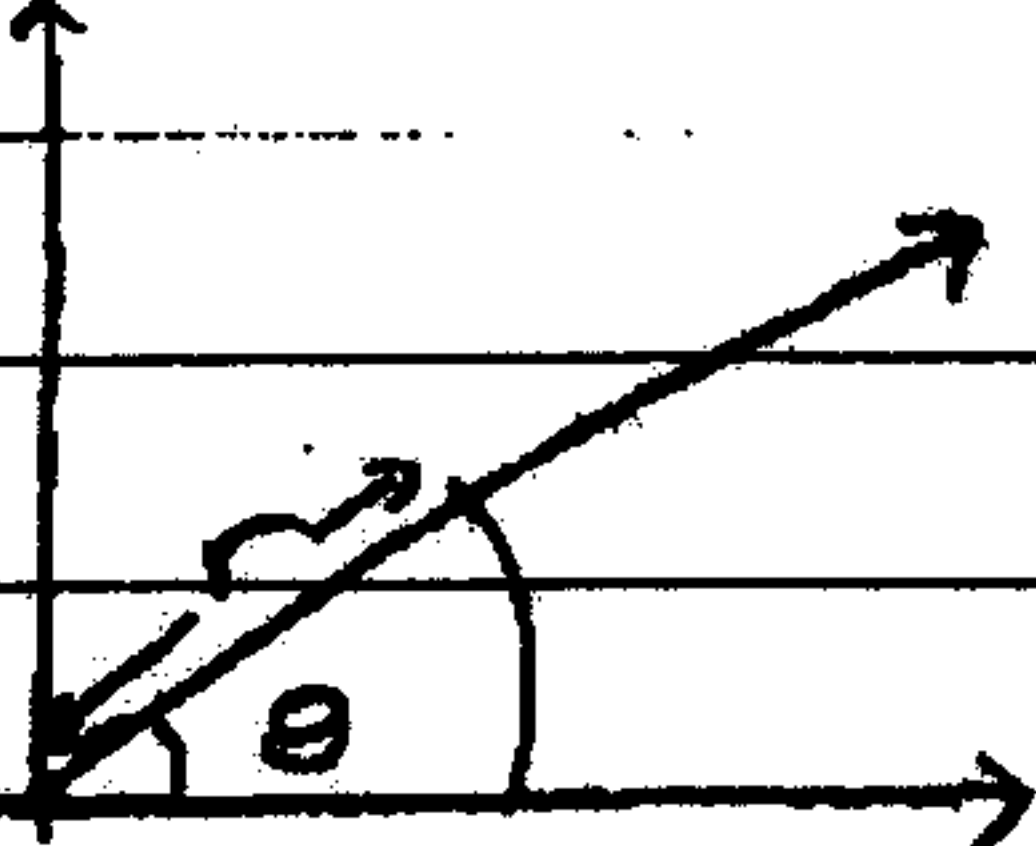
Figure 5.17(b): Sample of a reflection submitted by Wahidah

Reflection

→ When r that is the circle region or curves

→ Replace x and y with $x = r \cos \theta$ and $y = r \sin \theta$

→ From the region we get



r from the origin

The reflection was in response to the following questions:

Reflection

- When are polar coordinates more convenient?
.....
- How do you transform a double integral in rectangular coordinates into polar iterated integral?
.....
- How are the polar limits of integration determined?
.....

The students were at first persuaded through their own expectations of having to follow lecturers' instructions and to participate in any 'marks-producing' activities, to use and engage with all the elements in the workbook, including the *Reflection* section. However, my colleagues and I could see that their ability to focus, summarise and write down what they have learnt became better as the course progressed. The students were highly

motivated and they desired success in their study, thus their awareness of their ability became the incentive for further engagement with the book.

5.4.6.4 Feedback Questionnaire³

A feedback questionnaire was given out to the students at the beginning and at the end of the semester. The questionnaire was developed by my colleagues and I have been given permission to include their findings in this thesis. Some excerpts of students' comments about various elements of the workbook such as the *Structured Examples* and the use of the *prompts and questions* are given below. In the beginning, there were many negative sentiments about the strategies used in the book.

Lee

.....The question and prompts would be unnecessary. - As for mathematic
.....subject, it is not about theory! It is about practice and practical. I
.....think illustration should be included and not question and prompts.
.....

However when students were stuck with a problem, my colleagues reported that the students would be asking them questions similar to the *prompts and questions* that were provided. The students preferred to work on many examples as they found that this strategy was useful for success in answering examination questions.

Sam

.....more examples, more solution results in high
.....mark.
.....

In our experience that meant that they would like 'more of the same' until they could master the types of questions to be found in the examination papers. Nevertheless, in this questionnaire, my colleagues found that some students found the gradual increase in the

³ Findings has been presented at ICME 11 held at Monterrey, Mexico, 2008

levels of difficulties in the tasks given was just as helpful. They said that it gave them the chance to follow the conceptual development. Thus, there was indication that they were aware of the different levels of complexity and how it helps them develop their understanding of the concepts and problem solving.

Iza

..... This workbook is very useful for me. I can understand what is the same
..... topic about and in this workbook build myself confidence to take
..... Engineering Mathematics subject. This workbook also ^{guide} me to
..... understand the topic.
.....

Interestingly, students' comments were more favourable at the end of the semester. Many saw the usefulness of the *prompts and questions* and they said that these helped in their learning and in organising their thinking.

Hasan

..... The workbook is very useful for engineering
..... student. It can induce my thinking skill and
..... get well organized
.....

Students also liked the different levels of difficulties found in the examples given and that they had to do the *Structured Examples* for themselves in and out of the class.

Well, at first I was not ~~not~~ well, at first when I was ^{doing} this work but I totally neglected the prompt and question because I was able to understand the content through my lecturer and I did not need to depend on the prompt to ~~do~~ ^{solve} my problems. But lately I miss a few class but yet I was able to understand and proceed with the help from the prompt.

5.4.7 Issues and Discussion

Several issues were identified and discussed at the end of Chapter 4. Here, the same issues are re-visited and new knowledge is added to the discussion.

Issue 1: Change in attitudes

My perspective:

I have a strong motivation to change as I wanted to explore and implement strategies that would enhance my students' ability to recognise and use their own thinking powers. The students that I was teaching in the second part of the implementation were highly motivated for success, very hard working and intelligent. I thought that it would be easier to convince them to re-examine their beliefs and attitudes if I could create class tasks that would give them experience of success in thinking about mathematics. However, the students were already very successful in their academic performance; using their well tried and tested methods of study still yields success in terms of good examination results; it was not easy to convince them that they would need new ways of working with mathematics. I was thinking of the future demands in the course as they progress to more advanced topics in Engineering as well as in their career; students were more focused on the here and now. I had to adapt the ways of relating and dealing with the students so as to strengthen the connection they feel towards me as their lecturer. I would need to use this connection to

initiate discussions to increase my students' awareness of attitudes of successful engineers and not just focused on being successful students. My experience has shown that for the majority, the connection is temporary, only for the length of the course. However, during that time, their attention to what I teach or say was intense. In this aspect, there was a big difference between the Engineering Mathematics (SSE) students when compared to the Calculus II (SRI) students. My SSE students were independent but always looking for ways to get better and better results, with or without the lecturer's help. My SRI students were grateful just to pass the course and thought that only the lecturer can help them do that.

Students' perspective: The students embraced all the changes in the teaching methods, using the worksheets, working in groups, sharing knowledge verbally and in written form. They were active in the class, participated in discussions, presented their answers and thoughts in the class but in the beginning, there was no real change in what they believed or how they studied for themselves. In conversations and interviews, they still held on to their old ways of working with the mathematics.

With the last two groups, the implementation of my teaching strategies was supported by a workbook which had various elements to support students' independent work and thinking. Thus, they were consistently working in such an environment. Thus, at the end of the semester there were more positive comments and appreciation of the ways the subject was taught but more importantly, more awareness of their own abilities to work through the concepts and mathematical techniques.

Issue 2: Factors Influencing Students' Study of Mathematics

In terms of beliefs about their abilities, they were modest and do not claim to be excellent but would only acknowledge above average performance. However, in many cases, the real performance would be better than the stated evaluation of themselves. Motivation was still an important factor in determining students' participation and commitment to the classes.

All the students were highly motivated, had good results for their prior mathematics, had clear ambitions and were consistently studying for all the subjects that they were learning. Even with such a strong base, they still cited their need for a 'good' lecturer who was effective in transmitting knowledge. Some claimed that they were motivated by good lecturers but in the interviews, nearly all said that as a resource person, the lecturer was last on their list. However, they also said that to understand the subject while it was being taught was an important learning strategy for them, thus, the appreciation for effective presentation of the contents.

The teaching approach was based on student-centred principles, thus there was more opportunities for 'lecturer to student' and 'student to student' interaction. Thus, 'approachability' and 'openness' were factors cultivated explicitly in the teaching and learning environment. These were the supporting factors that promoted students' knowledge development in the classroom.

Issue 3: Working on Mathematical Thinking

In the first part of the implementation (Chapter 4) and the teaching of E05 and F05, the strategies were explicitly used for the first three topics only. For students G07 and H07, their teaching and learning environment supported them in doing the mathematics for themselves; encouraged communication and discussions in solving problems and was consistently used for the whole course. Thus, the strategies were more successful in changing mathematical learning behaviour.

Issue 4: Linking personal theories to public theories

Much of what I found could be explained by existing theories of learning although I will need to combine a few of them. I believe that description of my work and the findings would contribute to the understanding and appreciation of what the teacher does, the choices she has to make and how she has to extract or work out what could be practiced from the theories. I had to balance between meeting student desires and promoting what I

believe to be valuable, constantly reappraising the situation; keeping one eye on the mathematical themes etc. while students focus on the techniques for the next set of questions.

In terms of research methodology, a stint in the first semester of 2007/2008 as a complete observer helped to provide the balance and objectivity I needed to review and reflect upon the whole research process and its findings.

4.10 Conclusion

The same issues that were identified in the first part of the implementation were revisited and examined in the light of the second implementation. These issues will be further developed and discussed in Chapter 6.

CHAPTER 6

RESULTS AND CONTRIBUTION TO THEORETICAL CONSIDERATIONS

6.1 Introduction

The research on changing teaching practice was carried out over a number of years on various groups of students. However, essentially, it can be divided into three phases as follows: phase 1 covered the period from November 2001 – 2004 (see Chapter 4), phase 2 was the teaching carried out in semester 1, 2005/2006 (see Chapter 5) and phase 3 covered the period of semester 1 2007/2008 (see Chapter 5). In each phase, data concerned with changing the teaching practice and its impact on students' learning were collected. In this chapter, I will first put forward an analysis of the factors affecting my teaching practice and then an analysis of the factors affecting students' learning and their change in attitudes towards learning the teaching strategies implemented with the underlying focus of using their own mathematical thinking powers. In addition, I would like to present a model that would describe the change process, the roles of the teaching strategies in supporting and sustaining the change from my perspective and as observed from the students' behaviour.

6.2 Changing Teaching Practice

In each of the phases, I have identified the factors that were important in sustaining the changes that I wanted to effect in my teaching. My focus was on implementing teaching strategies that would invoke students' use of their own mathematical thinking powers. In Chapter 2, I had discussed the need to enhance students' appreciation and use of their thinking powers as to equip themselves with better problem solving skills. I had decided to use a model suggested by Cocking and Chipman (1988) and adapted by Norman and Prichard (1994) which described factors that would influenced a teaching and learning situation. Cocking and Chipman (1988) attempted to identify linguistic and cultural variables to explain the poorer performance of language minority students in mathematics

when compared to students who spoke English as a primary language. They proposed their model which categorised the factors that influenced mathematical learning at school expanded along the lines of Input to the children and Output or Mastery which was the children's performance. Norman and Prichard (1994) used the basic model in their research project that looked at the cognitive obstacles in the learning of Calculus and identified three major components of the teaching and learning situation, which were, Entry Mastery, Student Motivation and Opportunity to Learn. However, Norman and Prichard were primarily concerned with the Entry Mastery category as they believed that this is where many cognitive obstacles originated.

I have introduced the model as described by Norman and Prichard (1994) in Chapter 4 (see Figure 4.1). In the first phase of the research, I was working with different groups of students from the Industrial Design Course; I found the model appropriate in describing the learning situation in my class. However, I focused on the other two components: **Opportunity to Learn** and **Students' Motivation**. I had no influence in changing the Entry Mastery component but only gathered data through various tools (see Section 4.4) to identify the students' mastery of their prior mathematical knowledge. The nature of the changes in my teaching was to provide suitably designed educational opportunities for students to use their mathematical thinking powers and communicate their mathematical knowledge, and record how these activities supported changes in students' attitudes towards learning Calculus. In the following discussion, I will first identify the factors that shaped and affected my teaching practice and will propose some additional elements to the categories of Opportunity to Learn and Students' Motivation in the model. The data I have collected has added depth to the model and will present naturalistic data of the interaction between lecturer and students in classroom, out of it and how these interactions could influence students' motivation which in turn, could influence students' attitudes towards learning.

6.2.1 My Own Attitudes and Motivation

It was not easy at first to be aware of and to articulate my attitudes but engaging in critical self-reflection and discussions with my colleagues helped to strengthen the ability to do both. I had identified the issues associated to the changing of my attitudes and factors affecting my motivation.

I was very concerned about students' development which was build up from personal and professional experience. Being a mother concerned with educating my children to be good decision makers, independent learners, and if possible, life-long learners, contributed and strengthened my resolve to do the same for my students at the university as well. I felt that I was highly motivated in providing opportunities to support students' change in their learning. I felt that it would be easy to maintain my interest and my efforts, to adapt and adopt new ways to think about teaching and learning. However, when the changes were implemented, the situation was very different. I found that my attitudes and motivation were also affected by the dynamics and events during the teaching sessions from within the classroom, my interaction with students and colleagues out of it, the organisational and administrative components of the course implementation. I have articulated my basic beliefs about teaching and learning in Section 4.2.1. In the early part of Phase 1, I was confronted with groups of students who were unmotivated due to various factors (see section 4.5.4) that it also affected my responses. Thus students' self esteem and beliefs about the relevance of their courses had to be addressed. In the later period of Phase 1, I had students who were 'very happy' (see Section 4.8) to be in my class, which of course, made me happy when I was conducting the lessons. Although, I tried very hard to maintain professionalism and impartiality, teaching and learning occur in a setting with social, psychological and emotional interactions. I had to work hard to maintain objectivity in my responses to students' personal problems or problems with the mathematics.

In Phase 2, I had to reevaluate the personal beliefs and attitudes that I held as the students in my class were students' intake based on new policies of the university (see sections 5.2.2 – 5.2.4), they had different entry qualifications as well as had more mathematical learning before coming to UTM. In particular, findings based on research on the difficulties of UTM students was re-considered and some experience and observation was attained when I taught the new students in their first year (see Sections 5.22 & 5.2.3). Thus, although their prior mathematical knowledge was stronger, they still needed support to enhance their awareness of their own mathematical thinking powers and ability to communicate mathematical ideas not only in written form but verbally as well. However, experience in Phase 1 were also helpful in identifying changes that were necessary to the teaching and learning situation in terms of the strategies, tasks design and classroom environment (see Section 5.2.4). I found that I was also affected by students' resistance to the changes and other behaviours in particular situations but my personal convictions about the need to bring about changes were substantial in sustaining the determination to continue with the implementation of the teaching strategies. Data collected also highlighted that a considerable number of students in these two phases identified 'rapport' with lecturers and in particular, with me, as significant in motivating them to participate in my class. Thus, a factor that had to be examined was whether the strategies could be used by other lecturers. Consequently, in phase 3, I decided to observe the same teaching strategies implemented and taught by two colleagues which gave me the opportunity to evaluate the viability of the teaching strategies being used by other lecturers (see Section 5.4.4).

6.2.2 Teaching Acts and Task Design

Teacher's knowledge and training were categories already identified in Norman and Prichard's (1994) modified model and in this section, I will expand on the discussion to highlight the various factors that I needed to develop or learn to sustain a learning

environment that would promote thinking and communication identified during the study.

These factors were:

- **How to be a good facilitator of active learning?** – In the beginning of the research, I realized that good intentions alone were not enough; I needed skills to facilitate students' learning. There were many episodes where I regressed into the comfortable habit 'show and tell' (see Section 4.5.2). I needed to consciously think of ways to change old habits that were no longer suitable to support the changes that I wanted to effect. One particular way was to engage in the mathematical tasks myself, put myself in the position of the learner and thus experience the process of learning. However, knowing that students still preferred lectures, I had to think of ways to negotiate for change. Thus, I assigned some time for lectures and introduce sessions for students to engage with mathematical tasks during the classroom session. This was a change in the usual way classes were conducted. In the past, work on mathematical tasks was carried out in the tutorial sessions. Lectures were for presentation of mathematical content and examples. To promote communication, a short session to encourage students' reflection on their work was also introduced. The reflection was sometimes verbal or in written form. Thus the class interaction was usually of the nature, '*lecture – activity – review – activity – reflection – summary*'. Depending on the classroom situation, all or some of these were carried out. It was obvious that new skills had to be developed to support the teaching acts that I wanted to implement.
- **To pay attention to the students' responses, both verbal and non-verbal** – As my teaching became more student centred, I found that I had to take more notice of students' responses. I was introducing a way of teaching that required more students' participation and the students were not accustomed to such an environment. Some responded favourably but others appeared unresponsive but I soon realised that they were responding but through non-verbal mannerisms. It was

easier to respond to their verbal contributions; their comments, criticisms, requests, inquiries; but much harder to interpret their non-verbal responses. Again, I found that I am continually negotiating and mediating for change and had to ensure certain conditions to gain trust and coax students to participate in my class. Some of the techniques that I adopted were to adopt a more non-judgemental approach in working with the students. In terms of non-verbal responses, I decided that if and when I did make a judgement of what they meant by their gestures or body language, I would state these explicitly and request that the students verbalise their needs and feelings. I had explicitly stated that there will be no recrimination or punishment for honesty and these approaches had helped to foster better teacher – to – student communication and vice-versa.

In the first phase, there were some planning but many of the interactions were impromptu and had to be improvised. In the second phase and the third phase, I had make more efforts to plan for a teaching and learning environment that would support the changes in students' participation, increase in mathematical communication and working on mathematical thinking, which will be described in Section 6.3.3.

- **Spontaneous coping skills to deal with students' mathematical and affective difficulties.**

This study did not focus on the social and cultural factors affecting students' learning within the classroom setting. However, some of the more pervasive behaviour will be briefly addressed.

- Confronting students' overly deferential behaviour – on the surface, such deferential behaviour is considered the norm in my society, the respect for students to their teacher, however, it becomes painful to witness when your aim is to enhance students' confidence and independence. Their gratefulness to a supportive teacher implies certain issues in the way the

teaching and learning culture has developed but this is not within the research focus. I can only change the way I interact with my students and encourage them to be themselves without sacrificing courtesy and good manners. By using the teaching strategies, I have focused on students' mathematical abilities but I would always emphasise and draw the students' attention to their own awareness and achievements and refuse to accept credit for their own efforts and hard work. A student once said, "*Madam, you saved my life*" to which I replied, "*no, you saved your life*" in reference to the fact that he passed his Calculus II paper.

- Supporting students to be independent learners and increasing their confidence – the teaching strategies, classroom settings and social interaction were designed and implemented towards these very objectives but to those students who needed some personal or pastoral care, this was duly given but within the limits that I was comfortable with. Students who needed more help was referred to the Students' Support Services for more professional help.

- **Presentation of tasks**

The tasks were designed to allow students to work on mathematical thinking and changes were made as to how they were presented to students as I worked through the different phases. In each task in the first phase, I had focused on the mathematical powers of specialising and generalising to increase students' awareness of the mathematical processes. However, the students needed frequent 'prompting' to be able to work through the tasks and wanted to be able to refer to their work for review and revision. In addition, they preferred 'official' notes provided by the lecturer as they also needed assurance that their work would be graded and what was taught was to be examined (see Section 5.2.4). Thus in phase

2, I provided the students with worksheets so that the topics were presented in a structured manner and the prompts and questions were made explicit that it would be easier to refer to and to use during class work. The worksheets were collected for assessment for some of the work. They also used a textbook for extra reading and as a source of tutorial questions. However, I was only using the first three topics for the research and work was carried out in the more traditional mode for the other topics. In phase 3 (see Section 5.4), the lecturers used a workbook that would allow students to work using the strategies consistently for all the topics. Several other features were also added in the book to promote a learning environment that would encourage students' engagement with the mathematical tasks, increase students' mathematical communication and group work.

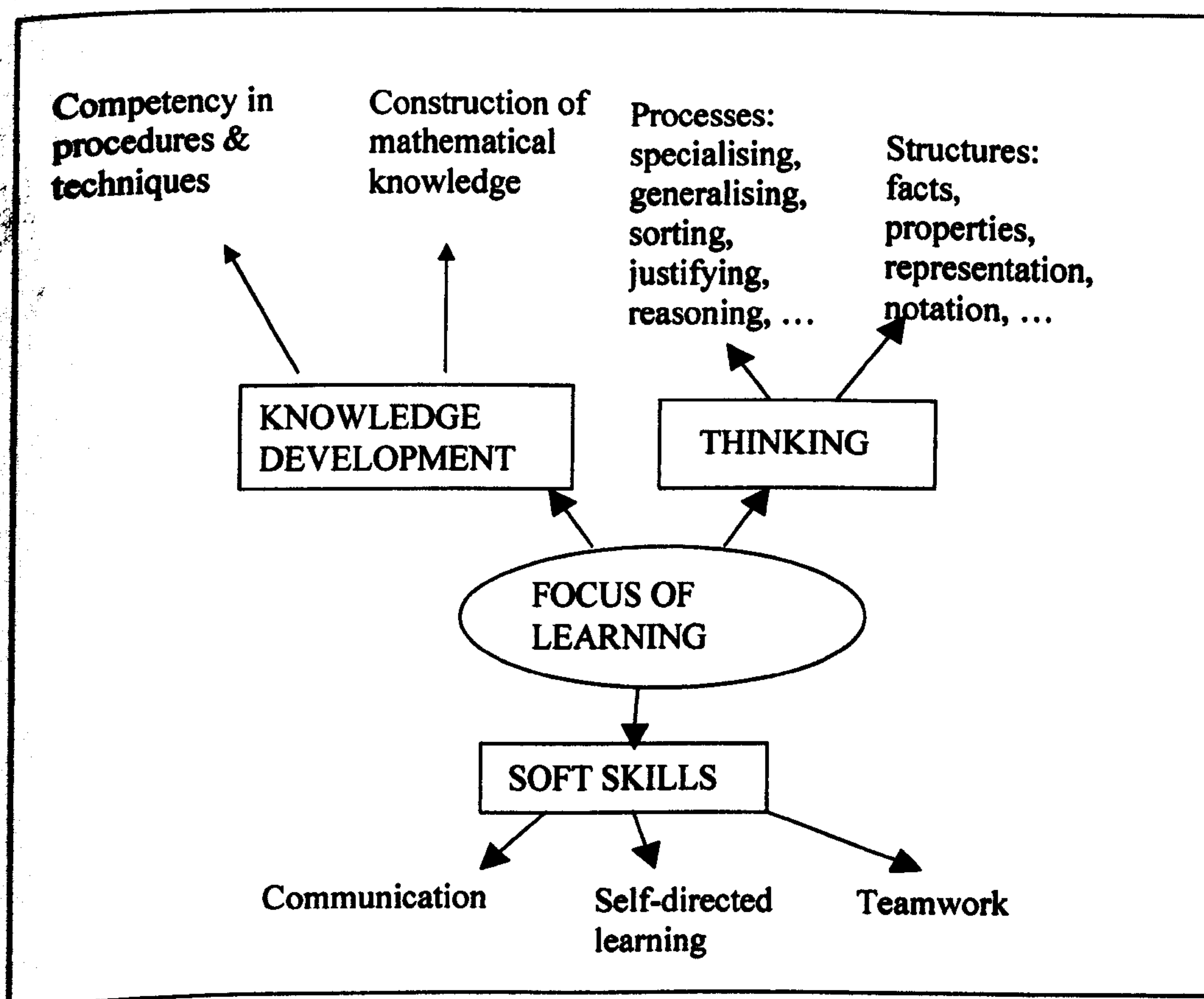
6.2.3 Classroom Environment

In describing the teaching and learning situation, I have not emphasised the social and cultural context. Briefly, the students were mainly Malaysians from the different major race groups, Malays, Chinese and Indians. I am of course, very aware of the various social mores and issues affecting the relationship between the races in the society and in the university. In fact, I use this knowledge in my interaction with the students but I strongly believe in promoting understanding, acceptance and respect between the different races and thus exercise these values in my classroom. Furthermore, my experience has shown that within the context of my mathematics class, the students were all focused on the learning of the subject and their desire to succeed. Within this setting, the students will generally accept and embraced the learning activities that were implemented and adopted the classroom culture so as to get on with their work.

However, it was clear that the teaching and learning environment had to support the changes that were made and thus changes were also introduced to create a more

conducive environment. In order to achieve the aims of providing a learning environment that can support students' awareness of mathematical thinking and problem solving skills, I have chosen two models to explain the teaching situation which has been described in Chapter 5 and are reproduced below. The first one (Figure 5.1) identified the focus of learning and the second model (Figure 5.2) identified the important elements of an active learning environment. In this way, the teaching and learning environment will support the aims of enhancing students' ability to take charge of their own learning, increase their understanding, able to communicate their mathematical learning and to increase their awareness of their own mathematical thinking. In choosing the focus of learning, elements consistent with the University's philosophy of teaching were identified and these are: Thinking, Knowledge Development and Soft Skills Development, which emphasised communication, independent learning and teamwork.

Figure 5.1: Focus of Mathematical Learning



Active learning theory was chosen as I thought that it would give students the opportunities to be interactive with the subject matter. The important elements to support effective active learning were talking, listening, reading, writing and reflecting (Meyers & Jones, 1993) and are included in the model (Figure 5.2).

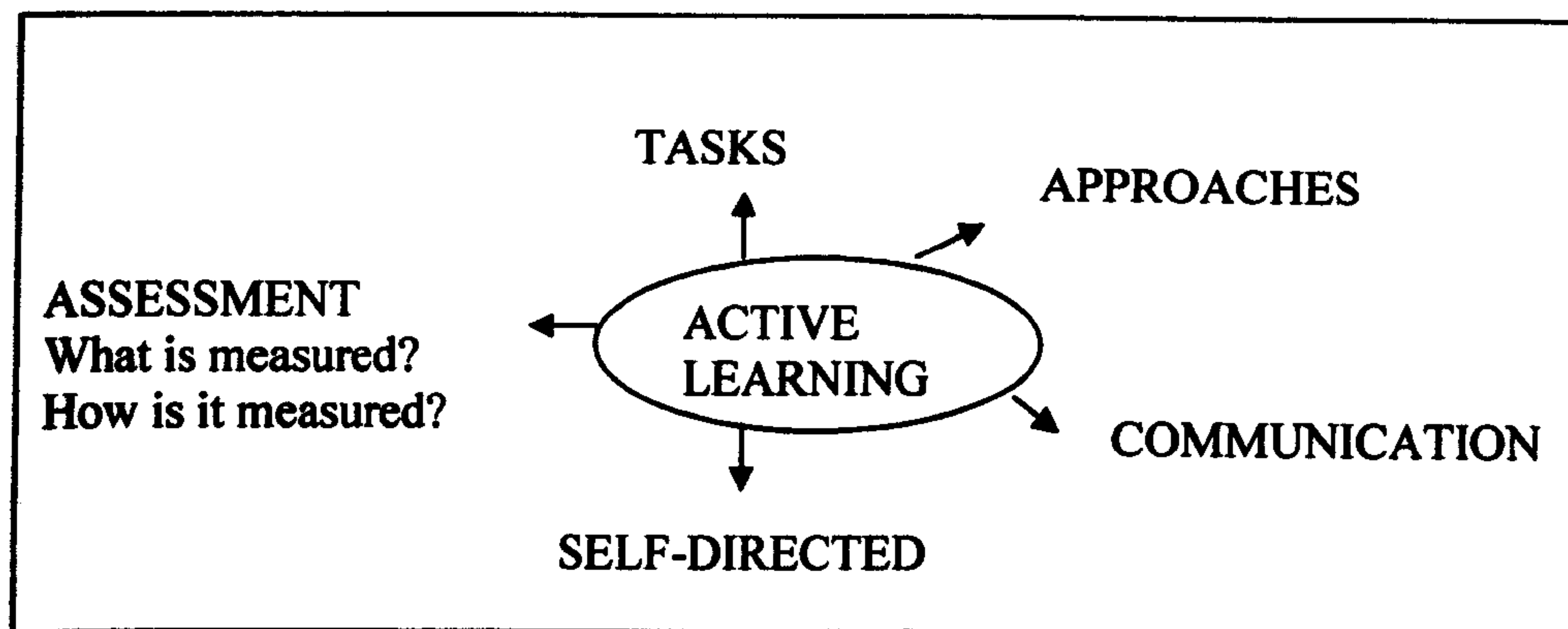


Figure 5.2: Model of Active Learning

6.2.4 Organisational Support

Administrative and management support was important and did have some impact in expediting or impeding the changes I wanted to implement. For example, there was some difficulties in arranging for a more suitable class to allow for more students' interaction as well as time-tabling hitches that saw my students having to walk long distances from another class to mine which meant that they were tired by the time they arrived (see Section 5.3). As I was a service lecturer thus my students' time-table were arranged by their Faculties as well as room allocation, it was more difficult to negotiate for better provisions in terms of room and time allocation. At the same time, the changes that I had implemented were not officially endorsed by the Department although I was given permission to carry out my strategies but they were more in the nature of support towards my intention of pursuing research towards a doctoral degree. I had to negotiate with my Department and at least two other Faculties in Phase 2. I had to share the objectives of the changes to be implemented with the relevant personnel so as to garner some support.

6.2.5 Obstacles to Change

The main obstacle that I had to confront was the need to change my paradigms of teaching and learning and sustaining the changes. I had to ‘*learn, unlearn and relearn*’ many of the ideas about teaching and how students learn. Reviewing literature about these ideas was not easy as they were many ideas and different perspectives to consider. Consequently, I had chosen some theoretical foundations and woven them together to support the strategies that I did implement as explained above.

External obstacles included common factors that governed and influenced any teacher’s life such as (i) time and curriculum demands and (ii) organisational and faculty support. Changing in isolation is not easy and trying to convince other members of the faculty is more difficult still. However, I have tried to implement some changes within the existing constraints and still believe that making an effort is important as not changing anything at all is unacceptable in terms of the development of the students.

6.3 Students’ Learning

There were two very different groupings of students that were studied as identified by their entrance qualifications. Pre-July 2000, students came in with SPM qualifications meanwhile post-July 2000, students had post-SPM qualifications which were mainly, Matriculation, STPM and Diploma qualifications. There were many differences in terms of their prior mathematical knowledge mastery, attitudes and affective demeanour. However, there were also similarities especially in terms of beliefs about mathematics, teaching, learning behaviour and criteria of success.

Generally, students had very little say in how a lecturer teaches and lecturers have no magical powers to change the way students learn, although, I believed that I could influence some of the views that would encourage students to change or consider changing their attitudes.

Regardless of mathematical background, my students were all very concerned about (i) how the changes in my teaching would influence their assessment and final examination, and (ii) getting good results for the course.

Students in Block 1 had a more varied background in terms of mathematical mastery of their prior knowledge. The students who had many difficulties were those whose prior SPM mathematical qualifications were in the credit or pass category (B4, C5, C6, P7 and P8, please refer to Chapter 4, p 99-100 for the explanations of grades) or who did not have any Additional Mathematics qualification. As their mathematics performance in UTM also suffered, many had to repeat the necessary subjects and thus, their self-confidence was also eroded. Assumed future prospects in their chosen career were always negative and hearsay was given the status of facts. They made no effort to find out more about the possible careers open to them nor was there an awareness of the necessary professional traits that they had to develop as Industrial Design engineers. Their focus was fixated on the present and since they were not doing too well in the mathematics, they further lowered their own expectations and most were quite satisfied with just getting through the course.

In contrast, the students with better prior qualifications could cope with the mathematics at UTM and thus had less difficulty in getting good results in the subjects. However, surprisingly, they also had similar views about future career prospects. There was a certain amount of despondency in the air, at least when I was teaching them, which was not only connected to the mathematics subjects but to the course itself.

For Block 2 students, their mastery of prior mathematics was excellent. The teaching strategies implemented were also supported with worksheets although the focus was still on the first three topics only. The students embraced all the changes in the teaching methods, used the worksheets, worked in groups, and willingly shared their knowledge verbally and in written form. They were active in class but from conversations and interviews, they still held fast to their views about how mathematics should be taught and

studied. Studying for the examination using “drill and practise” has served all the students well in the past. The nature of the examination paper, the unchanging format and routine questions, was easily surmounted with practise and more practise. They were also concerned with how the mathematics was taught and presented and were eager to share their views.

For Block 3 students, the implementation of my teaching strategies was supported by a workbook which made many of the strategies more explicit and thus increased their awareness of what was carried out. Thus, they were consistently working in such an environment for all the topics. Perhaps, due to the sustained work, they had more positive comments and appreciated how the subject was taught. They also had become more aware of their own abilities to work through the concepts and mathematical techniques.

Thus, in summary, I could say that for the students, some indicated their willingness to participate and appeared to engage in the class activities and tasks very early in the semester. Others took more time to appreciate that they needed to change and some were not moved to consider changing and clinging to their own ‘tried and tested’ viewpoints. It is not easy to measure change and difficult to get firm evidence that the teaching strategies have influenced students. Nevertheless, some episodes indicated that students thought that the teaching strategies helped them to understand better. A few also willingly talked about how they had changed their feelings about learning mathematics after following my classes. However, there were students who were not comfortable and thought the methods were too slow and complicated what was otherwise easy. They preferred the ‘traditional’ ways of teaching.

6.3.1 Factors affecting students’ learning

The study had provided data about students’ responses and attitudes towards changes in teaching practice and in this section, data that had contributed to a more detailed

description of the various factors in the model by Norman & Prichard (1994; Figure 4.1) will be discussed.

6.3.1.1 Prior Knowledge

The majority of the students coming into UTM came through local schools or other educational establishments. Thus, they all had to follow the standard curriculum at their institutions and this included the mathematics curriculum at SPM or post-SPM levels. The level of mastery might be different for each student but in UTM, there were bridging subjects that were designed to help students achieve a more homogenous mathematical background. Some Block 1 students were still facing difficulties after going through our own courses but as mentioned, these were students recruited under the old policies and thus, were no longer considered as 'typical' undergraduates. The bridging mathematics course was more successful for Blocks 2 and 3 students, recruited under new policies. As a consequence, I could say that, students had strong common background knowledge to prepare them for the Engineering Mathematics subject.

6.3.1.2 Motivation

The students identified several factors as important in their study. Firstly, they wanted to succeed, were willing to work hard and prepared to participate in the mathematics teaching and assessment processes. Even with Block 1 students, these attitudes were the same. However, I have chosen to relate stories of some individual students in this thesis and have included among them, students with less positive attitudes. There were not many. Even as they were trying to overcome their negative beliefs, they still wanted to pass the course. The drive to get a degree was strong. Most other students were very confident in their abilities and their main concern was to get a good 'score' for the subject.

-Secondly, they believed that their lecturer could influence the way they studied or learned the subject. The way the lectures were presented, how the lecturer connected or interacted with them was considered important and they claimed that these could increase or decrease

their enthusiasm for classroom work. Rapport with their lecturer was an important motivating element. Thirdly, they had to trust their lecturer as someone who was interested in their success and well-being as students. These notions were strongly held even if not supported by their own behaviour. For instance, in interviews, many students said that they would not seek help from the lecturer if they had any difficulty with the mathematics, or that the lecturer was a last resort resource person. Thus, their beliefs only came from their contact with the lecturer in the scheduled class meetings. They expected these to be conducted in such a way that they need not seek out the lecturer at all after class, a pervasive cultural norm. Clearly, motivation was wholly the students' domain of influence and choice. They can choose to participate and act upon the learning opportunities provided as well as respond to any overtures made by the lecturer for personal connection or support. Or, they can choose to ignore or limit their participation.

6.3.1.3 Learning Opportunities

Providing, designing and planning the learning opportunities were my domain of influence and opportunities for intervention. In providing the learning experiences, I have chosen to develop teaching strategies intended to increase students' awareness of their own mathematical thinking powers and to support them in using these powers in classroom work. To support the implementation of the teaching strategies, an active learning environment was chosen. Thus, the teaching also addressed other skills important for students' development such as team work and communication.

Working on Mathematical Thinking

The descriptions of the strategies that were used have been given in the various chapters in the thesis. As the experience in one class helped me to re-evaluate the strategies, modification and changes were made and implemented in the following class. Thus, the descriptions evolved and finally culminated in the story of Block 3 students. However, the strategies are still being carried out, reviewed, evaluated and modified, if and when

appropriate. The workbook itself was first printed under the Sprint Print program by Pearson (Malaysia), has been reviewed, edited and modified and is currently in the last stage of final editing to be printed by Pearson (Malaysia) for wider distribution. The workbook was only one item and as described in Section 2.5, there were other strategies that were used to make students' use their thinking powers.

As students had very little 'say' as to how a lecturer conducted her class, my students were 'coerced' to participate in the way I decide to teach them. In teaching Blocks 1 and 2, I had used the strategies explicitly for the first three topics and reverted to 'traditional' ways for the rest of the topics. Block 3 was taught based on the strategies throughout the semester. The difference was that the classroom tasks were designed to engage the students in thinking about the processes and structures of mathematics. Consequently, as they became involved in the tasks and activities, they became more aware of their own powers and how they could use their understanding of the processes to help in problem solving. The students have had years of successful learning using memorization, drill and practice methods. Thus, they do have very strong powers of identifying patterns, similarities and differences but many were not aware of their powers or the way they could be used more effectively. They also have a strong desire to succeed and in coping with the mathematics in the context of their own engineering subjects thus they also wanted methods of learning that could help them achieve this. Even though there was some resistance to changing they way they learn, many students realised that studying to understand was beneficial and much more effective in coping with solving non-routine problems.

I would like to share the stories of two students that illustrate the phases that they went through before they willingly participated in my lessons as well as showing some changes in the way they worked with the mathematics and some increase in their ability to communicate the mathematics that they were working on. The two students are Baba and Lily whom I taught in the first phase of the research.

Baba's Case:

He was in my Basic Calculus Group A01 (November 2001/02) and usually sat in the front row with a particular friend and they worked together most of the time during lessons. I first noticed him because I heard him speak in Mandarin while doing the mathematical tasks in the classroom. Baba was a Malay name and I found out that he went to a Chinese school, spoke fluent Mandarin and had a mixed Malay-Chinese parentage. He was very polite but during my rounds in class, he has never asked for help. If I sat next to him and queried about the work he was doing, he looked uncomfortable and usually said he was alright or he was getting on in his work. However, my observation of the mathematics he was doing showed that he did not know how to do his problems. He seemed to be learning from his friend, Ming. I could see that Ming was teaching him but they spoke in Mandarin so I could not be sure. He failed in the final examination whilst Ming scored an A.

Baba was in my Calculus II class (Group B02 – semester 1, 2002/03) and he was still working with Ming. In this class, Baba was more responsive and appeared to be more enthusiastic in engaging in the style of learning as he was more willing to talk to me and volunteered answers to the prompts and questions although he was not always clear what I was asking. He worked with Ming and usually got good grades for the group assignments but did not do well in the individual tests. I did offer Baba some extra tuition but he said that he preferred working with his friends. He only scored a D for the subject.

Baba was back in my class but now he was repeating his Basic Calculus (Group C02, Semester 2, 2002/2003). He was still enthusiastic and participated in the class activities. In this class, he was more ready to ask questions and was willing to share his ideas. It also appeared that he did not seem to have any particular friends but managed to do his assignments as paired work. At the end of the semester, I interviewed him using an open and unstructured setting. However, there were certain issues that I had prepared for us to talk about pertaining to his learning and my teaching. When asked to describe his study

techniques, he did not wish to reply but asked to write down the answers instead. In his written reply, he said that his preferred method of study was to work with a 'teacher' or mentor whereby he would like to be taught first and shown how to do the problems. He would then redo them before attempting new problems. He also claimed that he had poor memory retention and thus needed to do a question and redo it 2 to 3 times until he could get it right. His main problem was that, he would forget some of what he had learned to solve during the examinations. He said that he found the names of techniques difficult to remember. Commenting on my teaching, he said that there was a lot of group work in class but that it was time consuming. He said that it was important for 'teaching to be interesting' to maintain students' interest and that a number of easy questions should be given to help students score good results. He also said that for group work, it was not easy to find partners and sometimes it was embarrassing to work with first years as he was a repeat student. I also asked him to answer some questions on differentiation and to explain his working. The table below is a copy of his work.

Questions	Baba's answers	His comments
(i) $y = \sin 3x$	$y' = \text{kos } 3x(3) = 3 \text{ kos } 3x$ (note in Bahasa Malaysia 'cos' is written as 'kos')	<i>"can't remember the name for the formula"</i>
(ii) $y = \sin(3x^2 + 2)$	$\frac{dy}{dx} = \frac{d}{dx} \sin(3x^2 + 2) \frac{d}{dx} (3x^2 + 2)$ $= \text{kos}(3x^2 + 2)(6x + 2)$	<i>"there's brackets, differentiate the outside, differentiate the inside"</i>
(iii) $y = \ln(\sin 3x)$	$\frac{dy}{dx} = \frac{d}{dx} \ln(\sin 3x) \frac{d}{dx} (\sin 3x) \frac{d}{dx} (3x)$ $= \frac{1}{\sin 3x} \text{kos}(3x)(3) = \frac{3 \text{ kos}(3x)}{\sin 3x}$	

When asked to describe what he did, he used the words, *"there's brackets, differentiate the outside, differentiate the inside"*. He apologized for not remembering the names of the formulas because he said that he has not revised for the examinations. He also said that he chose to do Calculus II with me as *"he liked me because I was friendly"*. What transpired

during the interview showed how much Baba has changed in terms of his attempts to explain his work as well as his readiness to share his views.

Lily's Case:

I first Lily met in the Basic Calculus of Group A01, second semester of the academic session, November, 2001/02. She was quiet, uncooperative and usually sat alone in class. My efforts to establish a relationship with her were rebuffed with a polite, "*I'm okay, thank you*". I never saw her doing any of the problems and if I sat next to her and asked her to go through a problem; she would decline and said that she would do them later. I felt like there was an invisible barrier that had been put up. However, her class attendance was good although she failed in the final examination. She was also in my class of Basic Calculus (Group C02, Semester 2, 2002/2003). She was still aloof and generally unresponsive to any overtures made during the lessons in terms of help to do the mathematics.

I met her again as she was in the class of Calculus II of Group D04, Semester 1, 2004/2005. She still kept to herself although she was doing some of the work although poorly but I noticed that she was frequently absent. I met her by coincidence one afternoon at her Faculty and decided to sit with her for a chat. She had missed the morning class again. When asked about her poor attendance, she apologised and told me about various personal problems that she was facing mainly connected to her poor academic performance. She said that she had 'messed' up her first and second year and that she wanted to do better but was afraid that it was too late. She had forgotten many of the concepts in Basic Calculus and felt overwhelmed with the many concepts that needed to be revised. She told me she had got a book on Calculus and that she would try to study them. I offered her extra sessions to review her revision and also review all the lessons that she had missed. We were already in the sixth week but after this session, she used to come to see me for an extra hour every week to work on the mathematics. After a couple of

sessions, she appeared to be in better spirits and became more active in class. She asked questions or offered answers during lessons. She was also participating in her discussion group for the class tasks. She passed the subject for this session.

In Lily's case, my chance encounter and willingness to hear her personal problems was the gateway to mediate and negotiate with Lily to work on her mathematics. In each of the personal sessions, we worked on some of the worksheets that she had missed as well as time was also allocated for some revision of techniques and concepts that were taught in Basic Calculus but were needed to support the work in Calculus II. She had to prepare for these lessons by reading up the necessary mathematical topics. As she became more confident with her work, she was more willing to participate in the class as well as join in class discussion and group work.

I tried to summarize this experience of how students behaved and how they finally became engaged with the mathematical work. There was a cycle of (i) doing the tasks and activities because they had to, (ii) deciding to participate, (iii) getting involved in the tasks and activities, (iv) increasing their awareness and finally (v) increasing their understanding and appreciation of their own powers. These stages were not necessarily sequential as the students move from one to the other at their own choice and at their own time. I have decided to encapsulate the five stages as a non-directional cyclic process and label them as the stages of (i) **Enforced activity**, (ii) **Embracing the tasks and activities**, (iii) **Engaging with the tasks and activities**, (iv) **Increasing their Awareness**, and lastly, (v) **Appreciating their own abilities**.

However, there were some significant factors that determined students' participation and these factors were interconnected between their own motivation and how my interaction with them could help increase their motivation. Rapport and trust between the students and me were important in giving some room to negotiate or mediate with them to at least try to participate during the lessons as well as encourage them to share what their problems or

difficulties with the mathematics were. Not all the students were like this, some were more responsive and decided to participate in the lessons by themselves and they were a small number that did not appreciate the teaching changes although they went through the lessons without too many complaints.

In terms of measuring change, students who had come to appreciate that they had powers to use were considered as those who had changed their attitudes. The measures that were used to evaluate change was mainly based on students' own reflections, their responses in questionnaires and the way they worked on mathematical tasks. I have no way of knowing whether the changes were permanent or will be carried forward to other subjects.

6.3.2 Obstacles to Students Changing

The main obstacle to change was obviously the nature of motivation itself. As I have mentioned above, motivation was in the students' domain of influence. They decided what they would like to do, felt or believed in. Much of my data showed that there was a mismatch of objectives between what I wanted or believed in and what they wanted or believed in. For instance, I believed that understanding mathematical processes and developing thinking skills were important. Students believed that 'drill and practise' was enough for them to get through the course. All they needed was to do a lot of problems so that they could answer the tests and examinations papers. I was thinking of the skills that they might need as engineers, professional traits such as critical thinking, problem solving skills and the ability to work in a team. They only wanted to get through the course, and if they had to work in groups then they would work in groups. Some wanted 'formulas' for solving problems and I quote again, Midah's request, *'give a question, show the method, the formula. This question, this is the technique. This will help me remember the technique'*. Others had personal problems that hindered their progress in my class and needed some pastoral care to help them face these issues. Some students were quite good

but believed that they were not good enough. They set their own standards and against that, they labelled themselves, '*average*' or '*slightly above average*'. Through conversations, they were afraid to say 'good' or 'excellent' just in case they could not sustain their performance in the future.

Throughout my interactions with the different kinds of students, I had used their trust and their feelings of rapport to negotiate changes in their manner of study or the way they work on the mathematics. I will always bring back their attention to the mathematics I was teaching. I needed to get them through the subject. Good students who were already coping very well and did not want to participate in exploring their thinking powers, were coaxed to try out some of the tasks or perhaps try to verbalise their understanding so that they could increase their own awareness of their own skills. Students who always needed answers at the back of the book were persuaded that they could look at the working and discuss with their peers to try to make judgements whether they thought their answers were reasonably right. In the case of Lily, I persuaded her to spend more time revising and reviewing the mathematics with me as a means to catch up on the mathematics so that she could pass her course. I also had made some arrangements for her to get professional counselling. Thus, I used the students' own concerns and their motivation to '**negotiate and mediate**' for more mathematics learning in the environment that I initiated, sustained and established.

6.4 A model of the change process

The study had the objectives of examining both my own and my students' attitudes to Advanced Calculus though working on mathematical thinking. However, I am going to propose a model to describe the changing process from my students' perspectives. Nevertheless, the only way I could initiate changes in my class was by changing as well. Thus, as I worked towards developing strategies to teach my students, I needed to explore my own thinking powers, I needed to understand them and use them.

In the proposed model (Figure 6.1), the change process is depicted as a process in a time continuum with three different phases, Entry phase, Activity phase and Adopted phase. I have also identified the domains of lecturer's and students' influence as well as when both are influential.

Entry Phase – I have identified rapport and trust of the lecturer as elements that students thought were important and they believed affected their motivation. In addition, I have included students' own motivating beliefs as important factors to consider in encouraging them to participate in the learning environment that I designed and planned. I use the word 'plan' quite loosely as I have tried to portray how in a classroom situation, I had to be open to and sometimes to take advantage of opportunities of learning that occurred without planning. In this entry phase, I find myself negotiating and mediating with students to bring them towards the learning of mathematics. Thus, I have used the keywords, 'negotiation' and mediation'. I have drawn two boxes, in red and blue, around the words 'negotiation and mediation' to represent the lecturer's and students' domain of influence. The red box signifies the lecturer's domain of influence in providing learning opportunities to support and enhance students' learning as well as means of intervention to help them develop better thinking skills. The blue box signify students' domain of influence in that they can choose to take action upon my actions. Thus the words, 'rapport', 'trust' and 'motivation' are in blue boxes. The students decide to give trust and reciprocate any overtures that I make to establish a connection with them. However, I decide to make the overtures in the first instance.

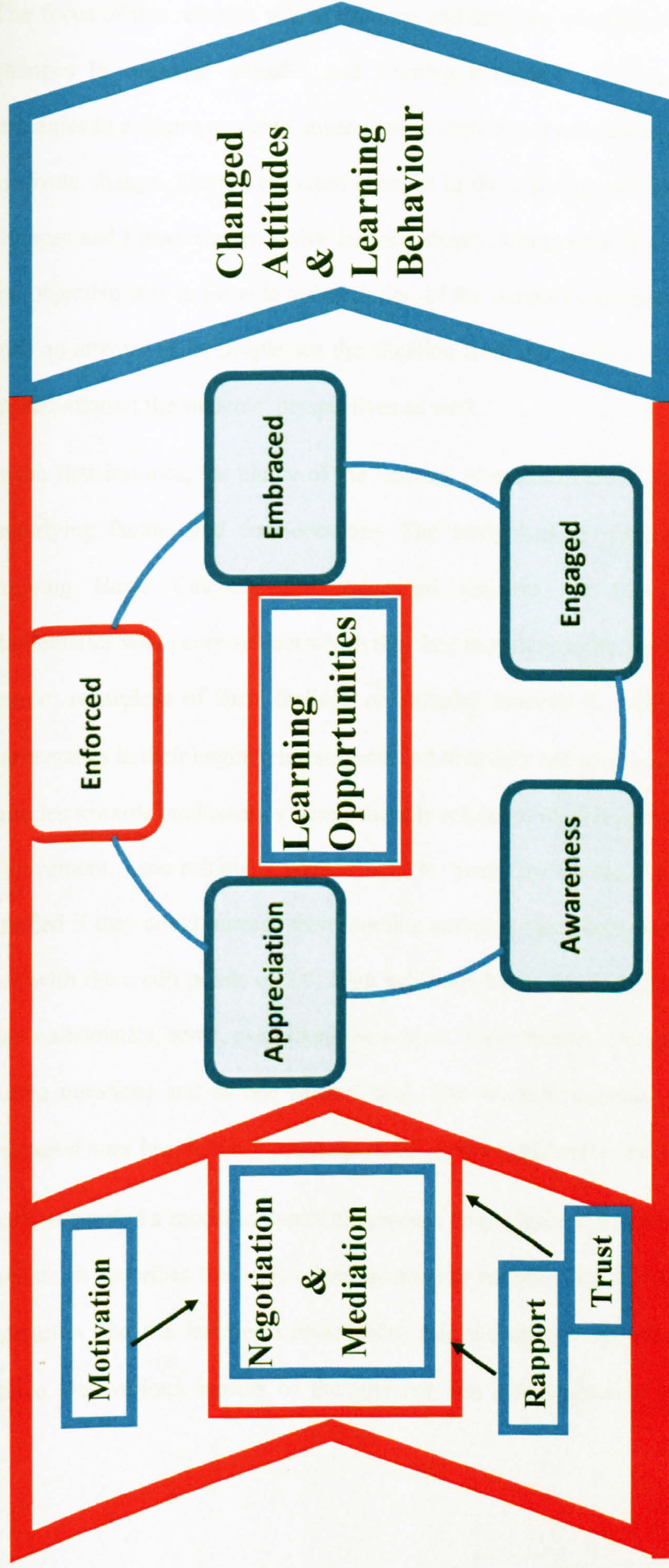
The meaning of the representation of the red and blue boxes is used consistently for the activity and adoption phases too.

Activity Phase – this is where students participate in the class tasks and activities. I have already described the different stages that my students went through. The stages are represented as cyclic but they are not sequential as students move from one stage to the

next based on their responsiveness, perceptiveness and understanding. I have chosen five keywords, enforced, embraced, engaged, awareness and appreciation, to identify the stages. The explanation has been given above. In this phase, my domain of influence is in providing, designing and executing my teaching strategies. I have more influence in the enforcement stage but other than this, the students decide the level of participation and engagement with the tasks and activities. For example, some students appeared to be actively engaged but there was no real commitment to the ideas of learning that I wanted to share.

Adoption Phase – I have chosen some criteria to indicate that they have gained some appreciation of the importance of being aware of their own thinking powers, more articulate in their mathematical communication as well as appreciative of the importance of team work. The first criterion is that they show support for these values in their reflections about working on the mathematics. Secondly, they can communicate their mathematics either verbally or in writing to me and to their peers. Thirdly, they have a better group work ethic and finally, they show an appreciation of the importance of independent learning.

Figure 6.1: Model of the Change Process



The focus of this research was to examine and improve teaching practice as well as study changes in students' attitudes and learning behaviour. I have chosen to implement strategies to enhance students' awareness of their own mathematical thinking as a mean to motivate change. There were other changes in the learning environment to support these changes and I have chosen active learning theory. Using an action research methodology, the objective was to provide a description of the dynamics of changing teaching practice with an attempt to let people see the situation from the teacher's perspective but with an appreciation of the students' perspectives as well.

In the first instance, the nature of the findings is specialised and localised with particular underlying factors and considerations. The study was on engineering undergraduates studying Basic Calculus and Advanced Calculus or Engineering Mathematics. Mathematics was a core subject which they had to pass as a requirement for obtaining their degree, regardless of their feelings or attitudes towards it. The students needed the mathematics in their engineering subjects and thus they had to engage with it. My students' attitudes towards mathematics were generally related to their level of past performance and achievement. Low achievers wanted only to 'pass' the course. Medium achievers were satisfied if they could manage their learning and pass the course with a minimum of a 'C' pass with the credit points of 2.0. High achievers knew they could succeed. They did not hate mathematics, some, even liked the subject. Nevertheless, a majority did not like non-routine questions and as one student said, *"the worked examples are ok, I can do the structured ones but suddenly you jump in the review and further exercises."*

I have suggested a model to depict the process of change that has occurred for most of my students. It describes the roles of the interaction between the lecturer and students, their motivation and the learning opportunities that the teacher provides for the students. I believe that various aspects of the research are independent of location and level of

learning such as the incorporation of strategies to invoke use of mathematical thinking powers, students' active participation and to increase their soft or generic skills in terms of communication, team work and independent learning.

There are similarities with many ideas already forwarded by researchers, some of whom have been included and others who had indirectly influenced the ideas in this research, namely, Vygotsky (1978), Leont'ev (1981), Gattegno (1987) and Brosseau (1997) because their ideas were used by the researchers on whose work I based my choice and design of teaching and learning strategies, tasks and activities. My research highlighted the teacher's perspective and dilemma as she found a myriad of theories and ideas to choose from. As a teacher, I had to choose some of the theories, and organise them within the context of my teaching situation, my difficulties and my students.

CHAPTER 7

CONCLUSION AND RECOMMENDATIONS

7.1 Review of the research objectives

My research was motivated by the objective of changing students' attitudes towards Calculus through implementing teaching strategies to invoke students' use of their own mathematical thinking powers. An action research methodology was adopted with the data presented in two parts. Chapter 4 presents data on students who were taught from November 2001 to November 2004. These were students who had entered UTM with entry qualifications based on entrance policies that were changed in 2002. Thus, another cycle of research was undertaken and Chapter 5 presents data on students who were taught from July 2005 to July 2007. For the sake of discussion, the students were identified in three blocks, Block 1, first and second year Industrial Design students; Block 2, second year Electrical and Civil Engineering students and finally, Block 3 consisting of Electrical Engineering students only.

The objectives of the research are:

- (a) What are the factors that affect students' learning behaviour?**
- (b) What are the students' attitudes and perceptions towards Calculus?**
- (c) What changes are required in the teaching practice?**
- (d) What are suitable strategies to invoke students' use of their own mathematical thinking powers?**
- (e) What are the changes in students' attitudes and learning behaviour?**

In this chapter, I would like to discuss each of the objectives and to what extent has each been achieved.

(a) What are the factors that affect students' learning behaviour?

In the beginning, previous research was reviewed and my own experience was used to provide information about factors that affected these students' learning behaviour. General findings on research of students' difficulties were reviewed as well as more specific research carried out on students in UTM. However, as the research progressed, the findings from new experience were used to augment or modify these assumptions and this process has been identified clearly in the presentation. Various factors were identified such as prior mathematical knowledge, beliefs about mathematics and how to study it, attitudes towards mathematics and Calculus in particular, students' learning styles, difficulties that they had with some mathematical concepts and motivation. However, in identifying the factors, I had focused and emphasised on motivation as an enabling factor to bring about change. At the beginning of my class, students' still held on to beliefs that Calculus was best learned by memorisation of concepts, facts and techniques. The questions that I developed in my teaching strategies, directed their attention to look at the structures and processes of mathematics so as to change this perception. The changes in the teaching practice as described and the underpinning theories that were identified was to invoke students' awareness and increased use of their own thinking powers. The teaching strategies that were drawn up, activities and tasks designed were implemented in the classroom for different groups of students over seven years. Every implementation provided information that was used to review, modify and adapt the teaching acts for the next group of students. The aim was to promote changes in students' attitudes through working on mathematical thinking. This was executed in a student-centred learning environment that also addressed other relevant elements of students' development.

The results have shown that the strategies were successful in promoting change in my attitude as well as that of my students'. In many ways, thinking about mathematics brought about many changes in my perception, my social relationship with my students as well as

knowledge about how my students learn. According to responses from the students, their experiences in following my course have also brought changes to their perceptions and ideas about how to work on mathematics. I had a better relationship with my students and there was better relationship between students to students.

(b) What are the students' attitudes and perceptions towards Calculus?

Information about students' attitudes and perceptions were collected through various means as described in chapters 3, 4 and 5. Generally, students thought that they could master the mathematics by doing a lot of examples and studying worked examples. Memorising concepts and familiarising themselves with the relevant techniques were considered effective ways of getting good results. Thus, the results showed that students displayed attitudes that were consistent with research findings discussed in Chapter 2.

(c) What changes are required in the teaching practice?

The thesis was about changing teaching practice and has described in great detail the journey of change from the lecturer's and students' perspectives. By choosing an action research perspective, I have tried to share the difficulties and the sense of fulfillment and satisfaction in working towards changing my practice and in turn supporting students' change in their ways of working with mathematics. I had identified that working with students to become more aware of their mathematical thinking was a key in increasing their understanding and ability to be more flexible with mathematical ideas and thus will help them in transferring their mathematical knowledge to their own specialist area.

However, this research only described students' learning behaviour within the Calculus class and no research was carried out to seek information about how the same students were relating to the mathematics in the Engineering subjects. As the research was carried out on a part-time basis, I had to focus on my classes and decided that I would add depth to the study. Thus, the strategies were used in a teaching period of seven years to various groups of engineering undergraduates. Changes made to the intake requirements meant that

the students I was teaching came from two very different backgrounds and I had to set apart their stories. Their data was presented in two chapters, Chapter 4 and Chapter 5.

I highlighted in particular my relationship with the students' who had various personal problems where they needed some personal counseling and support in order to focus on their study especially of the mathematics that I taught. The portrayal of the relationship was given in Chapter 4 and although I did make some efforts to provide support, I was not fully comfortable with their '*overwhelming gratitude*'. The occurrences described indicated that more work has to be done to increase students' self-esteem and stress management skills.

I strongly feel that my experience can help other lecturers to understand and prepare themselves to embark on changing teaching practice. To realise that although in turn, it could be difficult and enjoyable, helping our students to become better thinkers is in itself the *raison d'être* for higher education. Thus, change towards such a learning environment where thinking is allowed, encouraged and supported is important and necessary for the future success of my students in Malaysia.

(d) What are suitable strategies to invoke students' use of their own mathematical thinking powers?

The strategies that were used in the research have also been described in the thesis in the relevant chapters. Here, I would like to pick out on the development and production of the workbook (see Section 5.2). An important consideration was the prevalent learning culture at secondary school, high school and matriculation courses where learning was procedural and achievement was only measured in terms of examination results. Thus, my students were consistently worried about their performance in the examinations and they also needed to see the prompts and questions to become familiar with them. However, by using the workbook, there was now a tension between promoting creativity and the structured nature of the workbook. I considered the workbook as a 'transition tool' to encourage

students to engage with the mathematical activities and tasks, to become aware of the use of the prompts and questions towards working on mathematical thinking. Clearly, more work needs to be carried out and strategies to encourage the use of various powers of mathematics for problem solving of open-ended or 'realistic' problems will have to be designed.

(e) What are the changes in students' attitudes and learning behaviour?

The change process has been described and summarised in a model which in the first instance is only relevant to my classroom experience. However, I believe that the process described will be useful to other teachers who face similar circumstances in terms of the teaching and learning of students at tertiary level particularly engineering undergraduates. From observations, I could see that most of the students also displayed better learning behaviour. Using criteria described in Chapter 6, namely, (a) students' reflections showed their awareness of mathematical thinking; (b) they can communicate the mathematics verbally or in written form, (c) adopted independent learning behaviour (less dependence on answers at the back of the book, more self-reading, etc). Thus, the teaching strategies were successful in providing the environment and a medium to influence changes in attitudes and learning.

7.2 Review of the research process

I decided to embark on a formal research undertaking in carrying out improvements to my teaching practice working towards a doctoral degree. By doing so, it enhances my awareness of all the activities that contributed to the preparations and implementation of my teaching. I also became much more sensitive to my students' learning behaviour and gradually became better at noticing and recording what I noticed. Definitely, I will be a better teacher because of the experience. I chose action research methodology because it encapsulates the process of learning that a teacher herself must go through and

acknowledge to be a better teacher. A teacher who is a learner herself will understand and support better learning behaviour in her students. However, in hindsight, there are certain aspects that I would have modified such as:

(1) Observations – I would have co-opted colleagues to observe my teaching and my classes on a periodic basis so as to garner objective evaluation of what was happening in my class much earlier in the research.

(2) Recording of observations – I thought that the use of videos or tape recorders is intrusive as students become more aware of their views being recorded. However, current trends in ‘reality shows’ showed that if the equipment is used consistently, students might forget that they are there. Perhaps, it would have helped to have these forms of records of classroom events to compare with my own notes. I have to admit that I have much influence in deciding what to record and report. Nevertheless, by making explicit the reasons of choice, I hope that it will put the report in appropriate context.

(3) My interest in the development of mathematical thinking has become strengthened that I hope to work on activities to share with other teachers to cater for the whole range of educational levels, primary to tertiary level. However, I have to work harder at preparing and providing opportunities for my own students to experience a wider range of thinking activities towards a greater appreciation of mathematics and problem solving.

7.3 Recommendations

A definite recommendation aimed at UTM is to fully implement teaching of all the mathematics courses with strategies to invoke mathematical thinking and widening the experience of mathematical thinking so as to expose students to many other powers that they have to become aware of which will enhance their problem solving skills. Concerns towards developing students’ critical thinking were compelling enough for the Ministry of Higher Education to identify Critical Thinking as one of the necessary graduate attributes to be adopted by all higher education institutions. As the research was a study of real

practice, my teaching strategies have illustrated how thinking can be incorporated into current practice and have shown that students will benefit and be able to effect some changes in learning behaviour. The strategies not only addressed the cognitive development of mathematics but could deal with some affective factors related to reducing mathematical anxiety and confidence in mathematical skills.

A recurring concern amongst the students was the need for relevancy as they wanted to see where the mathematics was used in their own area of specialisation. A second recommendation is to augment my teaching strategies with Problem-Based Learning or include some sessions of solving real engineering problems within the curriculum so as to increase students' appreciation of the use of the mathematics in the real world.

A third recommendation is that the University's management and administration bodies should give due consideration to these ideas and support implementation of the teaching practice with a firm commitment to changing the assessment practice as well.

REFERENCES AND BIBLIOGRAPHY

1. Abdul Rahman, R., 1993. *A Study of a Mathematics Provision for First Year Engineering Undergraduates with Non-GCE A Level Entrance Qualifications*, Unpublished MPil. Thesis, University of Loughbrough, UK.
2. Adelman, C., Jenkins, D. & Kemmis, S. 1983. "Rethinking Case Study: Notes from the Second Cambridge Conference", in *Case Study: An Overview. Case Study Methods 1 (series)*, Victoria: Deakin University Press.
3. Ainley, J., 1999. "Who are you today? Complementary and conflicting roles in school-based research." *For the Learning of Mathematics*, 19, 1, p 39-47.
4. Anthony, (2000). Factors influencing first year students' success in mathematics. *Int Jnl of Math Edn in Sc & Tech*, Vol. 31 (1), 3-14.
5. Artigue & Ervynck, (eds) 1993, Proceedings Of Working Group 3 On Students' Difficulties In Calculus, ICME-7, Canada.
6. Artigue, M., 1991. Analysis. In Tall (ed), *Advanced Mathematical Thinking*, Kluwer Academic Publishers, Dordrecht
7. Baharun, S., Mohd. Yusof, Y., & Abdul Rahman, R., 2007, Facilitating Thinking and Communication in Mathematics, DG 24, presented at ICME 11, Mexico.
8. Bajpai, A.C. & James, D. J., 1985. Mathematical Education for Engineers - a Future Perspective. *European Jnl for Engineering Education*, Vo. 10, 277-83.
9. Ball, S.J., 1985. Participant Observation with Pupils. In Burgess, R.G. (ed), *Strategies of Educational Research: Qualitative Methods*. The Falmer Press, London.
10. Becker, H. S., 1968. "Social observation and social case studies", in *International Encyclopaedia of the Social Sciences*, Vol. 11, New York: Crowell.

11. Bennet, A. A., 1977. Concerning the Function Concept. In Grinstein, L. S. and Michaels, B. (eds), *Calculus. Readings from the Mathematics Teacher*, NCTM, Virginia.
12. Bishop, A., 1985. The social construction of meaning - A significant development for Mathematics Education, *For the Learning of Mathematics*, 5 (1), 24-28.
13. Bloom, B. S., 1956. *Taxonomy of Educational Objectives. Handbook 1: Cognitive Domain*. David McKay, New York.
14. Bogdan, R.C. & Biklen, 1982. *Qualitative Research for Education. An Introduction to Theory and Methods*. Allyn & Bacon, Inc., Boston.
15. Brosseau, G., 1997. *Theory of Didactical Situations in Mathematics: didactiques des mathématiques*, 1970-1990 in Balacheff, N., Cooper, M., Sutherland, R. and Warfield, V. (eds.), Dordrecht, Kluwer.
16. Brown, M. 1996. Planning and Change: The Michigan Calculus Project. In Roberts, A. W. (ed) *Calculus. The Dynamics of Change*, MAA.
17. Brown S., Collins A., & Duguid P., 1989. Situated cognition and the culture of learning. *Educational Researcher*, 18 (1), 32-41.
18. Burgess, R. G., 1982. "Approaches to Field Research". In Burgess, R. G. (ed), *Field Research: A Sourcebook and Field Manual* Boston: George Allen & Unwin.
19. Burton, L. (1984), Mathematical Thinking: the Struggle for Meaning, *Journal for Research in Mathematics Education*, vol 15, No 1, 35-49.
20. Cocking, R. R. & Chipman, S., 1988. Conceptual issues related to mathematics achievement of language minority children. In Cocking, R. R & Mestre, J. P. (eds.), *Linguistic and cultural influences on learning mathematics*, p 17-46, Hillsdale, NJ: Lawrence Erlbaum Associates.
21. Cornu, B., 1991. Limits. In Tall (ed), *Advanced Mathematical Thinking*, Kluwer Academic Publishers, Dordrecht

22. Davis, R. B. & Vinner, S., 1986. The Notion of Limit: Some Seemingly Unavoidable Misconception Stages. *Journal of Mathematical Behaviour*, 5 (3), 281-303.
23. Dewey, J., 1933. *How We Think: A Restatement of the Relation of Reflective Thinking to the Educative Process*. D.C. Heath and Co., London
24. Dick, B., 2000. Postgraduate programs using action research, In Zuber-Skeritt, O. (ed), Action learning, action research and process management: theory, practice, praxis. Brisbane, Australia: Action Research Unit, Faculty of Education, Griffith University.
25. Douady, R., 1986. 'Jeu de Cadres et Dialectique Outil-Objet', *Recherches en Didactique des Mathématiques*, 7 (2), p 5-32. Cited in Robert & Schwarzenberger, 1991, Research in Teaching and Learning Mathematics at an Advanced Level. In Tall, D. (ed.), Advanced Mathematical Thinking, Kluwer Academic Publishers, Dordrecht.
26. Driscoll, M. P., 1994. Psychology of learning for instruction, Allyn and Bacon, Boston.
27. Dubinsky, E., 1986. Teaching mathematical induction, *The Journal of Mathematical Behaviour*, 5, p 305-317.
28. Dubinsky, E. & McDonald, M.A., 1998. APOS: One Constructivist theory of Learning in Undergraduate Mathematics Education Research. *Pre-proceedings of the ICMI Study Conference*, Singapore, 8-12 Dec.
29. Dubinsky, E. (1991). *Reflective Abstraction in Advanced Mathematical Thinking*. In Tall, (Ed), Advanced Mathematical Thinking, pp 95 – 123, Kluwer Academic Publishers, Dordrecht.
30. Edwards 2000
31. Eisenberg, T., 1991. Functions and Associated Learning Difficulties. In In Tall (ed), *Advanced Mathematical Thinking*, Kluwer Academic Publishers, Dordrecht

32. Faridah Abd. Rahman, 1996, *Satu Analisis Prestasi Pelajar Tahun Satu Dalam Matapelajaran Kalkulus (An Analysis of First Year Students' Achievement in Calculus)*. Unpublished final year project report, UTM.
33. Gattegno, C., 1987. *The Science of Education, Part 1: Theoretical considerations*, New York, Educational Solutions.
34. Glaser, B. G. & Strauss, A. L., 1967. *The Discovery of Grounded Theory: strategies for qualitative research*. London: Weidenfeld & Nicolson.
35. Gold, R., 1958. "Roles in sociological field observation", *Social Forces*, Vol. 36, No. 3, p 217-223.
36. Gonzalez-Leon, E. 1980. Remedial Work in Mathematics for Students Entering Engineering Courses at Universities, *Int. Jnl. Of Math. Educ. In Sc. & Tech.*, Vol. 11, 81-89.
37. Gray, E. M., 1991. An Analysis of Diverging Approaches to Simple Arithmetic: Preference and its Consequences, *Educational Studies in Mathematics*.
38. Gray, E. & Tall, D.O. (1994). Duality, Ambiguity and Flexibility: A Proceptual View of Simple Arithmetic, *The Journal for Research in Mathematics Education*, 26 (2), 115– 141.
39. Gray, E. M. & Tall, D., 2001. Duality, Ambiguity and Flexibility in Successful Mathematical Thinking, *Proceedings of PME 15*, Assisi, 2, p72-79.
40. Griffiths, M. & Tann, S., 1992. Using Reflective Practice to Link Personal and Public Theories, *Journal of Education for Teaching* 18 (1).
41. Guba, E. G. & Lincoln, Y. S., 1981. *Effective Evaluation*. San Francisco: Josey-Bass.
42. Harel, G., 2000. Advanced Mathematical Thinking across the Grades. Paper presented at the 22nd Annual meeting of the PME – North American Chapter, Tucson, AZ.

43. Heid, K., 1984. *Resequencing Skills and Concepts in Applied Calculus through the Use of the Computer as a Tool*, Unpublished PhD Thesis, Pennsylvania State University.
44. Heid, M. K. Harel, G., Ferrini-Mundy, M. & Graham, K., 2000. Advanced Mathematical Thinking: Implications of Various Perspectives on Advanced Mathematical Thinking for Mathematics Education Reform. In Fernandez, M. L. (ed.), *Proceedings of the 22nd Annual meeting of the PME – North American Chapter*, Vol 1, p 33-37, Columbus.
45. Herr, K. & Anderson, G. L., 2005. *The Action Research Dissertation*, Sage Publications, London.
46. Howson, et al, 1988. *Mathematics as a Service Subject*, ICMI Study Series. Cambridge Univ. Press, UK.
47. Karst, O. J., 1977. The Limit. In Grinstein, L. S. and Michaels, B. (eds), *Calculus. Readings from the Mathematics Teacher*, NCTM, Virginia.
48. Katz, Leon, 1986. Differentials: a versatile notation for derivatives. *Int. Jnl. of Mathematical Education in Science and Technology*, vol. 17, no. 1, p 41-46.
49. Kemmis, S. & McTaggart, R. 1982 and 1998. *The Action Research Planner*. Geelong, Victoria: Deakin University Press.
50. Kenny, W.R. & Grotelueschen, A. D., 1980. *Making the Case for Case Study*. Occasional Paper, Office for the Study of Continuing Professional Education, Univ. of Illinois.
51. Keynes & Olson, 2000. Redesigning the Calculus Sequence at a Research University: Issues, Implementation and Objectives. *Int Jnl of Math Edn in Sc & Tech*, Vol. 31 (1), 71- 82.
52. Kurz, G., 1985. Remedial Courses in Mathematics - Scopes and Problems - a Survey for the Federal Republic of Germany. *Int Jnl of Math Edn in Sc & Tech*, Vol. 16, 211-225

53. Leder, G. C. & Forgasz, H. J., 1992. Perspectives on Learning, Teaching and Assessment. In Leder, G. (ed) *Assessment and Learning of Mathematics*, The Australian Council for Educational Research Ltd, Victoria, Australia, 1-23.
54. Leont'ev, A., 1981. 'The problem of activity in psychology'. In Wertsch, J. (ed.) *The Concept of Activity in Soviet Psychology*, Sharpe, Armonk.
55. Liew Su Tim & Wan Muhamad Saridan, 1991, Ke Arah Memahami Dan Mengurangkan Kesukaran Dalam Pembelajaran Matematik (Towards Understanding and Minimising Students' Diffciulties in Mathematics Learning). *Berita Matematik*, no. 38, 22-29
56. Lim, C. S., 2000. A Comparison between Malaysian and United Kingdom Teachers' and Students' Images of Mathematics. *Proceedings of the 24th Conference of the International Group for the PME*, Japan, July 23-27, Vol. 3, 223-230.
57. Maselan Ali, 1996. *Symbolic Manipulation Related To Certain Aspects Such As Interpretation Of Graphs*. Unpublished PhD thesis. University of Warwick.
58. Mason, J. H., 1988. *Learning and Doing Mathematics*. The Open University, UK
59. Mason, J. H., 1999. *Learning and Doing Mathematics*. QED, UK.
60. Mason, J. H., 1998. H581. *Guide to Teaching Mathematics D2*. Open University, Milton Keynes, UK.
61. Mason, J. H, 2000. Asking Mathematical Questions Mathematically. *Int Jnl of Math Edn in Sc & Tech.*, Vol. 31, No. 1, 97-111.
62. Mason, J. H., 2002 (a). *Mathematics Teaching Practice: Guide for university and college lecturers* . Horwood Publishing in association with The Open University, UK
63. Mason, J. H., 2002 (b). *Researching your own Practice. The Discipline of Noticing*. Routledge and Falmer, London and New York.

64. Mason, J., Burton, L. & Stacey, K., 1982. *Thinking Mathematically*. Addison-Wesley Publishing Company, Inc, Wokingham, England.
65. Mason, J. H & Johnston-Wilder, S, 2004. *Designing and Using Mathematical Tasks*. Open University, Milton Keynes, UK.
66. McDonald, B. & Walker, R., 1977. "Case Study and the Social Philosophy of Educational Research". In Hamilton, D. et al (eds.) *Beyond the Numbers Game*, London: Macmillan Education.
67. McNiff, J., Lomax, P. & Whitehead, J., 1996. *You and Your Action Research Project*, Routledge & Hyde Publications, London & New York.
68. Md. Nor Bakar, 1991. *What Do Students Learn About Functions? A Cross Cultural Study in England and Malaysia*. Unpublished PhD thesis. University of Warwick
69. Merriam, S. B., 1988. *Case Study Research in Education. A Qualitative Approach*. Josey-Bass, San Francisco.
70. Meyers, C. & Jones, T.B., 1993. *Promoting Active Learning: Strategies or the College Classroom*. San Francisco: Jossey-Bass Publishers.
71. Miles, M. B., & Huberman, A.M, 1994. *Qualitative Data Analysis*, 2nd edition, sage Publications, London.
72. Mohd. Yusof , Y. & Tall, D., 1999. *Changing Attitudes to University Mathematics Through Problem Solving*. Educational Studies in Mathematics, 37: 67-82
73. Mohd. Yusof, Y., 1995. *Thinking Mathematically: A Framework for Developing Positive Attitudes amongst Undergraduates*. Unpublished PhD Thesis, Univ. of Warwick, UK.
74. Mohd. Yusof, Y. & Abdul Rahman, R., 1997 (a). Undergraduate Mathematics Education: Towards an Innovative Classroom Practice, Presented at 21st Int. Conf. for the Psychology of Maths. Education (PME), Lahti, Finland
75. Mohd. Yusof, Y. & Abdul Rahman, R., 1997 (b). An Assessment of the Modular Approach in Teaching and Learning First Year Calculus at UTM, Presented at

Conference on Science and Technology Education, UTM, Kuala Lumpur, 15 – 16 December.

76. Mohd. Yusof, Y. & Abdul Rahman, R., 1998. A Decade of Mathematics Education: A Review and a Critical Appraisal of UTM's Experience, pre-conference proceedings, ICMI Study Conference, NIE Singapore, 8 - 12 December.
77. Mohd. Yusof, Y. et al, 1999. Overcoming Mathematical Learning Difficulties: A Case Study of Collaborative Research, 8th Southeast Asian Conference on Mathematics Education (SEACME-8), Ateneo de Manila University, 30th May – 4th June.
78. Morgan, A.T., 1988. *A Study of Difficulties Experienced by Engineering Students in Higher Education with Mathematics and Related Subjects and their Relevance to the Structure of Mathematical Ability*. Unpublished PhD Thesis, Brunel Univeristy, UK.
79. Norman, F. A., & Prichard, M. K., 1994. Cognitive Obstacles to the Learning of Calculus: A Kruketskiiian Perspective. In Kaput, J. J. & Dubinsky, E. (eds.) *Research Issues in Undergraduate Mathematics Learning. Preliminary Analyses and Results*, MAA Notes No 33, MAA.
80. OECD, 1966. The Mathematical Education of Engineers, Report of the Seminar, Paris
81. O' Shea & Senechal, 1993. Student Learning Difficulties and Calculus in Context. In Artigue & Ervynck, (eds) 1993, *Proceedings Of Working Group 3 On Students' Difficulties In Calculus*, ICME-7, Canada.
82. Prestage, S. & Perks, P., 2001. Adapting and Extending Secondary Mathematics Activities, New Tasks for Old, David Fulton Publishers Ltd, London.
83. Powney, J. & Watts, M. 1987. Interviewing in Educational Research. Routledge & Kegan Paul, London.

84. Polya, G., 1981, *Mathematical Discovery*, Combined edition, 3rd edition, John Wiley & Sons, New York.
85. Rasmussen, C.L., Zandieh, M., King, K. D., & Teppo, A. 2000. *Advanced Mathematical Thinking: Aspects of Students' Mathematical Activity*. In Fernandez, M. L. (ed.), *Proceedings of the 22nd Annual meeting of the PME – North American Chapter*, Vol 1, Columbus.
86. Robert, A. & Schwarzenberger, R., 1991. *Research in Teaching and Learning Mathematics at an Advanced Level*. In Tall (ed), *Advanced Mathematical Thinking*, Kluwer Academic Publishers, Dordrecht
87. Roberts, A. W. et al, (eds) 1996, *Calculus. The Dynamics of Change*, MAA.
88. Roselainy Abdul Rahman, et al, 2001. Unpublished Research Report (Vot 71162): *Alternative Strategies in the Teaching of Mathematics at Tertiary Level*, Universiti Teknologi Malaysia.
89. Roselainy Abdul Rahman, 2001. *Guidelines to integrate teaching strategies to invoke students' mathematical thinking in Basic Calculus*. Unpublished Sabbatical Report, Universiti Teknologi Malaysia.
90. Roselainy A. Rahman & Yudariah M. Yusof, 2000. A Naturalistic Model of Curriculum Development: A Study of Mathematics Education in UTM, National Conference on Teaching and Learning in Higher Education, Alor Star, 9-10 Oct.
91. Ross, S. C., 1996. In Roberts, A. W., et al (eds) 1996, *Calculus. The Dynamics of Change*, MAA.
92. Schoenfeld, A. H., 1985. *Mathematical Problem Solving*, Academic Press, Inc., Orlando.
93. Schoenfeld, A. H., 1989. Explorations of Students' Mathematical Beliefs and Behavior, *Jnl. For Research in Mathematics Education*, Vol. 20, No. 4, 338-355.
94. Schön, D., 1983. *The reflective practioner: How professionals think in action*. New York: Basic Books.

95. Selden, J., Mason, A., & Selden, A., 1994. Even Good Calculus Students Can't Solve Non-routine Problems. In Kaput, J. and Dubinsky, E. (eds), *Research Issues in Undergraduate Mathematics Learning*, MAA, 3, 19-26.
96. Simons, F., 1988. Teaching First Year Students. In Howson et al (eds). *Mathematics as a Service Subject*. ICM Study series, Cambridge University Press, UK.
97. Skemp, R., 1987. *The Psychology of Learning Mathematics*. Lawrence Erlbaum, Mahwah, NJ.
98. Skemp R. R., 1993. *The Psychology of Learning Mathematics*. Penguin, London.
99. Smith, B. J. 1979. *An Identification and Alleviation of the Mathematical Difficulties of First Year Engineering Undergraduates at Plymouth Polytechnic*. Unpublished Mphil. Thesis, Plymouth Polytechnic.
100. Schwarz, B., Hershkowitz, R. and Dreyfus, T., 2002. Abstraction in Context: Construction and Consolidation of Knowledge Structures. In Cockburn, A., Nardi, E. (Eds.) *Proceedings of the 26th Conference of the International Group for the Psychology of Mathematics Education*, Norwich, UEA. Vol. 1, 120-125.
101. Tall, D., 1986. *Graphic Calculus I, II, III, (BBC Compatible software)*, Glentop Press, London.
102. Tall, D., 1990. Inconsistencies in the Learning of Calculus and Analysis, *Focus*, 12 (3 & 4), p 49-63.
103. Tall, D., 1991. The Psychology of Mathematical Thinking. In Tall, D. (ed.), *Advanced Mathematical Thinking*, Kluwer Academic Publishers, Dordrecht.
104. Tall, D., 1993; The Plenary Address. In Artigue & Ervynck (eds) *Proceedings Of Working Group 3 On Students' Difficulties In Calculus*, ICME-7, Canada.
105. Tall, D., 1995. Cognitive Growth in Elementary and Advanced Mathematical Thinking, Plenary Lecture, Conf. of International Group of PME, Recife, Brazil, Vol. 1, p 161-175.

106. Tall, D., 1996. Functions and Calculus. In Bishop, A.J, Clements, K, Keitel, C, Kilpatrick, J & Laborde, C. (eds), *International Handbook of Mathematics Education*, Kluwer Academic Publishers, Dordrecht, 289-325.
107. Tall, D. O. & Vinner, S., 1981. Concept Image and Concept Definition in Mathematics with Particular Reference to Limits and Continuity, *Educational studies in Mathematics*, 12 (2), 151-169.
108. Tall, D. & Razali, M. R., 1993. Diagnosing Students' difficulties in Learning Mathematics, *Int Jnl of Math Edn in Sc & Tech*, Vol. 24, No. 2, 209-222.
109. Vygotsky, L., 1978. Mind in society: the development of the higher psychological processes, London, Harvard University Press.
110. Watson, A. & Mason, J., 1998. *Questions and Prompts for Mathematical Thinking*. ATM, Derby.
111. Wong, D., 1995. "Challenges confronting the researcher/teacher: conflicts of purpose and conduct", *Educational Researcher*, 24 (3), p 22-28.
112. Woods, P., 1986. Inside Schools. Ethnography in Educational Research. Routledge Kegan Paul, London.
113. Wood, D. Bruner, J. & Ross, G., 1976. The Role of Tutoring in Problem Solving, *J. Child Psychology*, 17, p89-100.
114. Yudariah Mohd. Yusof & Roselainy A. Rahman, 2001. Mathematics Education in UTM: Learning from Experience, *Jurnal Teknologi*, Universiti Teknologi.Malaysia, Johor.

APPENDICES

APPENDIX A:	PRE-CALCULUS TEST
APPENDIX B:	REVISION WORKSHEETS FOR FUNCTIONS, DOMAINS, RANGE AND GRAPHS
APPENDIX C:	EVALUATION QUESTIONNAIRE

APPENDIX A: PRE-CALCULUS TEST (Translated from Bahasa Malaysia)

Section A

Answer all questions. Circle the right answer.

1. This expression $(2x - 3)(2 - x)$ is equivalent to

(A) $-2x^2 - 7x + 6$

(B) $2x^2 + 7x - 6$

(C) $2x^2 - 7x + 6$

(D) $-2x^2 + 7x - 6$

2. Simplify $\frac{5}{4p} + \frac{7}{20p}$

(A) $\frac{12}{24p}$

(B) $\frac{8}{5p}$

(C) $\frac{128}{80p}$

(D) $\frac{32}{5p}$

3. Simplify $\frac{(m-1)}{4} \div (m^2 - 1)$

(A) $\frac{(m-1)}{4m^2 - 1}$

(B) $\frac{1}{4m + 4}$

(C) $\frac{m^3 - m^2 - m + 1}{4}$

(D) $\frac{(m-1)}{4(m+1)}$

4. Given $T = 2\pi\sqrt{\frac{l}{g}}$, then l in terms of π, T and g is

(A) $\frac{T^2 g}{4\pi^2}$

(B) $\frac{4T^2 \pi^2}{g}$

(C) $\frac{4\pi^2}{T^2 g}$

(D) $\frac{T^2 g}{2\pi^2}$

5. Given $p = \frac{2m-1}{m+2}$, then m in terms of p is

(A) $\frac{-2p-1}{2-p}$

(B) $\frac{2p+1}{p-2}$

(C) $\frac{2p+1}{2-p}$

(D) $\frac{2p-1}{2-p}$

6. Simplify $(p^2q^{-3}r^{-1})^2(q^3r)^3$

(A) $r^3p^4q^3$

(B) $r^2p^4q^3$

(C) rp^4q^3

(D) rp^3q^4

7. Find x given $x^3 = -125$

(A) 25

(B) No answer

(C) 5

(D) -5

8. Given $8 - 2x \geq 10$ and x is an integer. List all the elements of x

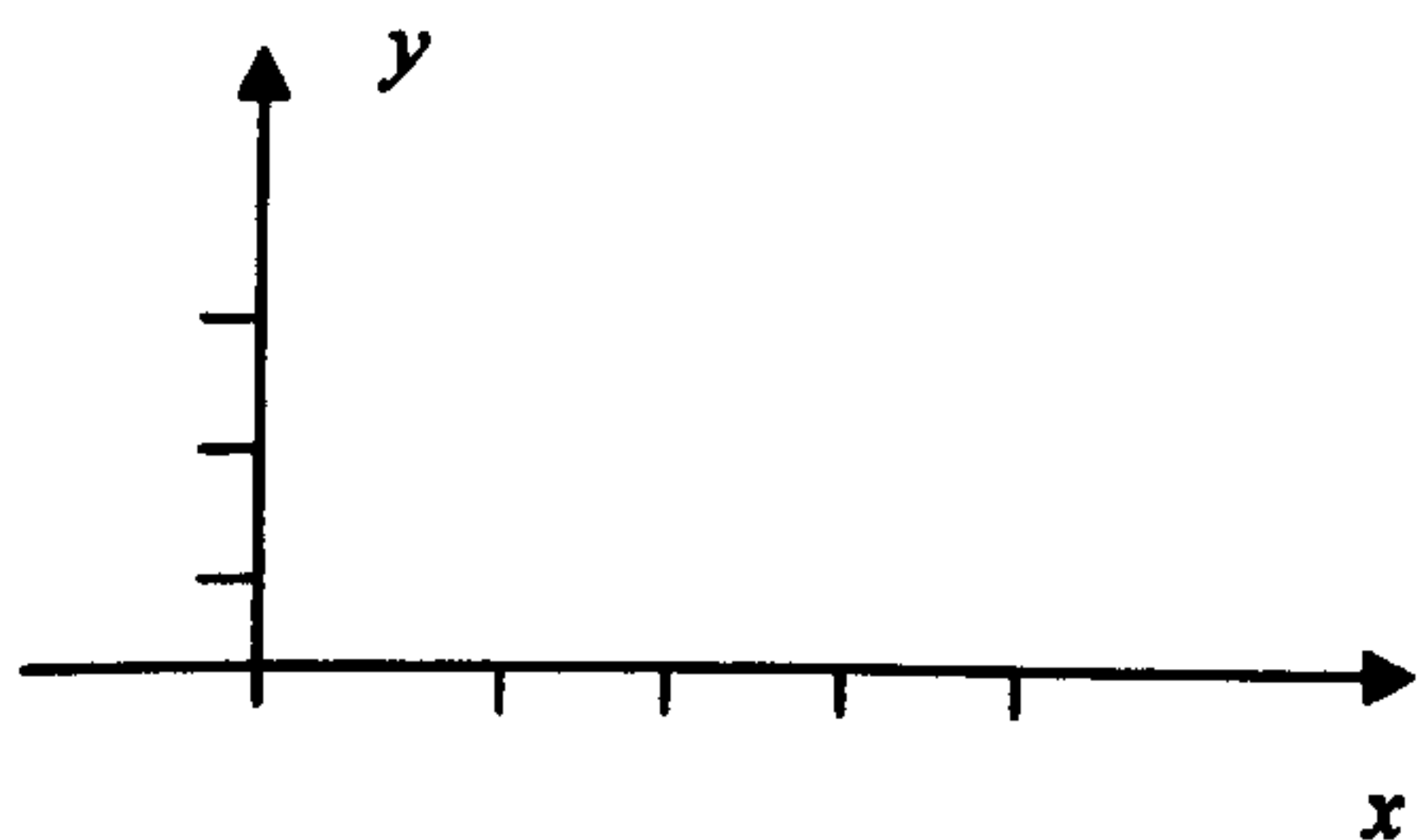
(A) $\{\dots -4, -3, -2, -1, 0\}$

(B) $\{\dots -4, -3, -2, -1\}$

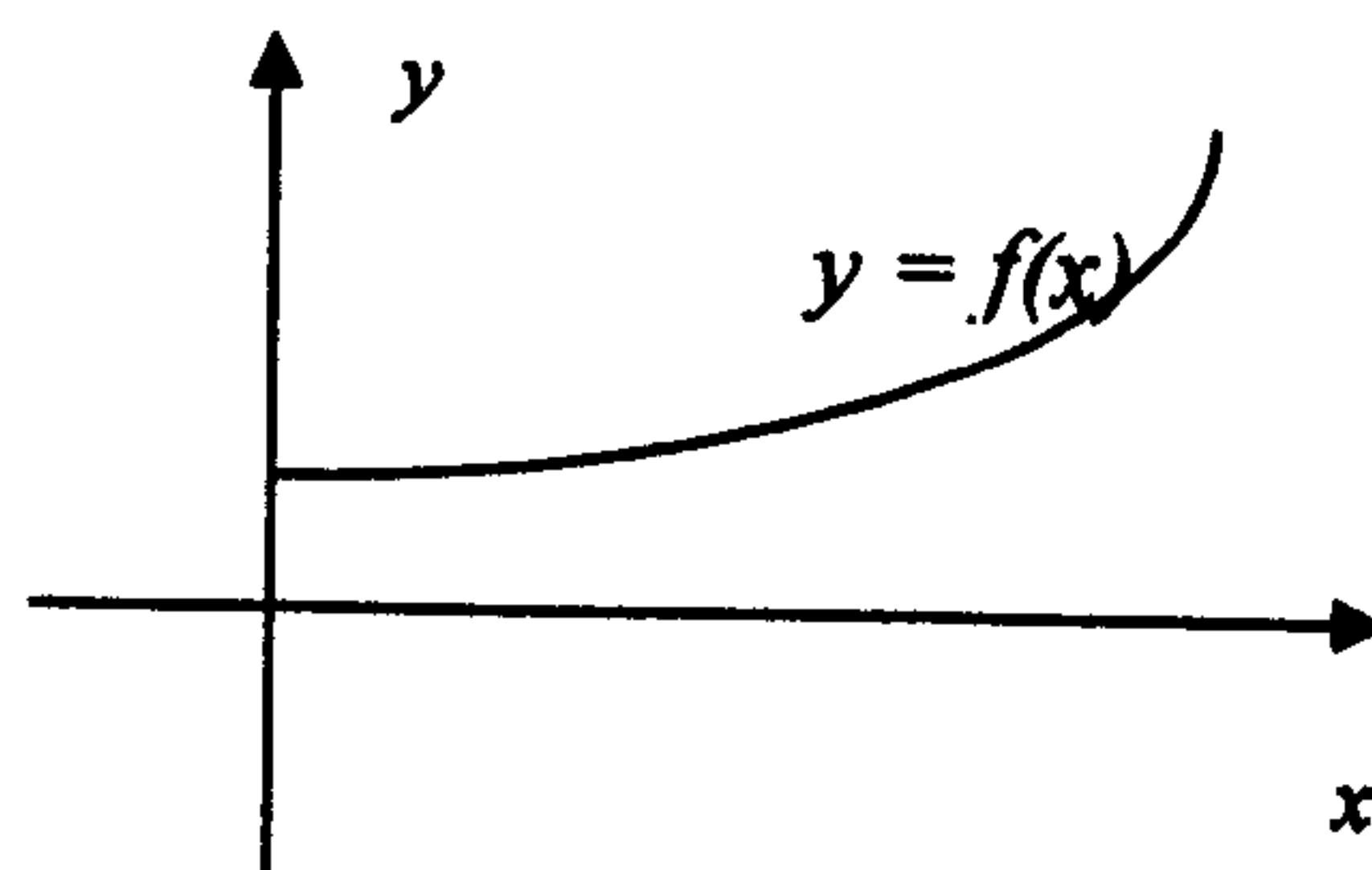
(C) $\{-1, 0, 1, 2, 3, \dots\}$

(D) $\{-4, -3, -2, -1, \dots\}$

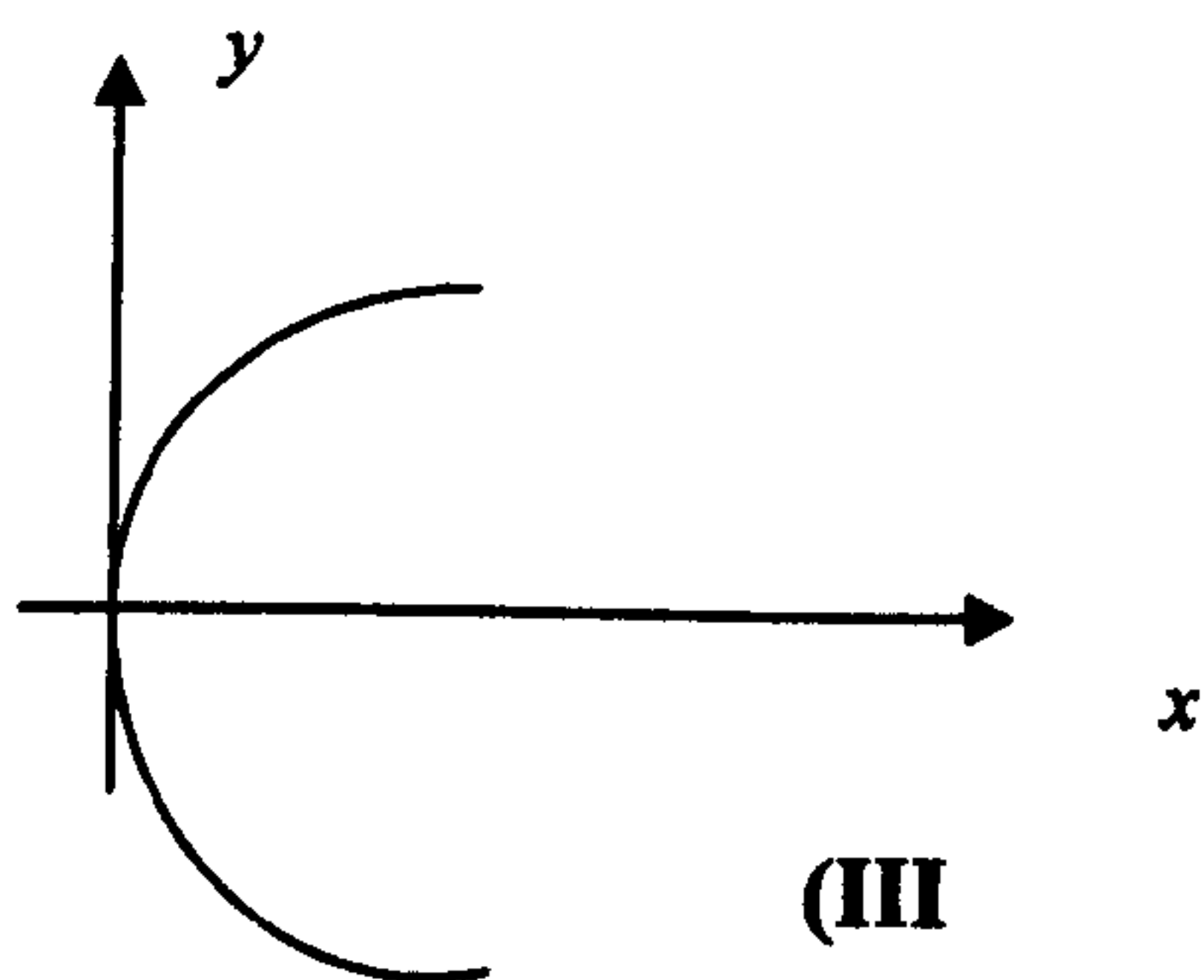
9. Which of the graphs below depicts a function?



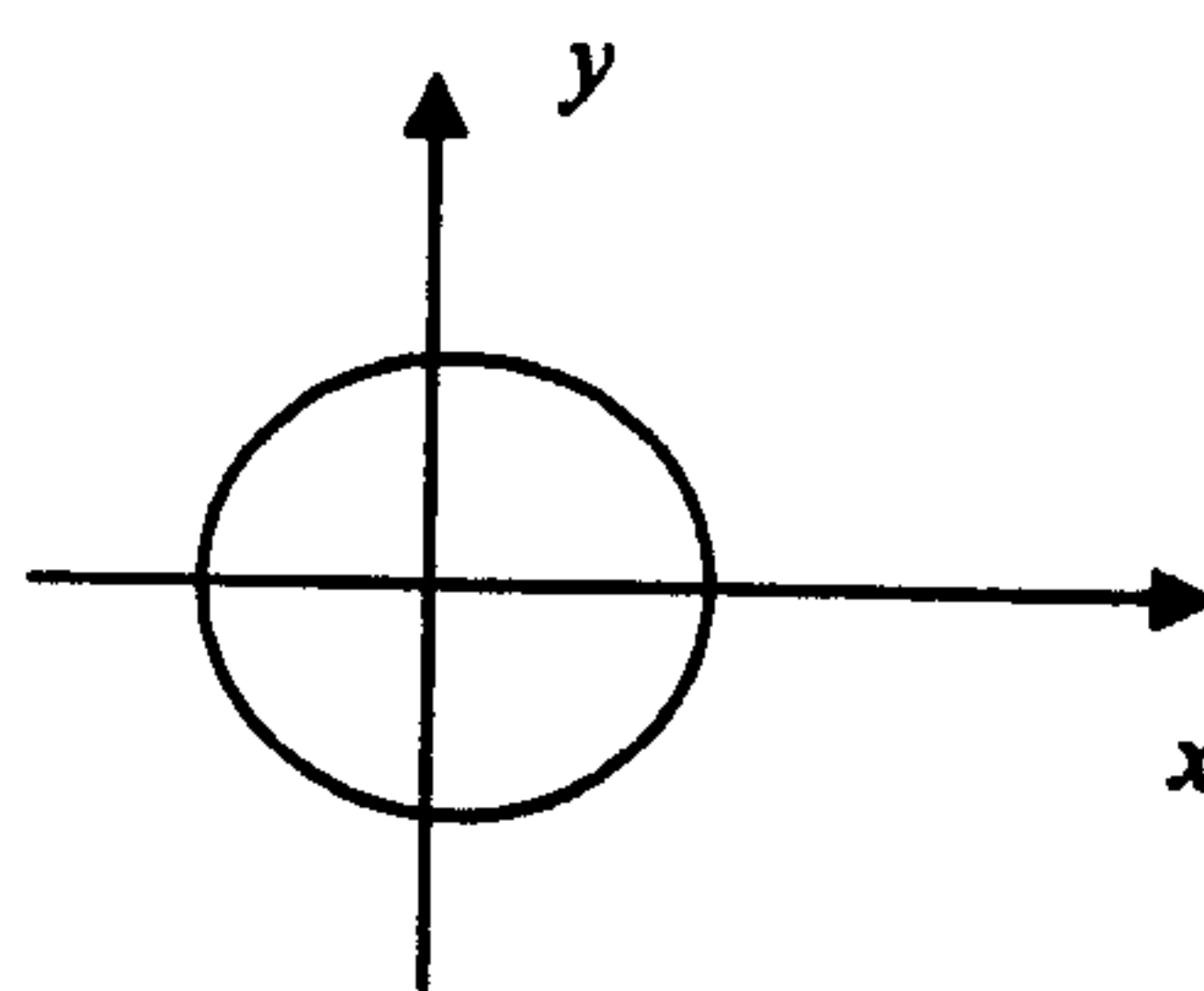
(I)



(II)



(III)



(IV)

(A) II, III and IV

(B) II only

(C) I and II

(D) Don't know

APPENDIX B: Revision Worksheets for Functions, domains, range and graphs.**(Translated from Bahasa Malaysia)****FUNCTIONS: DOMAIN, RANGE AND GRAPHS****1. Find the domains of the following functions:**

(a) $f(x) = 1 + x^2$

(b) $f(x) = 1 - \sqrt{x}$

(c) $F(t) = \frac{1}{\sqrt{t}}$

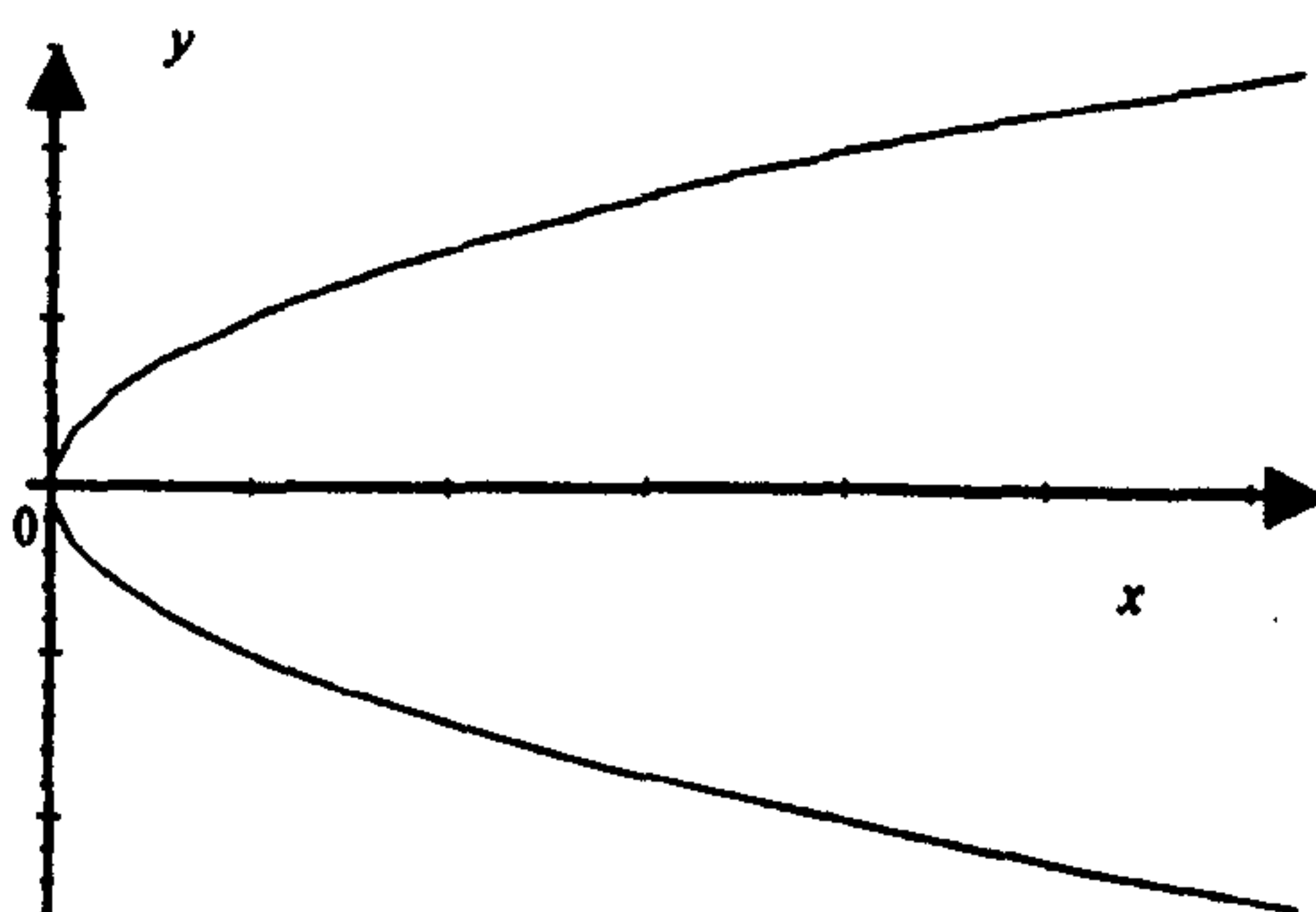
(d) $F(t) = \frac{1}{1 + \sqrt{t}}$

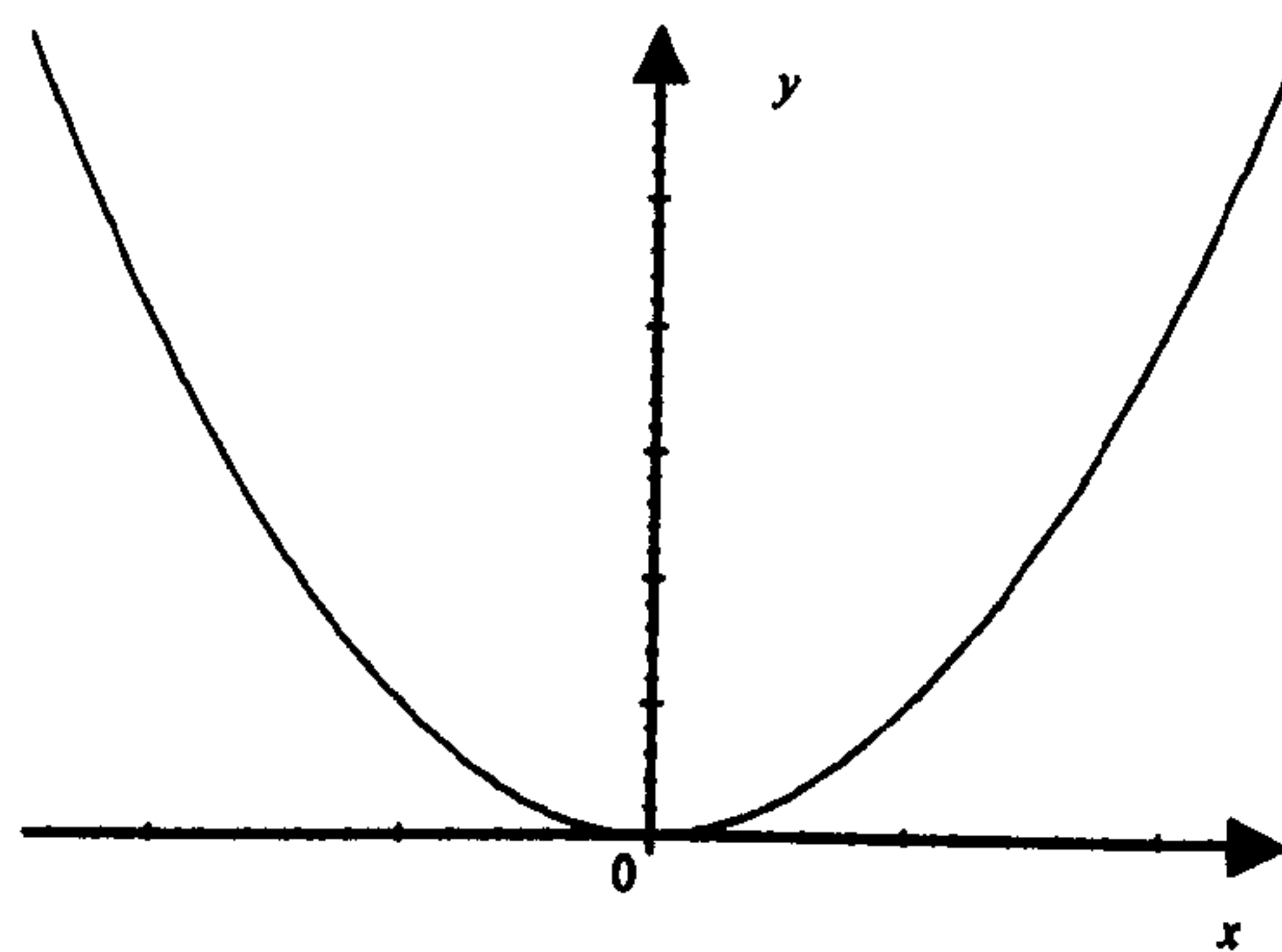
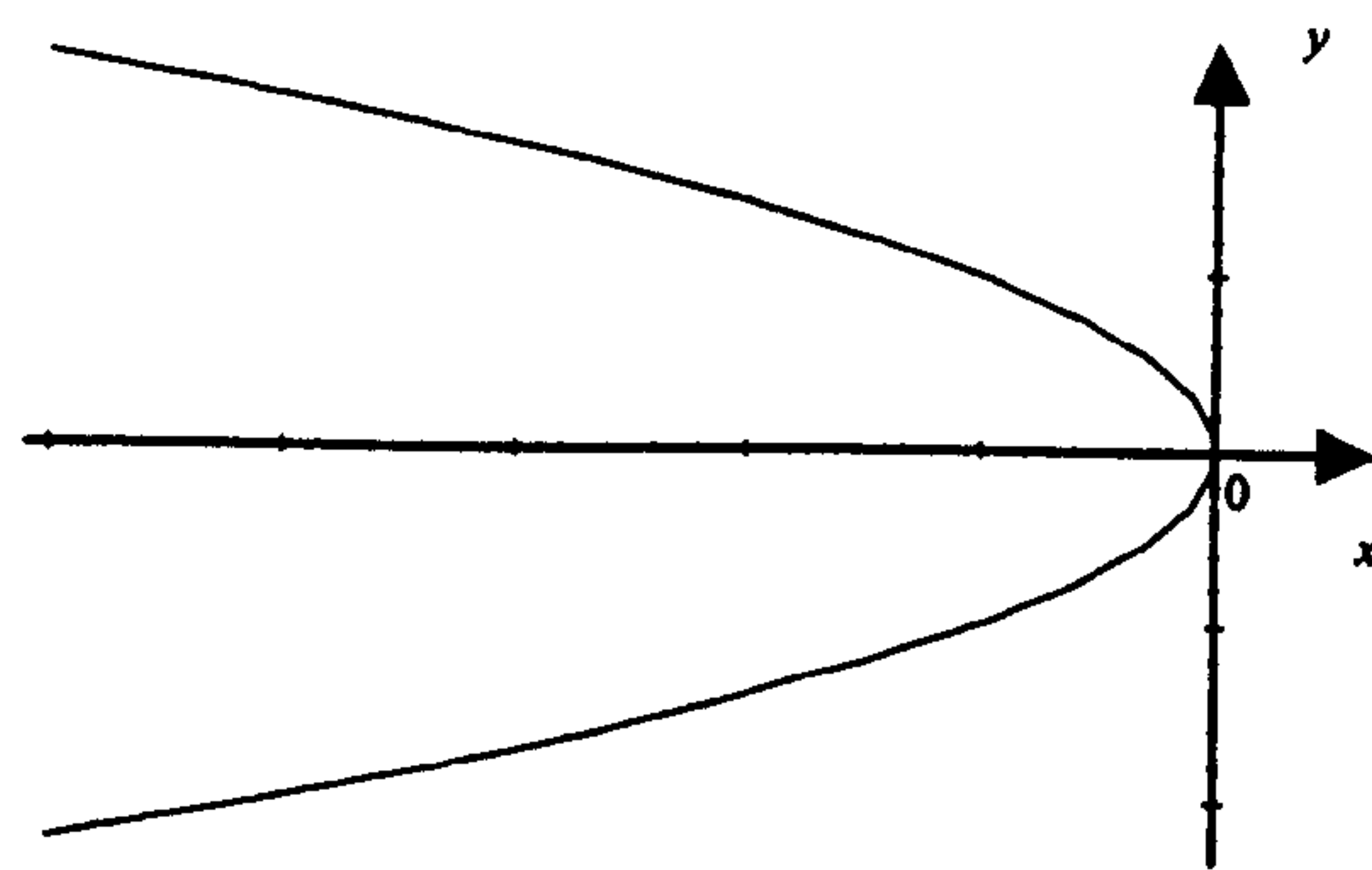
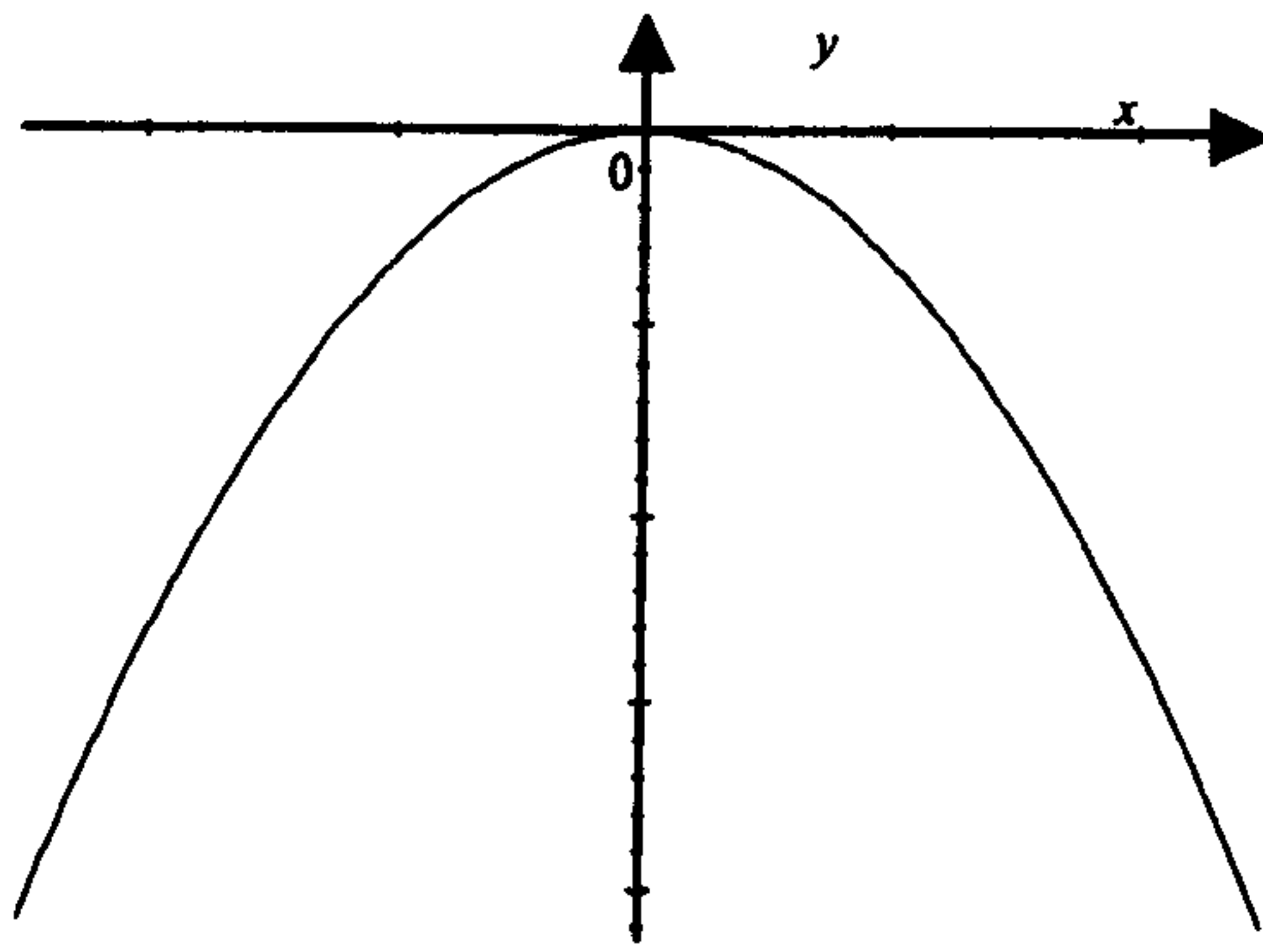
(e) $g(z) = \sqrt{4 - z^2}$

(f) $g(z) = \frac{1}{\sqrt{4 - z^2}}$

2. Sketch the graphs of each function in 1.**In exercises 3 – 6, some graphs are given. (a) Match the graphs to the given functions,****(b) which of these are functions?**

(a) $x^2 = 2y$ (b) $x^2 = -6y$ (c) $y^2 = 8x$ (d) $y^2 = -4x$





7. For each graph in Exercises 3-6 , find the domain and range.
8. State two ways of representing the function $y = 1 + x^2$

SECTION B

Answer all questions in the space provided.

Question	Answer
10. Solve $2^{2x+5} = 4^{-x}$	
11. Factorise (a) $-36 + 49r^2$ (b) $2x^2 + 5x - 3$	
12. Given the graph of $y = x^2 + bx + c$ as in the graph below. Find values of b and c .	
13. If $x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$. State c in terms of a, b and x .	

14. Given a line PQ parallel to the line $2y - x = 5$ and passes through the midpoint of AB where A (4, 5) and B (8, 11). Find the equation of PQ.	
15. What is a function?	
16. Give two examples of polynomials	
17. Give two examples of the exponential function with two different bases.	
18. Give two examples of the logarithmic function with two different bases.	
19. Give two examples of the rational functions	
20. Give two different examples of the trigonometric functions	
21. Give two examples of Hyperbolic Functions	
22. Give two examples of Inverse Trigonometric Functions	
23. Give two examples of Inverse Hyperbolic functions.	
24. Give examples of a piece-by-piece function	

Appendix C EVALUATION QUESTIONNAIRE

Course title :
Instructor :
Semester :

A. For each of the five areas below, answer the question by placing a check in the appropriate column

In this course, to what extent was the instructor	Very	Somewhat	Not at all
1. Knowledgeable about the subject?			
2. Able to communicate well			
3. Organised?			
4. Courteous and respectful to students?			
5. Successful in bringing a variety of voices and perspectives into a course?			

6. What would you most like to say about the instructor’s effectiveness as a teacher?

B. Please describe the way in which this course affected your learning in each of the following areas.

- 7. Knowledge of the content of the discipline.
- 8. Mathematical thinking
- 9. Critical thinking
- 10. Writing
- 11. Speaking
- 12. Teamwork
- 13. Other

C. Please complete the following statements.

Appendix C EVALUATION QUESTIONNAIRE

14. My learning in this class was helped most by

15. My learning in this class was hindered most by

16. The aspect of this class that most helped me to take responsibility for my own learning was

17. The factor that was most important in preventing me from taking responsibility for my own learning was

18. As a result of this class, I now understand the area of my development as a learner that I most need to work on is

D. Please complete the following statements by checking one of the alternatives and briefly answering each related question.

19. In this course, I found that

- a. _____ Many different teaching approaches were used.
- b. _____ Some different teaching approaches were used.
- c. _____ Very few different teaching approaches were used.

What are your feelings about the teaching approaches used?

20. In this course, I found that the instructor was responsive to students' concerns

- a. _____ Always
- b. _____ Sometimes.
- c. _____ Rarely.

What are your feelings about this level of responsiveness?

21. In this course, I found that the teacher was successful in bringing about students participation

- a. _____ Consistently.
- b. _____ Sometimes.
- c. _____ Rarely.

What are your feelings about the amount of participation by students in this course?

22. In this course, I found that I received information about my learning

- a. _____ Regularly.
- b. _____ Occasionally.

Appendix C EVALUATION QUESTIONNAIRE

c. _____ Rarely.

What are your feelings about the frequency with which you received information about your learning and the quality of that information?

23. In this course, I found that the democratic habits of equity, inclusion, and negotiation was practised

- a. _____ Regularly.
- b. _____ Occasionally.
- c. _____ Infrequently.

What are your feelings about the level or the lack of democracy in this class?

E. Please complete the following two statements.

24. Overall, the moments in this course when I was most engaged, excited and involved as a learner were when _____

25. Overall, the moments in this course when I was most distanced, disengaged, and uninvolved as a learner were when _____

Appendix C EVALUATION QUESTIONNAIRE

F. Please answer the following questions.

26. What would you like most to say about your experience as a student in this course?

27. What piece of advice would you most like to give the instructor on how to teach the course in the future.

28. If there is anything else you would like to say about the experience of being a student in this class that you have not already said in response to previous items, please write it below.